Reappraising Einstein's 1909 application of fluctuation theory to Planckian radiation

F. E. Irons[a)]

School of Aerospace, Civil and Mechanical Engineering, University College, The University of New South Wales, Australian Defence Force Academy, Canberra, ACT 2600, Australia

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Einstein's 1909 application of fluctuation theory to Planckian radiation is challenged by the fact that radiation within a completely reflecting cavity is not in thermal equilibrium and therefore should not qualify as a candidate for analysis by Einstein's theory. We offer an alternative interpretation wherein Planck's function, to which Einstein applied his theory, represents the source function in the wall material surrounding a real, partially reflecting cavity. The source function experiences thermal fluctuations and radiation within the cavity (which originates in the wall material and has an intensity equal to the source function) fluctuates in concert. That is, blackbody radiation within a real cavity exhibits the thermal fluctuations predicted by Einstein, but the fluctuations have their origin in the wall material and are not intrinsic to radiation.

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I. INTRODUCTION

A closed system in thermal equilibrium (such as a molecular gas in an adiabatic enclosure) is characterized by statistical fluctuations in energy. In 1909 Einstein advanced a theory for these fluctuations (see Sec. II),[1,2] leading to a formula that is usually expressed as in Eq. (1). Einstein hoped that his result would be more widely applicable than the underlying assumptions suggested and applied it to Planckian radiation within a completely reflecting cavity (see Sec. III). The resulting expression, Eq. (8), contains two terms, one of which Einstein identified with wave behavior and one with particle behavior, leading to the hypothesis of the wave-particle duality of light.

Although we do not dismiss Einstein's hope that his fluctuation formula is more widely applicable than suggested by the underlying assumptions, we emphasize that radiation within a completely reflecting cavity is not in a state of thermal equilibrium (Sec. IV), and we take the view that this radiation does not qualify as a candidate for analysis by Einstein's theory.

Yet, as demonstrated by Einstein's thought experiment involving a flat plate suspended in a cavity[1,2] (see Secs. V and VI), blackbody radiation in a real, partially reflecting cavity exhibits the fluctuations predicted by Einstein for a closed system. We make sense of this result in Secs. VII and VIII by asserting that Planck's function, to which Einstein applied his formula, should be interpreted as representing the source function in the wall material surrounding a real, partially reflecting cavity. The source function experiences thermal fluctuations and radiation within the cavity (which originates in the wall material and has an intensity equal to the source function) fluctuates in concert. That is, blackbody radiation within a real cavity exhibits the thermal fluctuations predicted by Einstein, but the fluctuations (with their "wave" and "particle" components) have their origin in the wall material and are not intrinsic to radiation.

The paper does not challenge what is currently understood by the wave-particle duality of light.[3] However, it does challenge the interpretation that the wave and particle terms which emerge from Einstein's theory are linked to what is currently understood by the wave-particle duality of light.

II. EINSTEIN'S THEORY OF FLUCTUATIONS

The texts cited in Ref. 4 provide a useful introduction to the theory of fluctuations in thermodynamic quantities. Gibbs[5] in 1902 and Einstein[6] in 1904 independently employed an ensemble average to obtain Eq. (1) which describes the statistical energy fluctuations within a subvolume of a closed, thermal system.[7] In 1909 Einstein did "not apply his canonical fluctuation formula, perhaps to forestall doubt about the applicability of this formula to radiation. Instead, he applied an equivalent formula based on Boltzmann's principle, a principle he held to be universally valid."[8]

Accounts of Einstein's theory may be found in Refs. 9–18. Einstein's theory is described here in Appendix A. Briefly, Einstein considered a system divided into two fixed volumes, one of which (the subvolume) is much smaller than the other (the remaining volume). The two regions can exchange energy (but not matter) freely, so that the energy lost from one region during a fluctuation is gained by the other. As explained in Appendix A, the mean square fluctuation in the energy in the subvolume is given by

$$\langle \epsilon^2 \rangle = kT^2 \left(\frac{\partial \langle E \rangle}{\partial T} \right)_v , \tag{1}$$

where $\langle E \rangle$ denotes the mean energy of the subvolume.

Equation (1) is similar to that given by Einstein in 1904 and will be familiar to most readers. It differs somewhat from that given by Einstein in 1909, although the latter readily reduces to Eq. (1). The division of Eq. (1) by $\langle E \rangle^2$ leads to

$$\frac{\langle \epsilon^2 \rangle}{\langle E \rangle^2} = -kT^2 \left(\frac{\partial}{\partial T} \left(\frac{1}{\langle E \rangle} \right) \right)_v . \tag{2}$$

Although not quoted by Einstein, Eq. (2) is an obvious consequence of the theory.

The statistical fluctuations described by Eq. (1) are characteristic of a system in thermal equilibrium, such as a molecular gas in an adiabatic enclosure. The derivation of Eq. (1) specifically cites the condition for thermal equilibrium [see Eq. (A3) in Appendix A]. Pippard gives the example of Johnson noise, which "shows clearly, what we have stressed before, that fluctuations are not to be regarded as spontaneous departures from the equilibrium configuration of a system, but are manifestations of the dynamic character of thermal equilibrium itself, and quite inseparable from the equilibrium state."[19]

The subvolume and remaining volume in Einstein's theory can exchange energy, but not matter (particles). As regards the application of Einstein's theory to radiation, one might wonder how an exchange of radiation energy is possible without a corresponding exchange of particles (photons).

