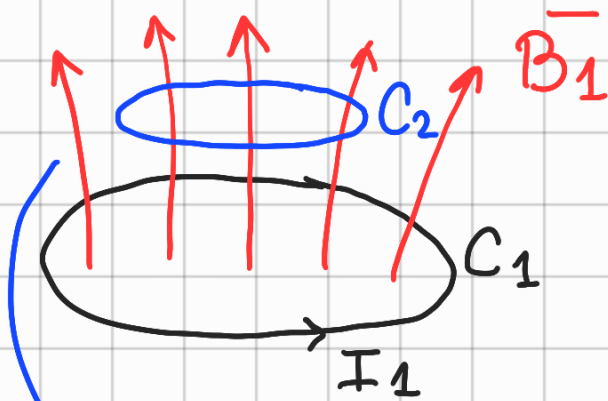


# Inductancia



$$\Phi_2 = \int \vec{B}_1 \cdot d\vec{S}_2 \quad / \quad \vec{R} = \vec{r} - \vec{r}'$$

$$\bullet \quad \vec{B}_1 = \frac{\mu_0}{4\pi} \cdot I_1 \oint \frac{d\vec{l}_1 \times \vec{R}}{R^3}$$

$$\rightsquigarrow B_1 \propto I_1 \Rightarrow \Phi_2 \propto I_1$$

$$\Phi_2 = M_{21} \cdot I_1$$

$M_{21}$ : depende de la geometría  
↓ Inductancia Mutua.

$$\vec{B}_1 = \vec{\nabla} \times \vec{A}_1 \quad , \quad \vec{A}_1 = \frac{\mu_0}{4\pi} \cdot I_1 \oint_{C_1} \frac{d\vec{l}_1}{R}$$

Reemplazando...

$$\Phi_2 = \int \underbrace{\bar{B}_1}_{\bar{\nabla} \times \bar{A}_1} \cdot d\bar{S}_2 = \frac{\mu_0}{4\pi} I_1 \oint \oint \frac{d\bar{l}_1 d\bar{l}_2}{R}$$

↓  
Stokes

$$\int \bar{\nabla} \times \bar{A} \cdot d\bar{S} = \oint \bar{A} \cdot d\bar{l}$$

$$M_{21} = \frac{\mu_0}{4\pi} \oint \oint \frac{d\bar{l}_1 d\bar{l}_2}{R} = M_{12}$$

Formula de Neumann

$$\Phi_1 = M_{12} \cdot I_2 \rightarrow \text{Flujo sobre } C_1.$$

$$\bullet \quad \mathcal{E}_2 = - \frac{d\Phi_2}{dt} \quad \text{FARADAY}$$

$$\mathcal{E}_2 = -M \cdot \frac{dI_1}{dt} \rightarrow I_1 = I_1(t)$$

# Auto inductancia

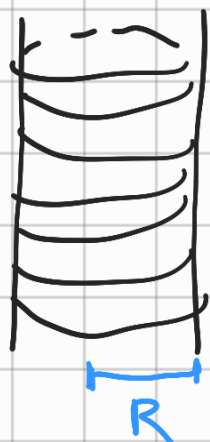
$$\Phi = L I$$

$$[L] = \text{Henry} = \frac{V \cdot s}{A}$$

$$\Rightarrow \mathcal{E} = - L \cdot \frac{dI}{dt}$$

## Ejercicio 8

a)



$$n = N/\ell$$



$$B_z = \begin{cases} \mu_0 \cdot n \cdot I & r < R \\ 0 & r > R \end{cases}$$

coord.  
cilíndricas

$$\Phi_i = \int \vec{B} \cdot d\vec{S} = \mu_0 \cdot n \cdot I \cdot \overbrace{\pi \cdot R^2}^{\text{AREA}}$$

$$N = n \cdot l$$

$$\Rightarrow \Phi = \sum_i^N \Phi_i = N \cdot \mu_0 \cdot n \cdot I \cdot \pi \cdot R^2$$

