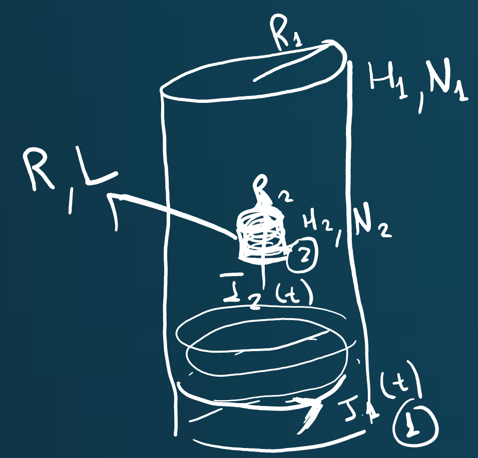
Problema 7 - Guía 5



$$I_1(t) = \alpha t + \beta \quad \begin{cases} I_1(0) = 5\,A \\ I_1(0.5\,s) = 1\,A \end{cases}$$

$$I_1(t) = 5A - 8\frac{A}{s}t$$

$$I_1(t<0) = 5A$$

- $$\boxed{\varepsilon_{em}} = -\frac{d\Phi}{dt} = -\frac{d\Phi_{21}}{dt} - \frac{d\Phi_{22}}{dt} = \boxed{-M_{21}\underbrace{\frac{dI_1}{dt}}_{\alpha} - L\frac{dI_2}{dt}}$$

$$M_{21} = \frac{\Phi_{21}}{I_1}, \quad L = M_{22} = \frac{\Phi_{22}}{I_2}$$

- $$\varepsilon_{em} = I_2 . R$$

$$\Rightarrow \boxed{\frac{dI_2}{dt} + \frac{R}{L}I_2 = -\frac{M_{21}\alpha}{L}}$$

Ec. dif. 1er orden no homogénea.

$$I_2 = I_2^H + I_2^P$$

H:

$$\frac{dI_2^H}{dt} + \frac{R}{L}I_2^H = 0$$

$$\int_{I_2^H(0)}^{I_2^H(t)} \frac{dI_2^H}{I_2^H} = -\int_0^t \frac{R}{L}\,dt'$$

$$\ln\left(\frac{I_2^H(t)}{I_2^H(0)}\right) = -\frac{R}{L}t$$

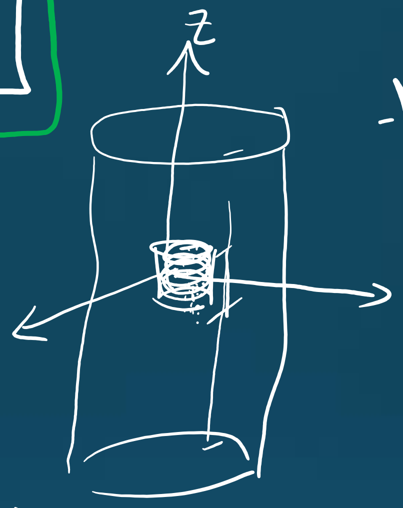
$$\Rightarrow I_2^H(t) = I_2^H(0)\,e^{-\frac{R}{L}t}$$

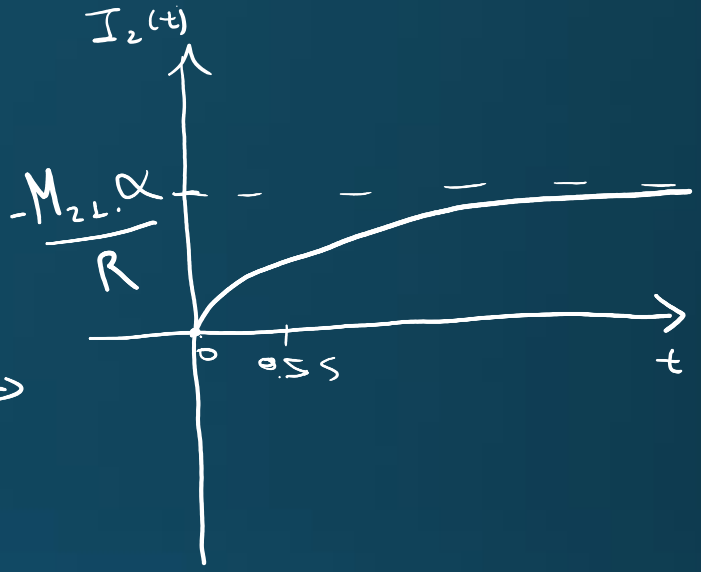
$$I_2^P = -\frac{M_{21}\alpha}{R}$$

$$I_2(t) = I_2^H(0)\,e^{-\frac{R}{L}t} - \frac{M_{21}\alpha}{R}$$

Pidiendo $I_2(0) = 0 \Rightarrow I_2^H(0) = \frac{M_{21}\alpha}{R}$

$$\boxed{I_2(t) = \frac{M_{21}\cdot\alpha}{R}\left[e^{-\frac{R}{L}t} - 1\right]}$$