III. THE CASE OF PLANCKIAN RADIATION

A. Einstein's formula applied to Planckian radiation

Assuming his fluctuation formula to be more widely applicable than suggested by the underlying assumptions, Einstein applied it to radiation within a closed space (one bounded by "diffusely, completely reflecting walls"[20]), the spectrum of the radiation being described by Planck's law. In Sec. IV we consider the validity of Einstein's procedure. For the moment we follow Einstein's analysis.

Within a subvolume V of a closed radiation space, the mean radiation energy within the frequency interval ν to $\nu+d\nu$ is

$$\langle E\rangle = V\rho\, d\nu, \tag{3}$$

where ρ denotes the radiation energy density per unit frequency interval. Our interest is the case where ρ is given by Planck's law:

$$\rho = \left(\frac{8\pi h\nu^3}{c^3}\right)\frac{1}{\exp(h\nu/kT)-1}, \tag{4}$$

so that

$$\langle E\rangle = V\left(\frac{8\pi h\nu^3}{c^3}\right)\frac{d\nu}{\exp(h\nu/kT)-1}. \tag{5}$$

If we substitute Eq. (5) into Eq. (2), we obtain

$$\frac{\langle \epsilon^2\rangle}{\langle E\rangle^2} = \frac{\exp(h\nu/kT)}{V(8\pi\nu^2/c^3)d\nu}, \tag{6}$$

which may be expressed as

$$\frac{\langle \epsilon^2\rangle}{\langle E\rangle^2} = \frac{\exp(h\nu/kT)-1}{V(8\pi\nu^2/c^3)d\nu} + \frac{1}{V(8\pi\nu^2/c^3)d\nu}. \tag{7}$$

We then use Eq. (5) to find

$$\frac{\langle \epsilon^2\rangle}{\langle E\rangle^2} = \frac{h\nu}{\langle E\rangle} + \frac{c^3}{V8\pi\nu^2 d\nu}, \tag{8}$$

which, using Eq. (3), becomes

$$\langle \epsilon^2\rangle = \left(h\nu\rho + \frac{c^3}{8\pi\nu^2}\rho^2\right)V d\nu. \tag{9}$$

B. The limit $h\nu \gg kT$

In the limit $h\nu \gg kT$, Eq. (5) becomes

$$\langle E\rangle = V\left(\frac{8\pi h\nu^3}{c^3}\right)\exp(-h\nu/kT)d\nu, \tag{10}$$

which indicates that (in this limit) Eq. (6) may be written as

$$\frac{\langle \epsilon^2\rangle}{\langle E\rangle^2} = \frac{h\nu}{\langle E\rangle}. \tag{11}$$

Equation (11) would have resulted had we calculated $\langle \epsilon^2\rangle/\langle E\rangle^2$ with ρ in Eq. (3) given by Wien's radiation law.

If we now let

$$\langle E\rangle = Nh\nu, \tag{12}$$

where [see Eq. (10)]

$$N = V\left(\frac{8\pi\nu^2}{c^3}\right)\exp(-h\nu/kT)d\nu, \tag{13}$$

then Eq. (11) becomes

$$\frac{\langle \epsilon^2\rangle}{\langle E\rangle^2} = \frac{1}{N}. \tag{14}$$

Equation (14) is the same as the fluctuations in the number density of an ideal gas with N equal to the mean number of particles,[21] which has prompted the hypothesis that N in Eq. (12) be interpreted as an integer and that cavity radiation in the limit $h\nu \gg kT$ behaves like a collection of independent particles, each of energy $h\nu$.[22] This hypothesis is not supported by Eq. (13), because the latter gives no hint that N is a quantity that takes only integer values. We will return to this point in Sec. VII D.

C. The limit $h\nu \ll kT$

In contrast, in the limit $h\nu \ll kT$, Eq. (6) becomes

$$\frac{\langle \epsilon^2\rangle}{\langle E\rangle^2} = \frac{c^3}{V8\pi\nu^2 d\nu}. \tag{15}$$

Equation (15) would have resulted had we calculated $\langle \epsilon^2\rangle/\langle E\rangle^2$ with ρ in Eq. (3) set equal to the Rayleigh–Jeans law.

The right-hand side of Eq. (15) has a wave interpretation. If (following Longair[21]) we assume that the electric field at a point in space is the superposition of the electric fields (with random phases) from a large number of sources, then the mean square fluctuation in the field, $\langle \epsilon^2\rangle$, is related to the mean energy, $\langle E\rangle$, by

$$\langle \epsilon^2\rangle = \langle E\rangle^2. \tag{16}$$

Equation (16) applies to waves of a particular frequency, corresponding to a single mode. When there are M separate modes per unit frequency interval, such that $\langle E\rangle$ varies as M and $\langle \epsilon^2\rangle$ also varies as M, then

$$\frac{\langle \epsilon^2\rangle}{\langle E\rangle^2} = \frac{1}{M}. \tag{17}$$

When we assign M a value in accordance with the number of modes in the frequency interval ν to $\nu+d\nu$ within a completely reflecting cavity of volume V, namely

$$M = V 8 \pi \nu^2 d\nu / c^3, \tag{18}$$

then

$$\frac{\langle \epsilon^2 \rangle}{\langle E \rangle^2} = \frac{c^3}{V 8 \pi \nu^2 d\nu}. \tag{19}$$

This result is the same as in Eq. (15), which has prompted the behavior of blackbody radiation in the limit $h\nu \ll kT$ to be likened to that of waves.