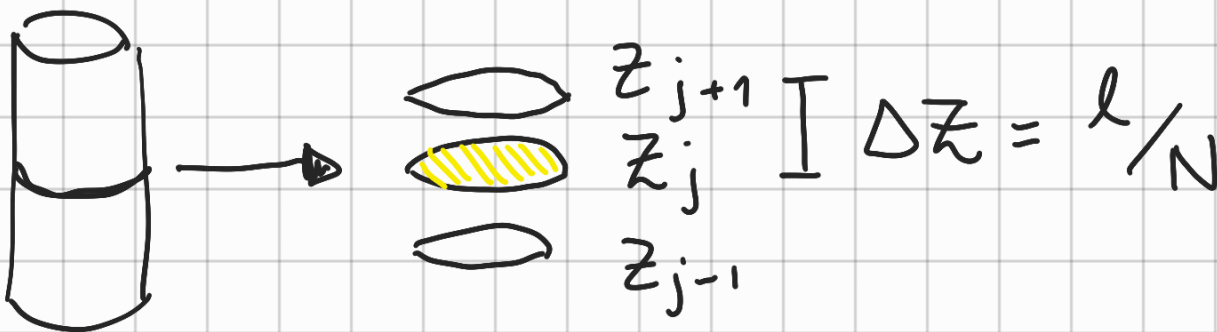
$$\Phi = \mu_0 \cdot n^2 \cdot l \cdot \pi \cdot R^2 \cdot I$$

$\parallel$   
 $L$ : autoinductancia

$$\frac{L}{l} = \mu_0 n^2 \pi \cdot R^2 \rightarrow \text{Autoinductancia por unidad de long. (finita)}$$

c)  $l, R$  y  $R \ll l$  con  $N$  vueltas

$$B_z = \frac{\mu_0 N \cdot I}{2l} \left\{ \frac{(z + l/2)}{\sqrt{R^2 + (z + l/2)^2}} - \frac{(z - l/2)}{\sqrt{R^2 + (z - l/2)^2}} \right\}$$



$$\Phi_j = \pi \cdot R^2 \cdot B_z$$

$$\Phi = \sum_{j=1}^N \Phi_j \cdot \frac{\Delta z}{\Delta z} = \frac{1}{\Delta z} \left[ \sum_{j=1}^N \Phi_j \Delta z \right]$$

\* Suma de Riemann

$$N/l = n$$

\* Si  $\Delta z \rightarrow 0$ , es una integral

$$\int_{-l/2}^{+l/2} \pi \cdot R^2 \cdot \mu_0 I \cdot \frac{N}{2l} \left\{ \frac{(z + l/2)}{\sqrt{R^2 + (z + l/2)^2}} - \frac{(z - l/2)}{\sqrt{R^2 + (z - l/2)^2}} \right\} dz$$

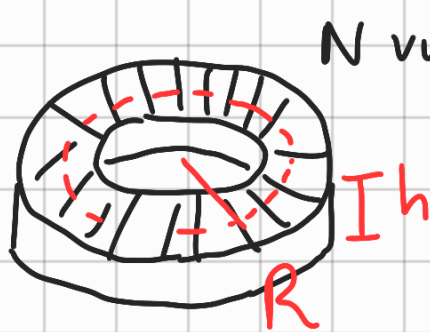
$$\int \frac{(z \pm \alpha)}{\sqrt{R^2 + (z \pm \alpha)^2}} dz = \sqrt{R^2 + (z \pm \alpha)^2} + C$$

$$\Phi = n^2 \pi R^2 \mu_0 I (\sqrt{R^2 + l^2} - R) \quad \text{--- } l = L$$

$$\Rightarrow \Phi = L \cdot I$$

$\frac{L}{l}$  del solenoide  $\propto$

b)



