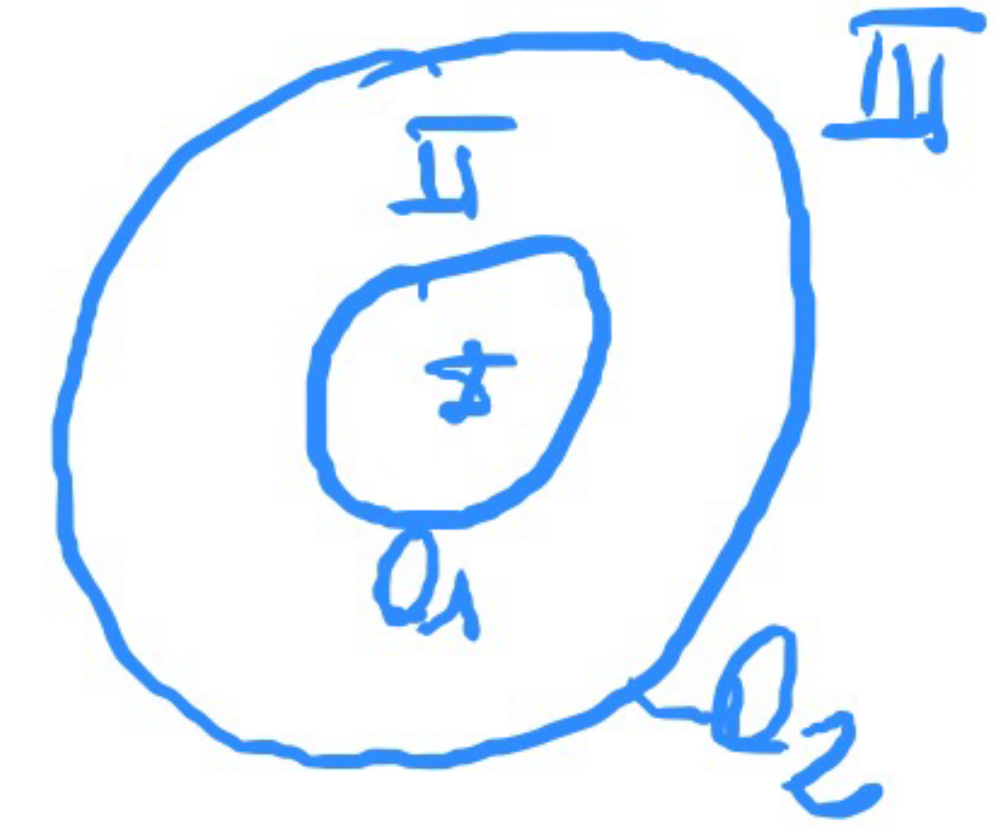
1

$$V(r) = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 R} & r < R \\[2ex] \dfrac{Q}{4\pi\varepsilon_0 r} & r > R \end{cases}$$



$$\lim_{r \to \infty} V(r) = 0$$

$$V(r=a) = 0$$

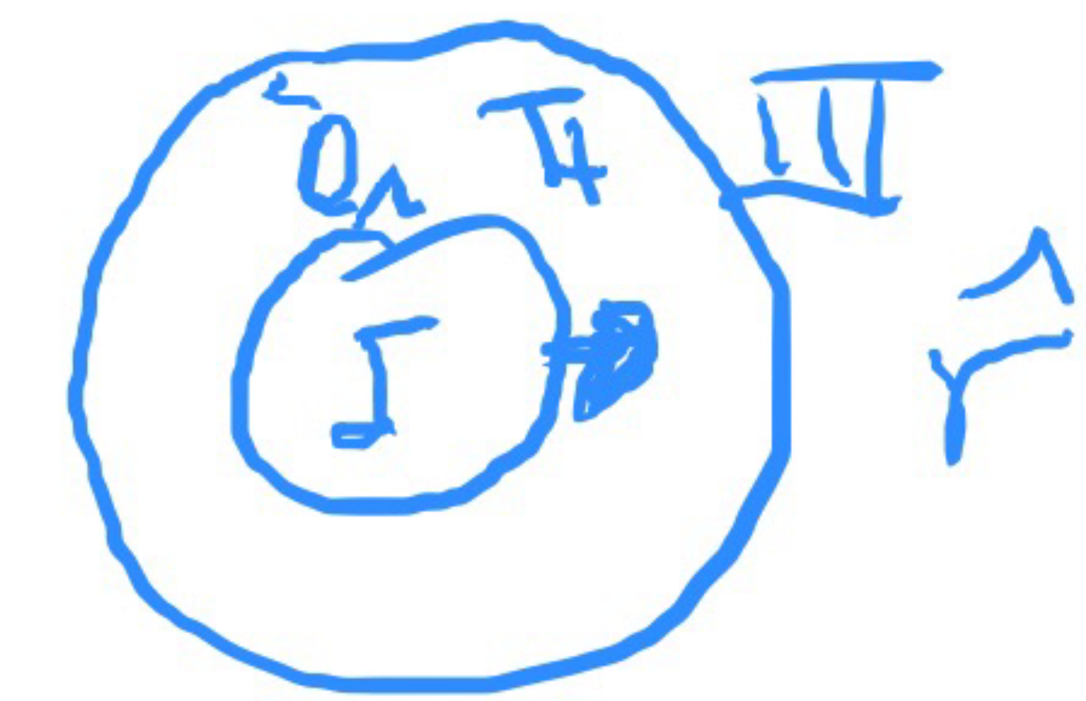
$$V(r=2a) = V_0$$

I $\quad \dfrac{Q_1}{4\pi\varepsilon_0 a} + \dfrac{Q_2}{4\pi\varepsilon_0 a 2} \qquad r < a$