IV. WHY A CLOSED RADIATION SPACE IS NOT A CANDIDATE FOR ANALYSIS BY EINSTEIN'S THEORY

A. Not in thermal equilibrium

A closed system in thermal equilibrium implies an internal mechanism for energy exchange and thermalization. Without such a mechanism we would not expect thermal equilibrium to prevail, in which case Einstein's theory (which analyzes fluctuations about thermal equilibrium in a closed system) would not apply. Unlike gases, which can thermalize by interparticle collisions, radiation within a closed space lacks an internal mechanism for thermalization. Electromagnetic waves can cause interference effects, but there is no transfer of energy from one wave to another. Likewise, photons do not exchange energy.[23] In 1909 we might have shared Einstein's reluctance to dismiss radiation as unreactive (''it must not be assumed that radiations consist of noninteracting quanta'').[24] However, we now know that there is no mechanism for the thermalization of radiation energy within a completely reflecting cavity free of matter. As a consequence, radiation within such a cavity is not subject to those fluctuations that are ''manifestations of the dynamic character of thermal equilibrium itself, and quite inseparable from the equilibrium state'' (see Sec. II). In other words, a closed radiation space is not a candidate for analysis by Einstein's theory.

We may assume that a completely reflecting cavity is filled with radiation via a hole that links the cavity to an adjoining isothermal enclosure. When the hole is closed, radiation is trapped within the completely reflecting cavity. The trapped radiation may be described as having a temperature T (the temperature of the adjoining enclosure), but having this temperature does not imply that the trapped radiation is in thermal equilibrium. An expansion or contraction of the walls of a completely reflecting cavity will cause the spectrum of radiation within the cavity to vary, while retaining a Planckian distribution; that is, it will cause the temperature to vary. Such a variation is a consequence of the Doppler shift associated with the moving walls,[25] and is not indicative of the radiation being able to thermalize.[26] Spanner[27] refers to radiation within a closed space as being in a state of metastable equilibrium, on the grounds that the introduction of a piece of matter brings about an irreversible transformation.

B. Thermalization through interaction with matter

The nonthermalizing character of radiation within a completely reflecting cavity is given *de facto* recognition by texts that refer to thermalization as proceeding via the interaction of radiation with matter. A small window in the wall of a cavity is sometimes cited in this regard. ''Through this small opening in the wall, the energy is exchanged between the hollow cavity and the reservoir [the wall material] to bring about eventually a state of thermal equilibrium.''[28] A small piece of matter inserted into an otherwise completely reflecting cavity also has been proposed as a means for effecting thermalization.[29] It has been suggested that the resonators employed by Planck in the derivation of his radiation law serve this purpose (''His [Planck's] resonators were imaginary entities, not susceptible to experimental investigation. Their introduction was simply a device for bringing radiation to equilibrium...''[30]).

The ''interaction-with-matter'' approach to reaching thermalization is commonly cited in derivations of Planck's law that draw on cavity modes of oscillation. It has no place where fluctuations within a closed radiation space are concerned. To permit an interaction with matter (that is, to add a source or sink of radiation energy to a closed system) would be to violate the premise on which Einstein's theory is based, namely [see Eq. (A2) in Appendix A] that a gain (or loss) of energy by radiation within the subvolume is accompanied by an equivalent loss (or gain) of energy by radiation within the remaining volume, *with no other options*.

V. A THOUGHT EXPERIMENT

We now turn our attention to the thought experiment described by Einstein in his 1909 papers[1,2] regarding a flat plate (perfectly reflecting on both faces) suspended in a real, partially reflecting cavity containing blackbody radiation and gas molecules. The radiation originates in the walls of the cavity, which ''have the definite temperature T, are impermeable to radiation, and are not everywhere completely reflecting toward the cavity.''[31] As a result of collisions with gas molecules, the plate executes an irregular (Brownian) motion. This motion leads to an imbalance in the radiation forces on the front and rear surfaces of the plate, which in turn resists the motion. Through this resistance (radiation friction), kinetic energy is transformed into radiation energy. However, equilibrium requires that the radiation energy be transformed back into kinetic energy. This transformation is achieved through fluctuations. According to Einstein, fluctuations in the radiation energy (or pressure) can, on average, return energy from the radiation field back to the plate (and then to the gas molecules), causing a balance to be established.[1,2]

Let Δ denote the increase in momentum of the plate during a time interval τ due to fluctuations in radiation energy.[32] Einstein related $\overline{\Delta^2}$ to P, the radiation resistance per unit velocity, and then related P to f (the area of the plate) and to ρ (the radiation energy density). With ρ equal to Planck's law, he arrived at the relation

$$\frac{\overline{\Delta^2}}{\tau} = \frac{1}{c}\left(h\nu\rho + \frac{c^3}{8\pi\nu^2}\rho^2 \right) f d\nu. \tag{20}$$

Equation (20) is remarkably similar to Eq. (9), the quantity in brackets being the same in both cases. However, Eq. (20) does not assume a closed radiation space and does not depend on Eq. (1).

VI. DISCUSSION

A. A puzzle

Einstein was well aware that his molecular-based theory might not be applicable to a radiation space. In 1904 he wrote that, ''Of course, one can object that we are not permitted to assert that a radiation *space* should be viewed as a *system* of the kind we have assumed, not even if the applicability of the general molecular theory is conceded [Einstein's italics]."[33] However, having (in 1904) applied fluctuation theory in an approximate manner to blackbody radiation and obtained a result in near agreement with Wien's displacement law (critically assessed in Appendix B),[6] Einstein was encouraged to persevere with the application of fluctuation theory to radiation. At a meeting in 1909 he invited the audience to view the wave-particle duality of light as a hypothesis needing further investigation. ("What I shall presently say is for the most part my private opinion or, rather, the result of considerations that have not yet been sufficiently checked by others."[34]) Klein remarks that Einstein "boldly applied Eq. (1) to calculate the energy fluctuations of thermal radiation, a system that *certainly did not satisfy the conditions of its derivation* [my italics]."[35]

The similarity between Eqs. (9) and (20) appears to support the result in Eq. (9), that is, appears to support the assumption that Eq. (1) can be applied to a closed radiation space. This view was taken by Einstein in 1909 and is recycled in the various accounts of his theory. In hindsight, the similarity is puzzling. Radiation within a closed space, not being in thermal equilibrium, is not a candidate for analysis by Einstein's theory. Yet Einstein's thought experiment shows that blackbody radiation in a real, partially reflecting cavity exhibits the fluctuations predicted by Einstein for a closed system. The resolution of this puzzle is the subject of Sec. VII.

