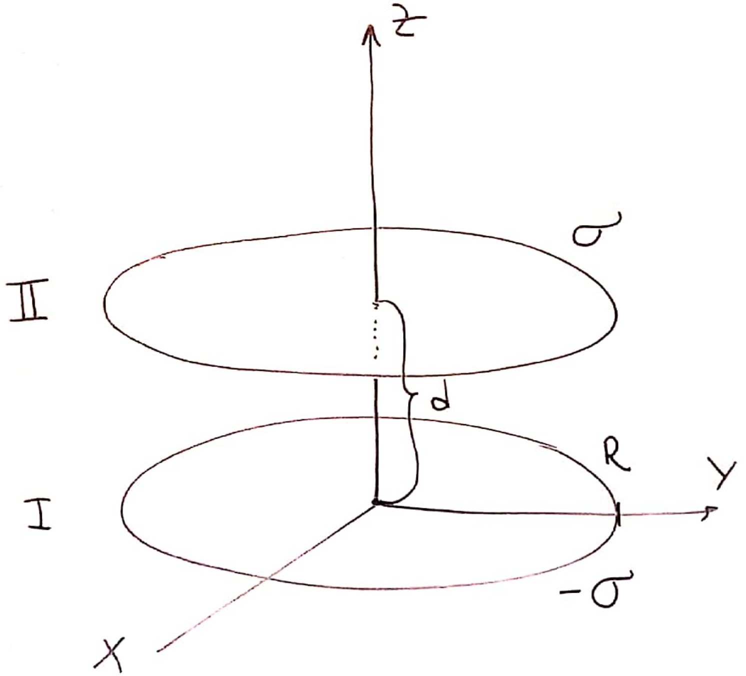
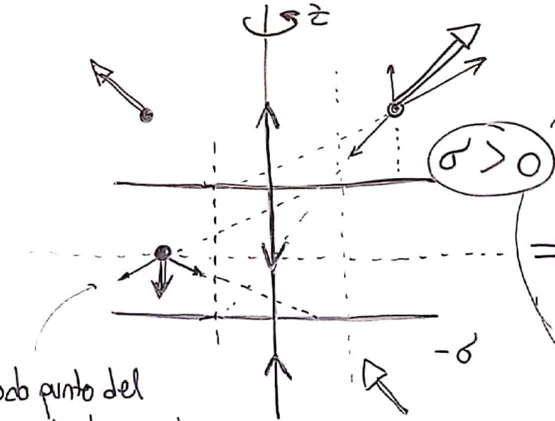
Problema 18



a) Líneas de campo en todo el espacio.

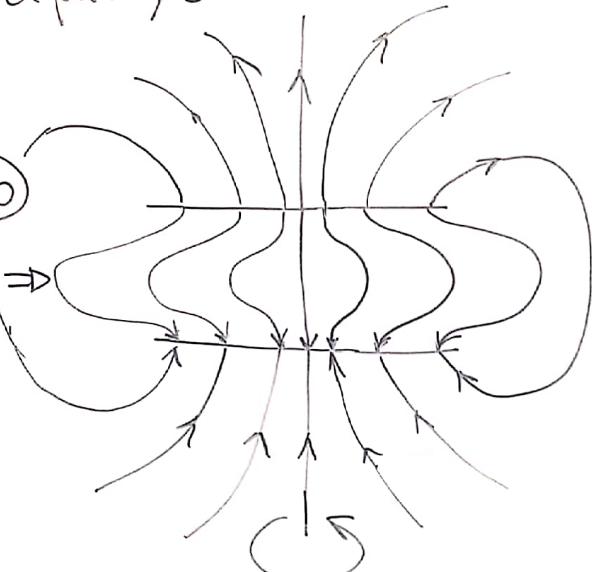
La configuración es simétrica frente a rotaciones

⇒ Alcanza con conocer las líneas en el plano yz:



En todo punto del eje apunta hacia abajo.