$N$  vueltas

$$\vec{B} = \frac{\mu_0 I \cdot N}{2\pi \cdot r} \hat{\varphi}$$

altura del  
toroide

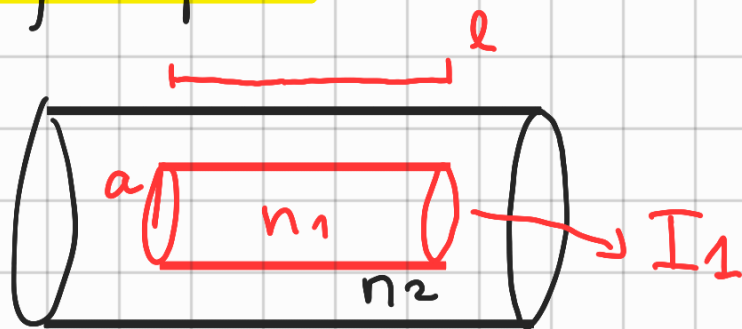
$$\Phi_1 = \int B \cdot \hat{\varphi} \cdot \hat{\varphi} \, dr \, dz = \frac{\mu_0 I N}{2\pi} \cdot h \int_a^b \frac{1}{r} \, dr$$



$$\Phi_1 = \frac{\mu_0 I N}{2\pi} \cdot h \cdot \ln(b/a)$$

$$L = \frac{\mu_0 \cdot N \cdot h \cdot \ln(b/a)}{2\pi}$$

## Ejemplo



Puedo suponer que circula  $I_2$  en el solenoide 2

$M$ : inductancia mutua

$$1) \Phi_2 = M \cdot B_1 = M \cdot \mu_0 n_1 I_1$$

$$2) \Phi_1 = M \cdot B_2 = M \mu_0 n_2 \cdot I_2$$

$$B_2 = \mu_0 \cdot n_2 \cdot I_2$$

$B_2$  a travésan.  
do sólo 1 esp.  
del Sol. 1.

$$\Phi_1 = \text{Area}_1 \cdot B_2 = a^2 \pi \cdot \mu_0 n_2 \cdot I_2$$

$$\Rightarrow M = a^2 \pi \cdot \mu_0 n_2 \cdot n_1$$

# Energía en un Campo Magnético

$$\mathcal{E} = -L \frac{dI}{dt} \quad \leftarrow$$

Trabajo realizado x unidad de carga para recorrer todo el circuito es  $-\mathcal{E}$

$$dW = -\mathcal{E} \cdot dq = -\mathcal{E} \cdot I \cdot dt = L I \cdot dI$$

$\downarrow$   
 $I = \frac{dq}{dt}$

$$\Rightarrow W = \int dW = L \int I dI = \frac{1}{2} L \cdot I^2$$

$$W = \frac{1}{2} \cdot L \cdot I^2$$

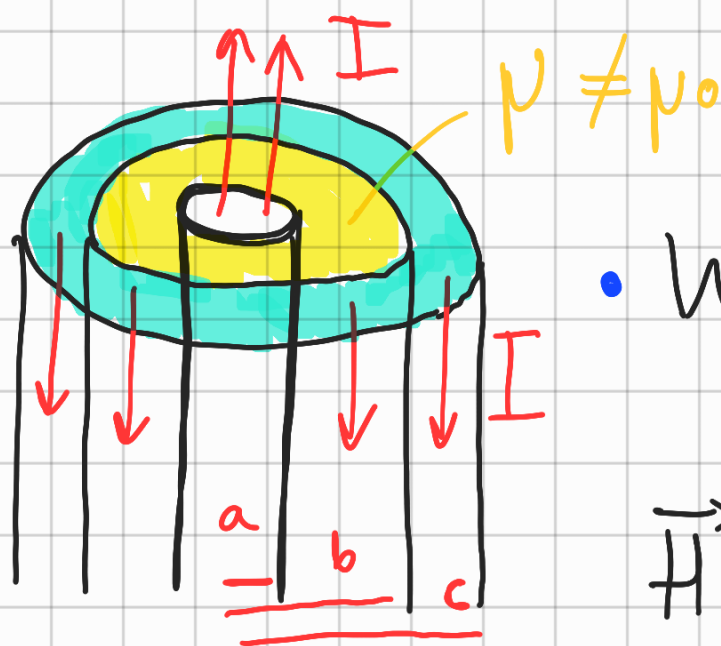
$$\Phi = \int \vec{B} \cdot d\vec{S}, \quad \vec{B} = \nabla \times \vec{A}$$