B. Living with inconsistency

Einstein might not have been unduly worried by the inconsistency surrounding the assumption of a closed radiation space as a thermal system. He had previously come to terms with the inconsistent mix of classical and quantum ideas in Planck's theory of blackbody radiation.[36] Inconsistency also was evident in Rutherford's contemporary model of the atom, wherein it was assumed that an electron could orbit a nucleus without spiraling inward, despite the fact that (according to classical theory) it loses energy by radiation. According to Wigner, "... it was clear in those days, and in fact it was clear even very much later to everybody, that the physics that we had was not consistent."[37]

The inconsistency surrounding the Rutherford atom was resolved by the advent of Bohr's quantum theory of the atom. In contrast, the inconsistency surrounding the assumption of a closed radiation space as a thermal system was not resolved by the advance of quantum theory. Admittedly, quantum statistics invests photons with the behavior characteristic of bosons, but there is no suggestion of a transfer of energy between photons. Do not be confused by the occasional reference in the literature to radiation as a "photon gas."[38] Although radiation has several properties also possessed by a gas (such as internal energy and pressure), the ability to self-thermalize is not one of them. To the extent that a photon gas can be considered thermal, it is because of its interaction with the material walls of the containing vessel.

VII. THERMAL FLUCTUATIONS IN A REAL CAVITY

A. Wall model and the source function

Radiation can exhibit fluctuations whose origin is to be found within the radiation source. Intensity fluctuations linked to temporal and spatial coherence are familiar examples.[39] Less well known are the intensity fluctuations that arise from thermally induced fluctuations in the quantum state number densities of those entities (atoms) responsible for the radiation, which we now consider.

The intensity of optically thick radiation within a material medium (such as the wall material surrounding a cavity) is determined by atomic and electronic processes in conjunction with the transport of radiation through the medium.[40] When thermal equilibrium and detailed balance prevail,[41] the intensity of radiation within the medium is blackbody, irrespective of the specific processes of emission and absorption. Let us elaborate with the aid of a model for the generation of radiation within a material medium.

With little loss of generality, we may assume that radiation of frequency ν corresponds to a transition between two quantum states of an entity (think of an atom) separated in energy by $h\nu$. Denote the number density of the emitting (excited) entities as N_2 and the number density of the absorbing (unexcited) entities as N_1, and define the total number density of entities as

$$N_{\mathrm{tot}}=N_1+N_2. \tag{21}$$

The specific intensity (power per unit frequency interval per unit area per unit solid angle) of optically thick radiation in a material medium may be written as[42]

$$I=n_w^2 S, \tag{22}$$

where n_w denotes the refractive index of the medium (the wall material). In Eq. (22), S denotes the *source function*, defined in our example by[42]

$$S=\left(\frac{2h\nu^3}{c^2}\right)\frac{N_2}{N_1-N_2}. \tag{23}$$

The factor $2h\nu^3/c^2$ arises from the ratio of the Einstein coefficients, A_{21} and B_{12}, which represent spontaneous emission and induced absorption respectively; also, we have ignored degeneracy and have let $B_{12}=B_{21}$ (the Einstein coefficient for induced emission). The ratio N_2/N_1 is determined from rate equations that account for all the relevant radiative and collisional processes. When thermal equilibrium prevails,

$$N_2/N_1=\exp(-h\nu/kT), \tag{24}$$

and

$$S=\left(\frac{2h\nu^3}{c^2}\right)\frac{1}{\exp(h\nu/kT)-1}, \tag{25}$$

which follows by substituting Eq. (24) into Eq. (23).

B. Linking cavity radiation to the source function

Radiation from within the medium and incident upon the surface of the medium will either be reflected back into the medium or will escape into the space beyond the medium. If the surface reflectivity is r, then the radiant power propagating along a ray will, at the point of reflection, be reduced below the intrawall value by the factor $1-r$; and the specific intensity will be reduced by a factor of $(1-r)n_c^2/n_w^2$, where n_c denotes the refractive index of the space beyond the medium. The latter result accounts for refraction at the wall-space interface.[43] If the space beyond the medium is a cavity, then multiple emissions and reflections from around the cavity wall would raise the intracavity specific intensity to the intrawall value, Eq. (22), multiplied by a factor n_c^2/n_w^2 (see Appendix C). In other words, for the cavity space, $I=n_c^2S$. Given that the energy density in the cavity space is related to the specific intensity by $\rho=4\pi In_c/c$, it follows that the energy density of radiation in the cavity space is

$$\rho=4\pi Sn_c^3/c, \tag{26}$$

which, with n_c assumed equal to unity, becomes

$$\rho=4\pi S/c. \tag{27}$$

With S given by Eq. (25), Eq. (27) is the same as in Eq. (4).

C. Application of Eq. (1) to the source function

Assume that the wall material surrounding a cavity is in thermal equilibrium and that energy is partitioned between the kinetic motion and the internal excitation of the atoms comprising the wall material (and other degrees of freedom which will not concern us). Just as the mean kinetic energy may fluctuate, so may the ratio N_2/N_1 and the quantity defined by Eq. (23) (the source function) also fluctuate. Such thermal fluctuations are amenable to analysis by Einstein's theory. Possible fluctuations in the source function from other causes[44] will not concern us.