⇒



Sim. de revolución.

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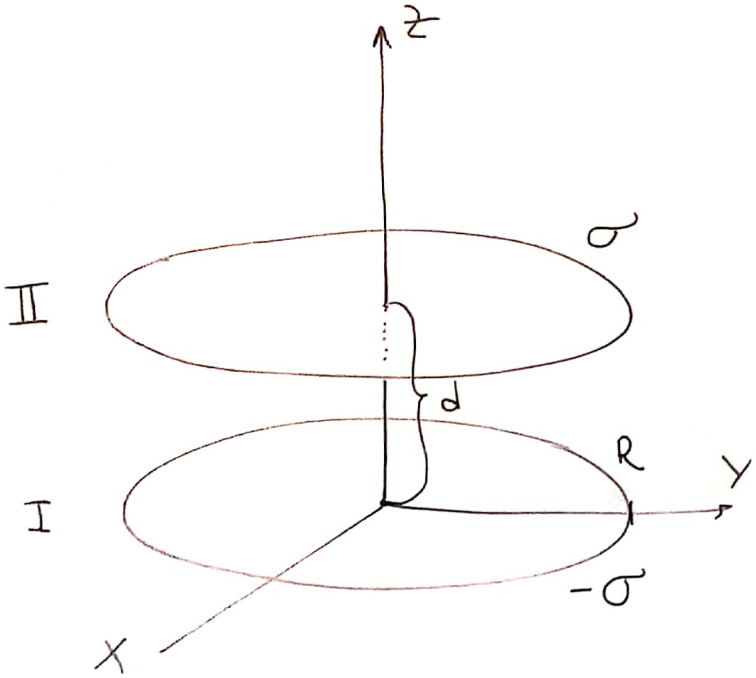
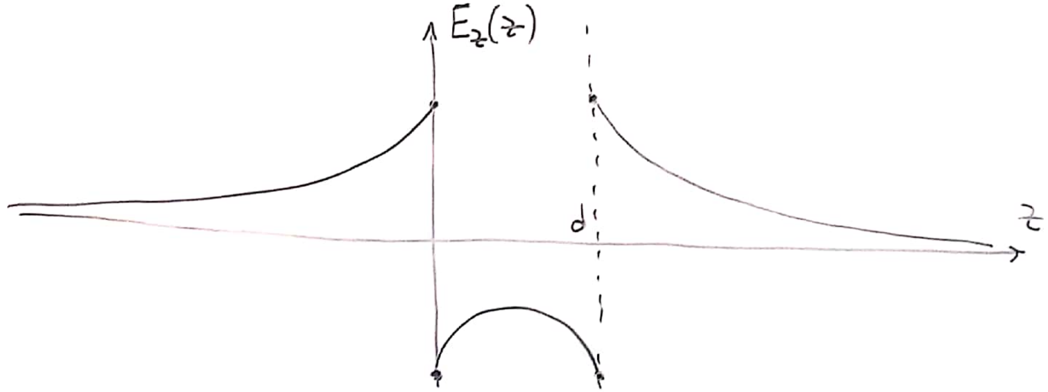


b) Calcular $V(z)$ y $\vec{E}(z)$ y graficar. Calcular $\vec{p}$.

Uso resultado del ejercicio 6 ⊕ superposición:

$$\vec{E}(z) = 2\pi k\sigma \left[(z-d)\left(\frac{1}{|z-d|} - \frac{1}{\sqrt{R^2+(z-d)^2}} \right) \quad (\text{II}) \right.$$

$$\left. - z\left(\frac{1}{|z|} - \frac{1}{\sqrt{R^2+z^2}} \right) \right] \hat{z} \quad (\text{I})$$