• $$M_{21} \doteq \frac{\Phi_{21}}{I_1} \cong \frac{N_2 \cdot \Phi_{esp}}{I_1} = \frac{N_2 \iint_{espira} \bar{B}(z)\cdot d\bar{s}}{I_1} \qquad d\bar{s} = ds\,\hat{z}$$

$$= \frac{N_2\, \mu_0\, \cancel{I_1}\, N_1}{\cancel{I_1}\, 2\sqrt{R_1^2 + \frac{H_1^2}{4}}}\, \pi R_2^2$$

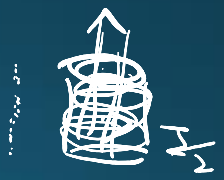
$$\bar{B}(z\hat{z}) = \frac{\mu_0 I_1 N_1}{2\cdot H_1}\left\{\frac{z+\frac{H_1}{2}}{\sqrt{R_1^2+\left(z+\frac{H_1}{2}\right)^2}} - \frac{z-\frac{H_1}{2}}{\sqrt{R_1^2+\left(z-\frac{H_1}{2}\right)^2}}\right\}\hat{z}$$

Guía 4 : prob. 9

$$\boxed{\bar{B}(0) = \frac{\mu_0 I_1 N_1}{2\cdot\sqrt{R_1^2 + \frac{H_1^2}{4}}}\,\hat{z}}$$

$$\boxed{M_{21} = \frac{\mu_0 N_1 N_2 \pi R_2^2}{2\sqrt{R_1^2 + \frac{H_1^2}{4}}}}$$

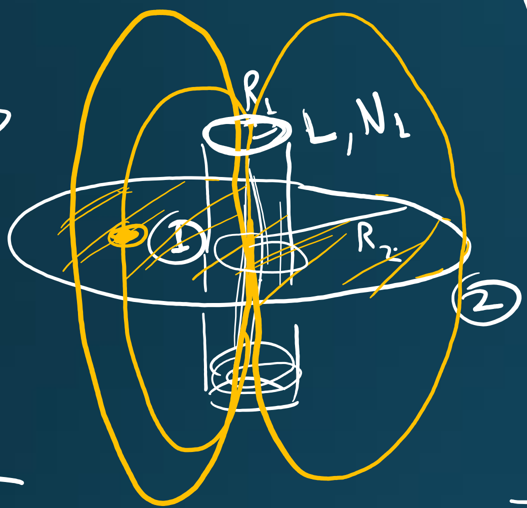
$$\overbrace{L}^{M_{22}} = \frac{\Phi_{22}}{I_2} \cong \frac{N_2 \cdot \Phi_{esp}}{I_2} = \frac{N_2 \iint \bar{B}(0)\cdot d\bar{s}}{I_2}$$



Problema L2:

¿M_{12} y M_{21}?

$$\begin{cases} + R_1 << L \\ + \underbrace{R_1 << R_2} \end{cases}$$



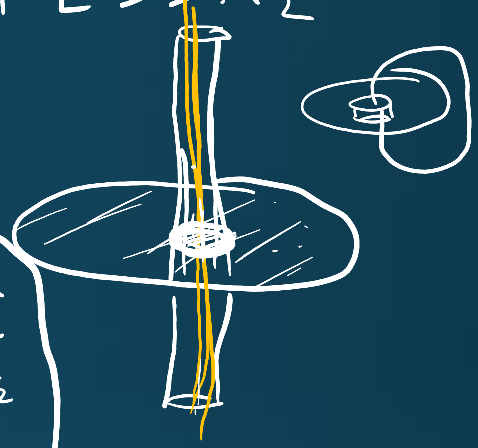
$$M_{12} = \frac{\Phi_{12}}{I_2} = \iint \bar{B} \cdot d\bar{S}$$

$$M_{21} = \frac{\Phi_{21}}{I_1}$$

Dos casos:

(1) $L << R_2$

(2) $L >> R_2$



$$\boxed{\bar{B}_{esp}(z\hat{z}) = \frac{\mu_0 I_2 R_2^2}{2(R_1^2 + z^2)^{3/2}} \hat{z}}$$

$$\int_{V(S)} dV \, \bar{\nabla} \cdot \bar{B} = 0 \Rightarrow \oint d\bar{S} \cdot \bar{B} = 0 \circledast$$

$$S = S_{sup} + S_{\delta S} + S_{INF}$$

$$\boxed{M_{21}^{(1)} = \frac{\Phi_{SS}}{I_1} + \frac{\Phi_{INF}}{I_1} \cong 0}$$

$$\circledast \underbrace{\int_{S_{sup}}}_{\sim 0} + \int_{S_{\delta S}} + \int_{S_{INF}} = 0 \Rightarrow \int_{S_{INF}} = -\int_{S_{SS}}$$

$$\Phi_{INF} = -\Phi_{\delta S}$$

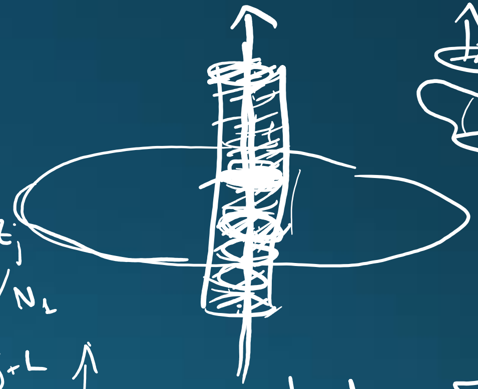
$$M_{21}^{(2)} = \frac{\Phi_{21}}{I_1} = \frac{\iint_{dS} \bar{B}_{sol}(0) \cdot d\bar{S}}{I_1}$$

$$M_{21}^{(2)} = \frac{\mu_0 N_1 I_1}{2 I_1 \sqrt{R_1^2 + \frac{L^2}{4}}} \pi R_1^2 = \frac{\mu_0 N_2 \pi R_1^2}{2\sqrt{R_1^2 + \frac{L^2}{4}}} \cong \boxed{\mu_0 \frac{N_1}{L} \pi R_1^2 = M_{21}^{(2)}}$$

$$M_{21}^{(1)} \cong \frac{\Phi_{fs}}{I_1} \cong \frac{\iint \bar{B}_{s1}(0)\cdot d\bar{S}}{I_1}$$

$$M_{21}^{(1)} \cong \frac{\mu_0 N_1 \cancel{I_1}\, \pi \cdot R_1^2}{2 \cancel{I_1} \sqrt{R_1^2 + \frac{L^2}{4}}} \cong \boxed{\frac{\mu_0 N_1 \pi R_1^2}{2L}} \cong M_{21}^{(1)}$$

$$M_{12} = \frac{\Phi_{12}}{I_2} = \frac{\sum_j \Phi_j}{I_2} \cdot \frac{\Delta z}{\frac{L}{N_1}} = \frac{N_1}{I_2 L} \int_{-\frac{L}{2}}^{\frac{L}{2}} dz\, \Phi(z)$$

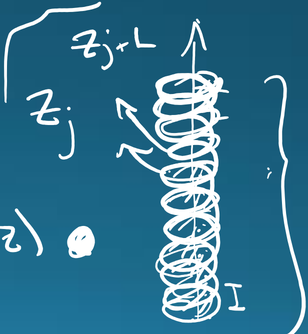


$$\Delta z = z_{j+1} - z_j = L/N_1$$

$$= \frac{N_1}{I_2 L} \int_{-\frac{L}{2}}^{\frac{L}{2}} dz \cdot \iint \bar{B}_{esp}(z)\cdot d\bar{S}$$

Espira del solenoide.

$$= \frac{N_1}{I_2 L} \int_{-\frac{L}{2}}^{\frac{L}{2}} dz\, B_{esp}(z) \cdot \iint dS = \frac{N_1 \pi \cdot R_1^2}{I_2 L} \cdot \int_{-\frac{L}{2}}^{\frac{L}{2}} dz\, B_{esp}(z)$$



$$F_{em}^{total} = \sum F_{em\ espira}.$$

$$M_{12} = \frac{N_1 \pi R_1^2}{I_2 L} \cdot \int_{-\frac{L}{2}}^{\frac{L}{2}} dz \, \frac{\mu_0 I_2 R_2^2}{2 \cdot (R_2^2 + z^2)^{\frac{3}{2}}}$$

$$= \frac{\mu_0 N_1 R_1^2 \pi R_2^2}{2L} \cdot \frac{1}{R_2^2} \cdot \frac{z}{\sqrt{R_2^2 + z^2}} \Bigg|_{-\frac{L}{2}}^{\frac{L}{2}}$$

$$= \frac{\mu_0 N_1 R_1^2 \pi}{2 \cancel{L}} \cdot \frac{\cancel{L}}{\sqrt{R_2^2 + L^2/4}}$$

$$\boxed{M_{12} = \frac{\mu_0 N_1 \pi R_1^2}{2\sqrt{R_2^2 + L^2/4}}}$$

(1) $L \ll R_2$

$$M_{21}^{(1)} \cong \frac{\mu_0 N_1 \pi R_1^2}{2 R_2}$$

(2) $L \gg R_2$

$$M_{21}^{(2)} \cong \frac{\mu_0 N_1 \pi R_1^2}{L}$$