We assume the medium containing the cavity to be a closed system (insulated from its surroundings). Because optically thick radiation emerging from a medium originates primarily in matter that is close to the surface of the medium, our concern is primarily with matter located close to the surface of the cavity wall. Such matter constitutes a subvolume V of the medium as a whole. Our interest is in the source function, or rather in the product $4\pi S/c$, representing an energy density. For the subvolume V and frequency interval ν to $\nu+d\nu$, we may define a mean energy

$$\langle E\rangle=V4\pi Sd\nu/c. \tag{28}$$

Clearly [with S given by Eq. (25)] Eq. (28) is the same as Eq. (5), and the substitution of Eq. (28) into Eq. (2) leads to the same fluctuation spectrum as discussed in Sec. III A.

An additional result is obtained by substituting Eq. (24) into Eq. (6):

$$\frac{\langle\epsilon^2\rangle}{\langle E\rangle^2}=\left(\frac{c^3}{V8\pi\nu^2}\right)\frac{N_1}{N_2}. \tag{29}$$

The ratio $\langle\epsilon^2\rangle/\langle E\rangle^2$ increases with decreasing excitation (decreasing values of the ratio N_2/N_1), that is, increases with decreasing temperature. This result is linked to the fact that excitation of atomic quantum states involves collisions primarily with particles at the high energy end of the Maxwellian spectrum. With decreasing temperature the high energy particles able to effect excitation become relatively fewer in number and, correspondingly, exhibit relatively greater fluctuations.

D. The limit $h\nu\gg kT$

In the limit $h\nu\gg kT$ we have from Eq. (11)

$$\frac{\langle\epsilon^2\rangle}{\langle E\rangle^2}=\frac{h\nu}{\langle E\rangle}. \tag{30}$$

In the same limit, $N_2\ll N_1$ and (to a good approximation) $N_{\text{tot}}=N_1$ [where N_{tot} is defined in Eq. (21)], so that from Eqs. (23), (24), and (28)

$$\langle E\rangle=V\left(\frac{8\pi h\nu^3}{c^3}\right)\frac{N_2}{N_{\text{tot}}}d\nu. \tag{31}$$

If we proceed as in Sec. III B and let

$$\langle E\rangle=Nh\nu, \tag{32}$$

where $h\nu$ denotes a quantum of excitation energy, then again [compare to Eq. (14)]

$$\frac{\langle\epsilon^2\rangle}{\langle E\rangle^2}=\frac{1}{N}, \tag{33}$$

except that N now may be expressed as

$$N=V\left(\frac{8\pi\nu^2}{c^3}\right)\frac{N_2}{N_{\text{tot}}}d\nu. \tag{34}$$

Observe that in Eq. (34) the product VN_2 (equal to the number of excited atoms in the volume V) is an integer and N is thus proportional to an integer quantity (N_{tot} is a constant of the material). It is no surprise, therefore, to find that the fluctuation spectrum in Eq. (33) has a form analogous to that of a particle system. This result contrasts with the procedure in Sec III B, where the expression for N, Eq. (13), gives no hint as to an integral character and where the analogy with a particle system requires the *hypothesis* of N as an integer.

In the limit $h\nu\gg kT$, there are very few excited atoms compared to unexcited atoms. In the sense that excited atoms are like occasional islands in a sea of unexcited atoms, the source function may be said to have a certain point-like or particle quality.

E. The limit $h\nu\ll kT$

In the limit $h\nu\ll kT$, we have from Eq. (15)

$$\frac{\langle\epsilon^2\rangle}{\langle E\rangle^2}=\frac{c^3}{V8\pi\nu^2d\nu}. \tag{35}$$

In the same limit, $N_2=N_1$, the mean energy per atom is $\langle E\rangle=h\nu N_2/(N_1+N_2)=h\nu/2$, and the mean square energy deviation per atom is

$$\langle\epsilon^2\rangle=\frac{(0-h\nu/2)^2+(h\nu-h\nu/2)^2}{2}=\langle E\rangle^2. \tag{36}$$

Equation (36) is the same as that derived previously for the superposition of electric fields (with random phases) from a large number of sources [Eq. (16)]. Interestingly, Eq. (36) invites a wave interpretation of its own. When $N_2=N_1$, an atom is equally likely to be excited (with energy $h\nu$) as unexcited, with no other option. Along any one direction in space, the ups and downs of excitation energy have a certain wave-like quality (a square wave). Over a suitably long distance, there are as many excited as unexcited atoms, and as is apparent from Eq. (36) the corresponding ''wave'' is characterized by $\langle \epsilon^2 \rangle = \langle E \rangle^2$.

If we allow that atoms of excitation energy $h\nu$ contribute to $\langle E \rangle$ in accordance with the weight $M = V8\pi\nu^2 d\nu / c^3$ [see Eqs. (23) and (28)], then [with an eye to Eq. (17)] we have

$$\frac{\langle \epsilon^2 \rangle}{\langle E \rangle^2} = \frac{c^3}{V8\pi\nu^2 d\nu}. \tag{37}$$

Equation (37) is the same as in Eq. (35), which suggests that the behavior of the source function in the limit $h\nu \ll kT$ can be likened to that of (square) waves.