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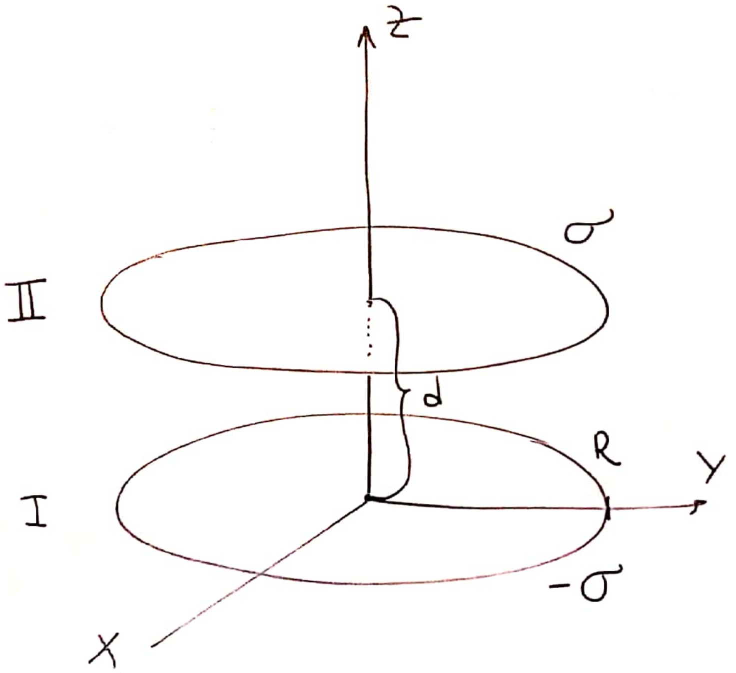
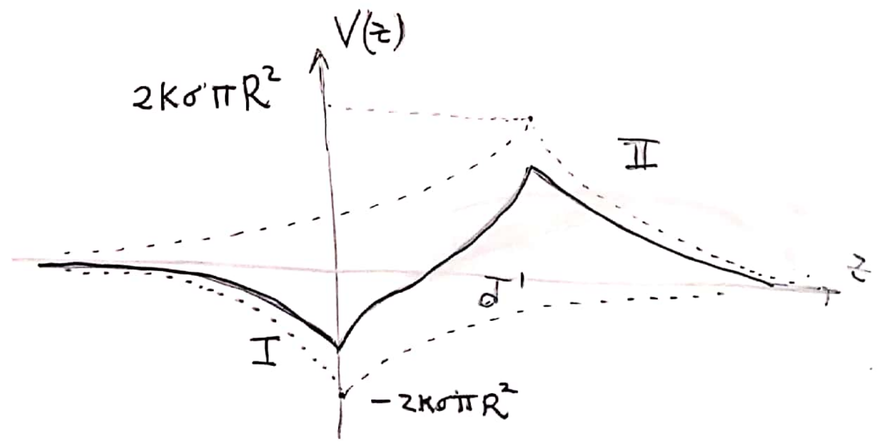


b) Calcular $V(z)$ y $\vec{E}(z)$ y graficar. Calcular $\vec{p}$.

Uso resultado del ejercicio 6 ⊕ superposición:

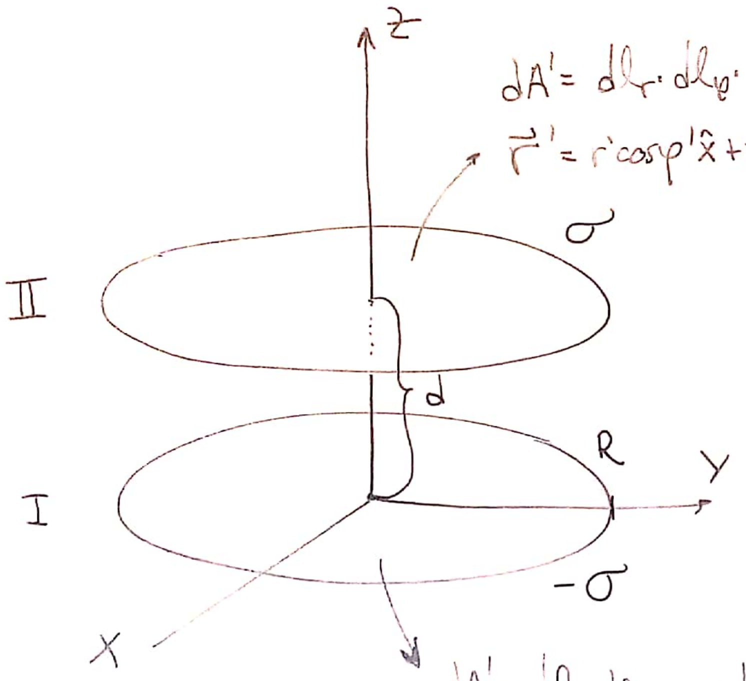
$$V(z) = 2\pi k\sigma \left\{ \sqrt{R^2 + (z-d)^2} - |z-d| \right. \quad (II)$$

$$\left. - \sqrt{R^2 + z^2} + |z| \right\} \quad (I)$$



Problema 18

b) Calcular $V(z)$ y $\vec{E}(z)$ y graficar. Calcular $\vec{p}$.



$dA' = dl_{r'} \, dl_{\varphi'} = r' dr' d\varphi'$

$\vec{r}\,' = r'\cos\varphi' \hat{x} + r'\operatorname{sen}\varphi' \hat{y} + d\hat{z}$

$dA' = dl_{r'} \, dl_{\varphi'} = r' dr' d\varphi'$

$\vec{r}\,' = r'\cos\varphi' \hat{x} + r'\operatorname{sen}\varphi' \hat{y}$

$$Q = Q_{I} + Q_{II} = -\sigma \pi R^2 + \sigma \pi R^2 = 0$$

$\Rightarrow \vec{p}$ no depende del sistema de referencia.

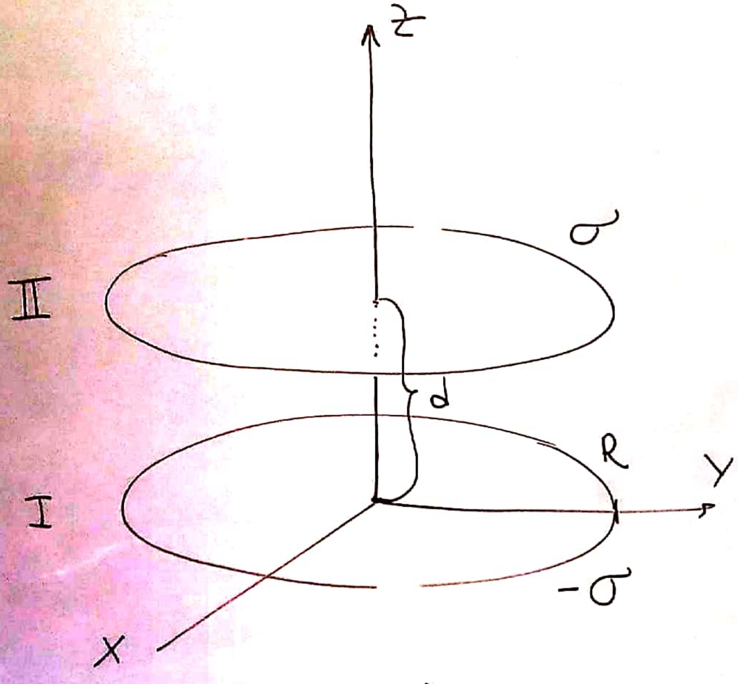
$$\vec{p} = \int dA' \sigma(\vec{r}\,') \vec{r}\,' = \int_0^{2\pi}\int_0^R r'dr'd\varphi' (-\sigma)\left[r'\cos\varphi' \hat{x} + r'\operatorname{sen}\varphi' \hat{y}\right] \quad (I)$$

$$+ \int_0^{2\pi}\int_0^R r'dr'd\varphi' \, \sigma \left[r'\cos\varphi' \hat{x} + r'\operatorname{sen}\varphi' \hat{y} + d\hat{z}\right]$$

$$\Rightarrow \boxed{\vec{p} = \sigma \pi R^2 d \hat{z} = p \hat{z}} \qquad \boxed{p = Q_{II} \cdot d}$$

Problema 18

b) Calcular $V(z)$ y $\vec{E}(z)$ y graficar. Calcular $\vec{p}$.