II $\quad \dfrac{Q_1}{4\pi\varepsilon_0 r} + \dfrac{Q_2}{8\pi\varepsilon_0 a} \qquad a < r < 2a$

III $\quad \dfrac{Q_1}{4\pi\varepsilon_0 r} + \dfrac{Q_2}{4\pi\varepsilon_0 r} \qquad r > 2a$

$$\left.\begin{array}{lll} \mathrm{I} & 0 & r<a \\ \mathrm{II} & \dfrac{A_1}{r}+B_1 & a<r<2a \\ \mathrm{III} & \dfrac{A_2}{r}+B_2 & r>2a \end{array}\right\} = V(r)$$



$$Q_1 = -8\pi\varepsilon_0 a V_0$$

$$Q_2 = 16\pi\varepsilon_0 V_0 a$$

continuidad en $r=2a$ $\cancel{r=a}$

$$V(r=a)=0 \qquad V(r=2a)=V_0$$

ELIJO $\lim_{r\to\infty} V(r) = 0 \Longrightarrow B_2 = 0$

$\hookrightarrow A_1, A_2, B_1 \Longrightarrow$ $\vec{E} = -\vec{\nabla} V(\vec{r})$

$$(\vec{E}_2 - \vec{E}_1)\cdot\hat{n}\Big|_{r=a} = \frac{\sigma_1}{\varepsilon_0}$$

$\hat{n}$ normal I a II

$$\vec{\nabla}\times\vec{D} = \vec{\nabla}\times\vec{P} \qquad \vec{\nabla}\cdot\vec{D} = \rho_L$$

$$\vec{\nabla}\times\vec{E} = 0 \qquad \vec{\nabla}\cdot\vec{E} = \frac{\rho_T}{\varepsilon_0} = \frac{\rho_L + \rho_P}{\varepsilon_0}$$

$\underline{\underline{\alpha r \hat{r}}}$

$0 < r < 2a$

$\vec{P} = 0 \Longrightarrow \vec{\nabla}\times\vec{P} = 0$ x que no hay medios materiales

$2a < r < 3a$

medio L. I. H $\qquad \vec{P} = \varepsilon_0 \chi \vec{E} \Longrightarrow \vec{\nabla}\times\vec{P} = \varepsilon_0 \chi \vec{\nabla}\times\vec{E} = 0$

$r > 3a$

$$\vec{P} = \alpha r \hat{r} \qquad \vec{\nabla}\times\vec{P} = \frac{1}{r\,\mathrm{sen}\,\theta}\underbrace{\left[\frac{\partial(P_\varphi\,\mathrm{sen}\,\theta)}{\partial\theta} - \frac{\partial P_\theta}{\partial\varphi}\right]}_{=0}\hat{r} +$$

$$\frac{1}{r}\left(\frac{\partial P_r}{\partial\varphi}\frac{1}{\mathrm{sen}\,\theta} - \frac{\partial(P_\varphi r)}{\partial r}\right)\hat{\theta} + \frac{1}{r}\underbrace{\left[\frac{\partial(P_\theta r)}{\partial r} - \frac{\partial P_r}{\partial\theta}\right]}_{=0}\hat{\varphi}$$

$$\iint \vec{D} \cdot \vec{dS} = Q_{L\,enc} \qquad \iint \vec{E} \cdot \vec{dS} = \frac{Q_T}{\varepsilon_0}$$

$$\vec{D} = D(r)\,\hat{r}$$

$$\int_0^{2\pi}\int_0^{\pi} D(r)\,\hat{r} \cdot \hat{r}\, r^2 \sin\theta\, d\theta\, d\varphi = 4\pi r^2 D(r) = Q_{L\,enc}$$

$$Q_{L\,enc} = \begin{cases} 0 & r < a \\ Q_1 & a < r < 2a \\ Q_2 + Q_1 & r > 2a \end{cases}$$

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \longrightarrow$ siempre

$\vec{D} = \varepsilon \vec{E} \longrightarrow$ L.I.H medio

$\varepsilon_1 = \varepsilon_0 (1 + \chi_1)$

$\vec{D} = \varepsilon_0 \vec{E}$

$0 < r < 2a \longrightarrow$ vacío $\quad \vec{E} = \dfrac{\vec{D}}{\varepsilon_0}$

$2a < r < 3a \quad$ L.I.H $\quad \vec{E} = \dfrac{\vec{D}}{\varepsilon_1}$

$\vec{P} = \varepsilon_0 \chi_1 \vec{E}$

$r > 3a \Longrightarrow \vec{E} = \dfrac{\vec{D} - \vec{P}}{\varepsilon_0}$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \Longrightarrow \vec{P} = \vec{D} - \varepsilon_0 \vec{E}$$

$$\rho_p = - \vec{\nabla} \cdot \vec{P}$$

$$\vec{\nabla} \cdot \vec{P} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 P_r \right)$$

$$= \frac{1}{r^2} \frac{\partial}{\partial r} \left[\cancel{r^2} \frac{Q_1 + Q_2}{4\pi \cancel{r^2}} \left(1 - \frac{\varepsilon_0}{\varepsilon_1}\right) \right] = 0$$

$$\vec{P} = \begin{cases} 0 & r < 2a \\ \dfrac{Q_1 + Q_2}{4\pi r^2} \left(1 - \dfrac{\varepsilon_0}{\varepsilon_1}\right) \hat{r} & 2a < r < 3a \\ A r \hat{r} & r > 3a \end{cases}$$

$$2a < r < 3a \Longrightarrow \rho_p = 0$$

$r > 3a$

$$\rho_p = - \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 A r \right) = - \frac{1}{r^2} A 3 r^2 = -3A$$

$$\rho_p = -3A$$