The names ''excitons'' and ''excitation waves'' suggest themselves as descriptors for the particle and wave behavior described above, except that these names are already used to describe the excitation or polarization states in solids.[45]

VIII. ANOTHER VIEWPOINT

Might it be (as suggested by an anonymous referee) that Einstein's application of fluctuation theory to Planckian radiation circumvents the fact that the origin of the fluctuations is within the walls, and is it possible that the theory in Sec. VII simply identifies the wall material as the source of the fluctuations? In other words, do the ''subvolume'' and ''remaining volume'' in Einstein's theory correspond to the cavity and wall material, respectively, in the theory outlined in Sec. VII, with the wall material serving the same role (being a reservoir of Planck radiation) as does the ''remaining volume?''

I can find nothing in Einstein's papers of 1904[6] and 1909[1,2] to support this viewpoint (indeed, Einstein concluded that the fluctuations were driven by properties—wave and particle properties—intrinsic to the radiation itself). Nor do any of the Einstein commentators cited in this paper hint at such an interpretation. More significantly, the interpretation fails on physical grounds. The wall material contains atoms that exchange energy with the radiation. The wall material may well serve as a reservoir of radiation, but it also serves as a source and sink of that radiation: and for a source or sink of radiation to be part of the system is to violate the premise on which Einstein's theory is based, namely [see Eq. (A2) in Appendix A and the second paragraph of Sec. IV B] that, during a fluctuation, a gain (or loss) of energy by radiation within the subvolume is accompanied by an equivalent loss (or gain) of energy by radiation within the remaining volume *with no other options*.

In the theory outlined in Sec. VII, it is the energy held as particle excitation that is fluctuating (and there is no source or sink of particles). In this theory, cavity radiation simply reflects the fluctuations in atomic excitation, in much the same way that the modulation of a radio wave reflects the modulation imposed by the transmitter.

If, as suggested in the last paragraph of Sec. II, an exchange of radiation energy between the subvolume and remaining volume is necessarily accompanied by an exchange of particles (photons), then are we not advised to use a fluctuation formula based on a grand canonical ensemble rather than a canonical ensemble? The answer would be yes, were it not for the fact that radiation within a closed space is not in a state of thermal equilibrium. Whatever fluctuations may be exhibited by radiation within a closed space, they are unlikely to be correctly described by a theory (based on either a canonical or grand canonical ensemble) which analyzes fluctuations about a point of stable equilibrium.

IX. CONCLUDING REMARKS

By applying his fluctuation formula to radiation within a closed space (a system that does not satisfy the condition of thermal equilibrium assumed when deriving the formula), Einstein deduced certain properties for light. We have offered an alternative interpretation along the lines that Planck's function, to which Einstein applied his formula, should be seen as representing the source function in the wall material surrounding a real, partially reflecting cavity. The source function experiences thermal fluctuations, and radiation within the cavity (which originates in the wall material and which has an intensity equal to the source function) fluctuates in concert. That is, blackbody radiation within a real cavity exhibits the thermal fluctuations predicted by Einstein, but the fluctuations (with their ''wave'' and ''particle'' components) have their origin in the wall material and are not intrinsic to radiation.

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APPENDIX A: EINSTEIN'S 1909 DERIVATION OF HIS FLUCTUATION FORMULA

The following is a summary of Einstein's 1909 derivation of his fluctuation formula.[1] It draws to a large extent on the accounts of Klein[11] and Pais.[46] Consider a closed, thermal system of fixed energy, and assume that the system is subdivided into two fixed regions or volumes that can exchange energy (but not matter) freely. Denote the energies of the two volumes as E_1 and E_2, and the entropies as S_1 and S_2. Let ϵ_1 and ϵ_2 denote the amounts by which E_1 and E_2 deviate from their equilibrium values, respectively. The amount by which the total entropy deviates from its equilibrium value may be expanded as

$$\Delta S = \left(\frac{\partial S_1}{\partial E_1}\right)_o \epsilon_1 + \left(\frac{\partial S_2}{\partial E_2}\right)_o \epsilon_2 + \frac{1}{2}\left(\frac{\partial^2 S_1}{\partial E_1^2}\right)_o \epsilon_1^2 + \frac{1}{2}\left(\frac{\partial^2 S_2}{\partial E_2^2}\right)_o \epsilon_2^2 + \cdots, \tag{A1}$$

where the terms in brackets correspond to equilibrium values. The partial derivatives are performed with the volumes held constant and with the number of particles in each volume held constant.[47] Because

$$\epsilon_1 = -\epsilon_2 \quad \text{(constant energy)} \tag{A2}$$

and

$$\left(\frac{\partial S_1}{\partial E_1}\right)_o = \left(\frac{\partial S_2}{\partial E_2}\right) = \frac{1}{T} \quad \text{(thermal equilibrium)}, \tag{A3}$$

the first-order terms in Eq. (A1) cancel. Moreover,

$$\left(\frac{\partial^2 S_1}{\partial E_1^2}\right)_o = \left(\frac{\partial}{\partial E_1}\left(\frac{1}{T}\right)\right)_o = -\frac{1}{T^2}\left(\frac{\partial \langle E_1 \rangle}{\partial T}\right)_v^{-1}, \tag{A4}$$

where $\langle E_1 \rangle$ denotes the mean or equilibrium value of E_1; similar considerations apply for $(\partial^2 S_2/\partial E_2^2)_o$. If we assume volume 1 (the subvolume) to be very much smaller than volume 2, then $\langle E_1 \rangle \ll \langle E_2 \rangle$ and correspondingly $\partial\langle E_1 \rangle/\partial T \ll \partial\langle E_2 \rangle/\partial T$. Thus Eq. (A1) reduces to

$$\Delta S = -\frac{\epsilon^2}{2T^2}\left(\frac{\partial\langle E \rangle}{\partial T}\right)_v^{-1}, \tag{A5}$$

where we have dropped the subscript (1) as no longer necessary.