V en $\left[\begin{array}{l} z \gg d \\ z \gg R \end{array}\right]$ vía Taylor de la solución exacta:

$$\boxed{\begin{array}{l} \varepsilon_R \equiv \frac{R}{z} \ll 1 \\ \varepsilon_d \equiv \frac{d}{z} \ll 1 \end{array}}$$

$$V(z>d) = 2\pi k\sigma \left[\sqrt{R^2+(z-d)^2} - \sqrt{R^2+z^2}\right] \cancel{+cte}$$

$$= 2\pi k\sigma z \left[\sqrt{\underbrace{\varepsilon_R^2+(1-\varepsilon_d)^2}_{1+(-2\varepsilon_d+\varepsilon_d^2+\varepsilon_R^2)}} - \sqrt{\varepsilon_R^2+1}\right]$$

$$= 2\pi k\sigma z \left[1 + \frac{1}{2}(\underbrace{-2\varepsilon_d}_{cte}+\varepsilon_d^2+\varepsilon_R^2) - \frac{1}{8}(-2\varepsilon_d+\varepsilon_d^2+\varepsilon_R^2)^2 + \frac{1}{16}(-2\varepsilon_d+\dots)^3 - \left[1+\frac{1}{2}\varepsilon_R^2\right] + \dots\right]$$

$$= 2\pi k\sigma z \left[-\frac{1}{8}\, 2\, (-2\varepsilon_d)(\varepsilon_d^2+\varepsilon_R^2) + \frac{1}{16}(-8)\varepsilon_d^3 + \mathcal{O}(\varepsilon_i^4)\right]$$

$$= 2\pi k\sigma z\, \frac{1}{2}\varepsilon_d \varepsilon_R^2 = \boxed{\frac{\pi k\sigma d R^2}{z^2} = V_{lejos}(z)} \checkmark$$

$$\left.\begin{array}{l} \left(\sqrt{1+\varepsilon}\right)' = \frac{1}{2\sqrt{1+\varepsilon}} \to \frac{1}{2} \\ \left(\frac{1}{2}(1+\varepsilon)^{-1/2}\right)' = -\frac{1}{4}(1+\varepsilon)^{-3/2} \to -\frac{1}{4} \\ \left(-\frac{1}{4}(1+\varepsilon)^{-3/2}\right)' = \frac{3}{8}(1+\varepsilon)^{-5/2} \to \frac{3}{8} \end{array}\right\} \sqrt{1+\varepsilon} \approx 1 + \frac{1}{2}\varepsilon - \frac{1}{8}\varepsilon^2 + \frac{1}{16}\varepsilon^3$$