$$W = \frac{1}{2} \oint (\vec{A} \cdot \vec{I}) d\ell \xrightarrow{\vec{I} \cdot d\vec{\ell} \rightarrow \vec{J} dV} \frac{1}{2} \int_V \vec{A} \cdot \vec{J} dV$$

$$\Rightarrow W_M = \frac{1}{2} \int_{\text{todo el esp.}} \frac{|\vec{B}|^2}{\mu_0} dV$$

$$W_E = \frac{1}{2} \epsilon_0 \int |\vec{E}|^2 dV$$

## Ejercicio 9



$$\bullet W = \frac{1}{2} \int \vec{H} \cdot \vec{B} dV$$

$$\vec{H} = \frac{1}{\mu_0} \vec{B} \quad \text{material lineal}$$

$$\bullet W = \frac{1}{2} L \cdot I^2$$

¿Cómo es  $\vec{B}$ ?  $\vec{B} = B(r) \hat{\varphi}$  (Lo mismo  $\vec{H}$ )

$$\vec{B}_\varphi = \begin{cases} \frac{\mu_0 \cdot I}{2\pi \cdot a^2} r & r < a \text{ (1)} \\ \frac{\mu \cdot I}{2\pi \cdot r} & a < r < b \text{ (2)} \\ \frac{\mu_0 \cdot I}{2\pi \cdot r} \left( 1 - \frac{c^2 - r^2}{c^2 - b^2} \right) & b < r < c \text{ (3)} \\ 0 & c > r \end{cases}$$

$$W = \frac{1}{2} \int \vec{H} \cdot \vec{B} dV$$

$$\text{(1)} = \frac{1}{2} \int_0^l \int_0^a \int_0^{2\pi} \frac{I^2 \mu_0}{(2\pi)^2 a^4} \cdot r^2 \cdot r d\varphi dr dz$$

$$= \frac{I^2 \cdot 2\pi \cdot l \mu_0}{(2\pi)^2 \cdot a^4} \cdot \frac{a^4}{4} \cdot \frac{1}{2} = \frac{I^2 \cdot l}{16\pi}$$

$$W_1 = \frac{\mu_0 I^2 l}{16\pi}$$

$$(2) = \frac{1}{2} \int_0^l \int_a^b \int_0^{2\pi} \frac{\mu^2}{\mu_0} \cdot \frac{I^2}{(2\pi^2)} \cdot \frac{1}{r^2} \cdot r \, d\varphi \, dr \, dz$$

$$= \frac{\mu^2}{\mu_0} \cdot \frac{I^2 \cdot 2\pi \cdot l}{(2\pi)^2 \cdot 2} \ln(b/a)$$

$$= \frac{\mu^2}{\mu_0} \cdot \frac{I^2 \cdot \ln(b/a) \cdot l}{4\pi} = W_2$$

$$(3) = \frac{1}{2} \frac{\mu_0 \cdot I^2 \cdot l}{(2\pi)} \int_b^c \frac{1}{r} \left( 1 - \frac{c^2 - r^2}{c^2 - b^2} \right) r \cdot dr$$

$$= \frac{1}{2} \mu_0 \frac{I^2 l}{2\pi} \left[ (c-b) - \int_b^c \frac{c^2 - r^2}{c^2 - b^2} dr \right]$$

$$= \frac{1}{2} N_0 \frac{I^2 l}{2\pi} \left[ (c-b) - \left[ \frac{c^2}{c^2-b^2} \right] (c-b) + \right. \\ \left. \frac{c^3}{3(c^2-b^2)} - \frac{b^3}{3(c^2-b^2)} \right]$$

$$L = \frac{(W_1 + W_2 + W_3) \cdot 2}{I^2}$$