Equation (A1) gives the entropy fluctuation, ΔS, corresponding to an energy fluctuation ϵ. The probability of an energy fluctuation ϵ is linked to ΔS by (Boltzmann's principle)

$$P(\epsilon) = \alpha \exp(\Delta S/k), \tag{A6}$$

where α is a normalization constant. With the aid of Eqs. (A5) and (A6), we can calculate the mean square energy fluctuation, defined by

$$\langle \epsilon^2 \rangle = \int_{-\langle E \rangle}^{\infty} \epsilon^2 P(\epsilon)\, d\epsilon, \tag{A7}$$

where $\epsilon = E - \langle E \rangle$ and E varies from 0 to ∞. If $\langle \epsilon^2 \rangle \ll \langle E \rangle^2$, the lower limit in the integral in Eq. (A7) may be replaced by $-\infty$, in which case the integral reduces to

$$\langle \epsilon^2 \rangle = kT^2\left(\frac{\partial\langle E \rangle}{\partial T}\right)_v. \tag{A8}$$

The restriction of the result in Eq. (A8) to $\langle \epsilon^2 \rangle \ll \langle E \rangle^2$ does not arise when Eq. (A8) is derived as an ensemble average.[7] The actual formula derived by Einstein in 1909 differs somewhat from Eq. (A8), but readily reduces to Eq. (A8).

APPENDIX B: APPLICATION OF EINSTEIN'S FORMULA TO THE TOTAL RADIATION ENERGY

In his 1904 paper, Einstein reasoned that "if the space volume containing the radiation has the linear dimensions of a wavelength, then the energy fluctuations will have the same order of magnitude as the radiation energy contained in the space volume of radiation."[48] This reasoning prompted the following calculation.[49] Define the mean energy

$$\langle E \rangle = V \int \rho\, d\nu = VaT^4, \tag{B1}$$

where ρ is Planck's function and a is related to the Stefan-Boltzmann constant, σ, by $a = 4\sigma/c$. The substitution of Eq. (B1) into Eq. (2) leads to

$$\frac{\langle \epsilon^2 \rangle}{\langle E \rangle^2} = \frac{4k}{VaT^3}. \tag{B2}$$

If we equate the left-hand side of Eq. (B2) to unity, we obtain (with $k = 1.3804 \times 10^{-16}$ erg deg^{-1} and $a = 7.56 \times 10^{-15}$ erg cm^{-3} deg^{-4})

$$V^{1/3} = \frac{1}{T}\left(\frac{4k}{a}\right)^{1/3} = \frac{0.42}{T}. \tag{B3}$$

If we now identify $V^{1/3}$ with λ_m (the wavelength corresponding to the maximum of Planck's function), we see that Eq. (B3) compares with a known property of Planck's function, namely Wien's displacement law

$$\lambda_m = \frac{0.29}{T}, \tag{B4}$$

with λ_m in cm. We read that "this agreement must not be ascribed to chance,"[50] and that it "confirms the applicability of statistical concepts to radiation."[51]

We concur that the agreement must not be ascribed to chance, but dispute that it confirms the applicability of statistical concepts to radiation. A closed radiation space is not a system in thermal equilibrium. That we can substitute $\langle E \rangle$, Eq. (B1), into an equation, (2), designed for a closed system in thermal equilibrium and obtain a sensible result does not change the fact that a closed radiation space is not a system in thermal equilibrium. The answer to the question "how is it, then, that we get a sensible result?" must be sought elsewhere, and I would suggest the following.

If we replace ρ in Eq. (B1) by the equivalent quantity $4\pi S/c$ [Eq. (27)], and remember that ρ and $4\pi S/c$ are both equal to Planck's function, then Eqs. (B2) and (B3) still follow, and we still have agreement between Eqs. (B3) and (B4). [Equation (B4) is a property of Planck's function, and is as much a characteristic of S as of ρ.] The difference is that V now refers to a subvolume of a material medium.

The calculation just outlined might be prompted by the following line of reasoning (compare the quotation in the opening paragraph of this Appendix). "If the material volume containing the source function has the linear dimensions of a wavelength, then the energy fluctuations will have the same order of magnitude as the source function energy contained in the material volume of the source function." The agreement between Eqs. (B3) and (B4) may now be seen as confirming the applicability of statistical concepts to the source function—a conclusion that we would not dispute, given the analysis in Sec. VII.

APPENDIX C: THE SPECIFIC INTENSITY WITHIN A CAVITY

Pursuant to the argument in Sec. VII B, consider (for simplicity) a cavity with specularly reflecting walls with reflectivity r. Assume that Planckian radiation of specific intensity $I_0 (= n_w^2 S)$ reaches the wall surface from within the wall material and that the specific intensity of radiation passing into the cavity is I_1. Then $I_1 = (1-r)I_0 n_c^2/n_w^2$. After a reflection from within the cavity, the specific intensity, supplemented by radiation entering the cavity at the point of reflection, is

$$\begin{aligned} I_2 &= (1-r)I_0 n_c^2/n_w^2 + rI_1 \\ &= (1-r)I_0 n_c^2/n_w^2 + r(1-r)I_0 n_c^2/n_w^2, \end{aligned} \tag{C1}$$

and after $m-1$ such reflections,

$$\begin{aligned} I_m &= (1-r)(1 + r + \cdots + r^{m-1})I_0 n_c^2/n_w^2 \\ &= (1-r^m)I_0 n_c^2/n_w^2. \end{aligned} \tag{C2}$$

Because $r < 1$, the specific intensity of radiation within the cavity (obtained by letting $m \to \infty$) is thus equal to $I_0 n_c^2/n_w^2$, and because $I_0 = n_w^2 S$, it becomes equal to $n_c^2 S$, which equals S when $n_c = 1$.