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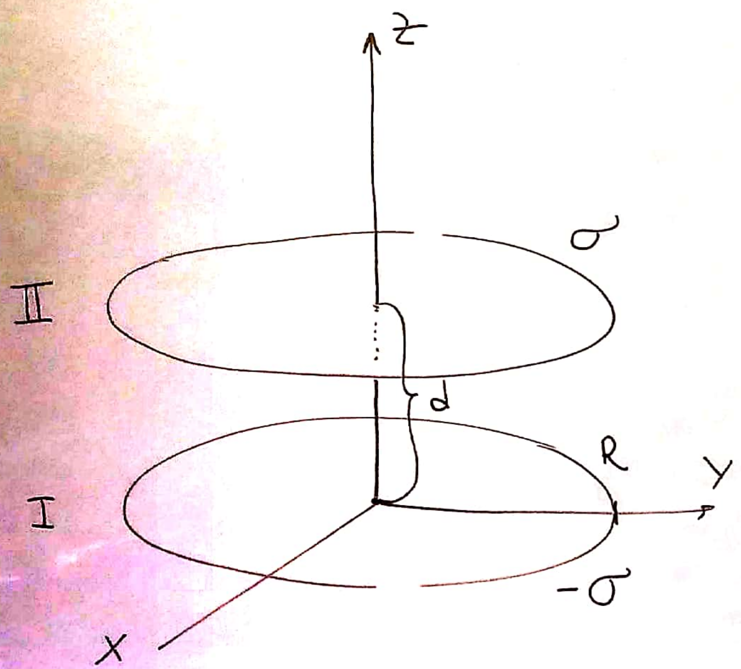

Potencial lejos vía desarrollo multipolar:

$V(\vec{r})$ lejos en todo el espacio: Uso cilíndricas.

$$\vec{r} = r\hat{r} + z\hat{z} \quad , \quad \vec{p} = \pi R^2 \sigma d \hat{z}$$

$$V(\vec{r}) = K \frac{\vec{r}\cdot\vec{p}}{|\vec{r}|^3} = K\pi R^2 \sigma d \frac{z}{[r^2+z^2]^{3/2}} \xrightarrow[z \text{ lejos}]{} \boxed{\frac{K\pi R^2 \sigma d}{z^2} = V_{lejos}(z)}$$

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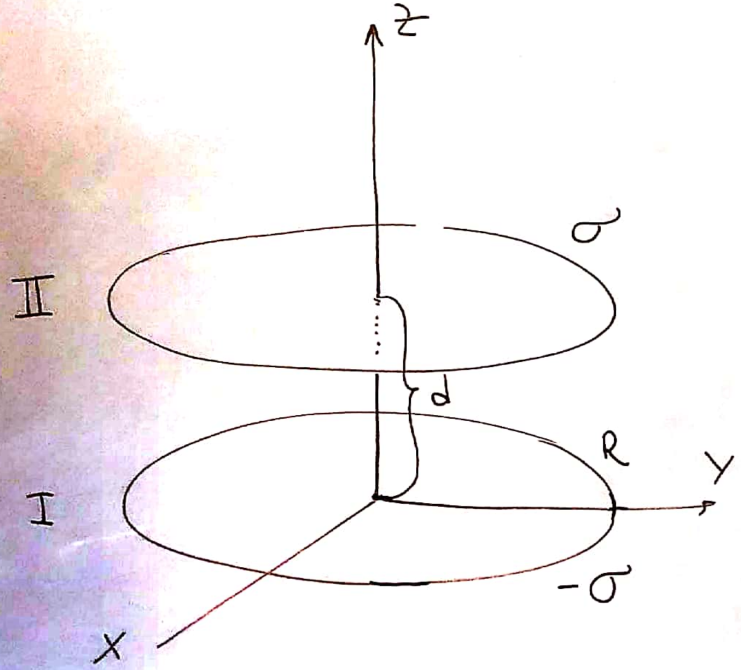
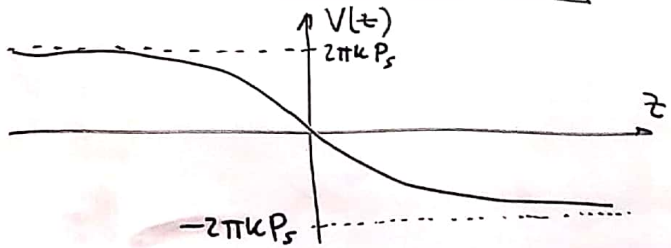


c) Distribución superficial de momento dipolar $\begin{pmatrix} d \to 0, \sigma \to \infty \\ d\sigma = P_s = cte \end{pmatrix}$

$$V(z) = 2\pi k \sigma \left[\sqrt{R^2+z^2-2zd} - \sqrt{R^2+z^2} - |z-d| + |z| \right]$$

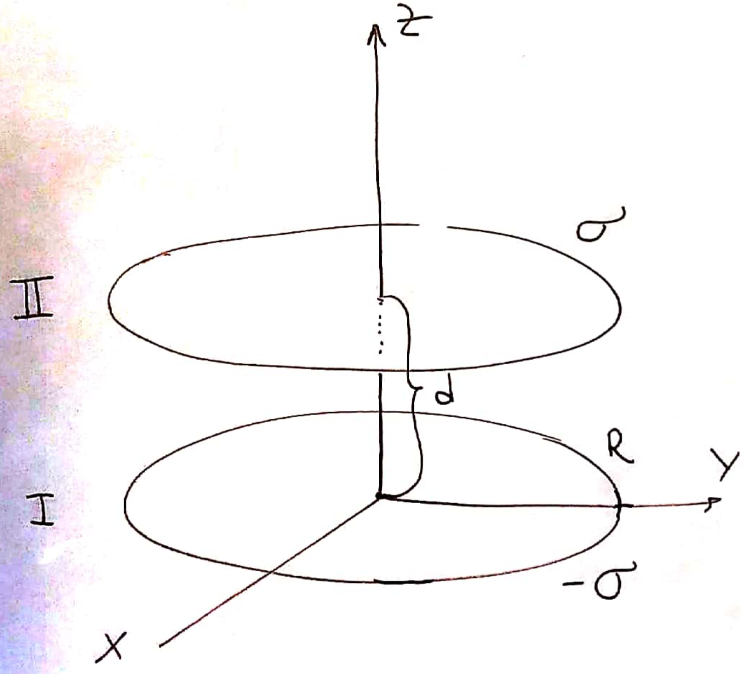
$$= 2\pi k \sigma \left[\sqrt{R^2+z^2} \left(\sqrt{1 - \underbrace{\frac{2zd}{R^2+z^2}}_{\varepsilon}} - 1 \right) - |z-d| + |z| \right]$$

$$\approx 2\pi k \underbrace{\sigma d}_{P_s} \left[\frac{-z}{\sqrt{R^2+z^2}} \underbrace{\pm}_{z \gtrless d} \overbrace{1}^{\cancel{cte}} \right] \Rightarrow \boxed{V(z) = -2\pi K P_s \frac{z}{\sqrt{R^2+z^2}}}$$



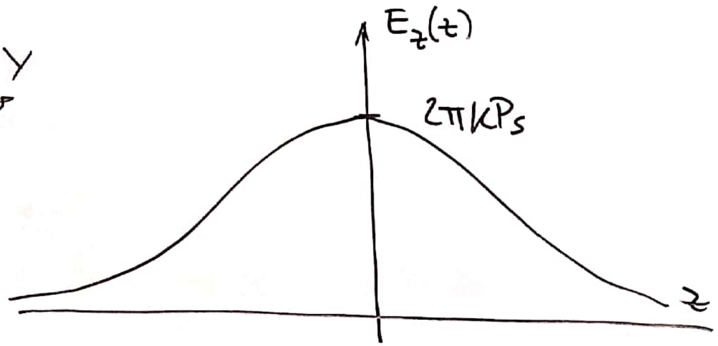
Problema 18

c) Distribución superficial de momento dipolar $\left(\begin{array}{l} d \to 0,\ \sigma \to \infty \\ d\sigma = P_s = \text{cte} \end{array} \right)$



$$\bar{E}(z) = -\frac{\partial V}{\partial z}\hat{z} = 2\pi k P_s \frac{R^2}{(R^2+z^2)^{3/2}}\hat{z}$$

se puede x argumento de simetría



$$\boxed{V(z) = -2\pi K P_s \frac{z}{\sqrt{R^2+z^2}}}$$