[a]Electronic mail: f.irons@adfa.edu.au

[1]A. Einstein, "On the present status of the radiation problem," Phys. Z. **10**, 185–193 (1909). Reprinted in A. Einstein, *The Collected Papers of Albert Einstein, Vol. 2, The Swiss Years: Writings 1900–1909*, edited by J. Stachel *et al.* (Princeton U.P., Princeton, 1989), Doc. 56. For an English translation, see *The Collected Papers of Albert Einstein*, translated by A. Beck (Princeton U.P., Princeton, 1989), Doc. 56.

[2]A. Einstein, "On the development of our views concerning the nature and constitution of radiation," Phys. Z. **10**, 817–826 (1909). Reprinted in A. Einstein, *The Collected Papers of Albert Einstein, Vol. 2, The Swiss Years: Writings 1900–1909*, edited by J. Stachel *et al.* (Princeton U.P., Princeton, 1989), Doc. 60. For an English translation, see *The Collected Papers of Albert Einstein*, translated by A. Beck (Princeton U.P., Princeton, 1989), Doc. 60.

[3]See, for example, G. Greenstein and A. G. Zajonic, *The Quantum Challenge* (Jones and Bartlett, Boston, 1997), Chap. 2, particularly Sec. 2.2.

[4]Y. Rocard and C. R. S. Manders, *Thermodynamics* (Isaac Pitman, London, 1961), Chap. 35; W. Greiner, L. Neise, and H. Stöcker, *Thermodynamics and Statistical Mechanics*, translated by D. Rischke (Springer-Verlag, New York, 1995), pp. 191–194; J. R. Waldram, *The Theory of Thermodynamics* (Cambridge U.P., Cambridge, UK, 1985), Chap. 16.

[5]J. W. Gibbs, *Elementary Principles in Statistical Mechanics* (Dover, New York, 1960), Chap. VII.

[6]A. Einstein, "On the general molecular theory of heat," Ann. Phys. (Leipzig) **14**, 354–362 (1904). Reprinted in A. Einstein, *The Collected Papers of Albert Einstein, Vol. 2, The Swiss Years: Writings 1900–1909*, edited by J. Stachel *et al.* (Princeton U.P., Princeton, 1989), Doc. 5. For an English translation, see *The Collected Papers of Albert Einstein*, translated by A. Beck (Princeton U.P., Princeton, 1989), Doc. 5.

[7]For a recent derivation of Eq. (1) as an ensemble average, see W. Greiner, L. Neise, and H. Stöcker, Ref. 4; A. Pais, *Subtle is the Lord: The Science and the Life of Albert Einstein* (Clarendon, Oxford, 1982), p. 69.

[8]Editorial note in A. Einstein, *The Collected Papers of Albert Einstein, Vol. 2, The Swiss Years: Writings 1900–1909*, edited by J. Stachel *et al.* (Princeton U.P., Princeton, 1989), p. 146.

[9]W. Pauli, "Einstein's contributions to quantum theory," in *Albert Einstein: Philosopher-Scientist*, edited by P. A. Schilpp (Tudor, New York, 1951), pp. 149–160.

[10]R. C. Tolman, *The Principles of Statistical Physics* (Oxford Univ., Oxford, 1959), pp. 636–641.

[11]M. J. Klein, "Einstein and the wave-particle duality," Natural Philos. **3**, 1–49 (1964).

[12]A. Kastler, "Albert Einstein and the photon concept," in *Relativity, Quanta, and Cosmology in the Scientific Thought of Albert Einstein*, edited by F. De Finis (Johnson Reprint, New York, 1979), Chap. 11.

[13]M. J. Klein, "Einstein and the development of quantum physics," in *Einstein: A Centenary Volume*, edited by A. P. French (Heinemann, London, 1979), pp. 133–151.

[14]M. J. Klein, "No firm foundation: Einstein and the early quantum theory," in *Some Strangeness in the Proportion: A Centennial Symposium to Celebrate the Achievements of Albert Einstein*, edited by H. Woolf (Addison–Wesley, Reading, MA, 1980), pp. 161–185.

[15]A. Pais, Ref. 7, Secs. 4c, 4d, 21a, 21c.

[16]J. Mehra and H. Rechenberg, *The Historical Development of Quantum Theory* (Springer-Verlag, New York, 1982), Vol. 1, Part 1, Sec. I.3.

[17]P. W. Milonni, "Wave-particle duality of light: A current perspective," in *The Wave-Particle Dualism*, edited by S. Diner, D. Fargue, G. Lochak, and F. Selleri (Reidel, Dordrecht, 1984), pp. 27–67.

[18]M. S. Longair, *Theoretical Concepts in Physics* (Cambridge U.P., Cambridge, 1984), Chap. 12.

[19]A. B. Pippard, *The Elements of Classical Thermodynamics* (Cambridge U.P., Cambridge, 1957), pp. 83–84.

[20]A. Einstein, Ref. 1, p. 364 of the English translation.

[21]Reference 18, pp. 245–249.

[22]References 1 and 2; Ref. 18, pp. 245–249; Ref. 11, p. 12.

[23]For a comment to this effect, see R. E. Kelly, "Thermodynamics of blackbody radiation," Am. J. Phys. **49**, 714–719 (1981); C. V. Heer, *Statistical Mechanics, Kinetic Theory, and Stochastic Processes* (Academic, New York, 1972), p. 234. The photon-photon interaction predicted by quantum electrodynamics is too small to be of concern. See J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Springer-Verlag, New York, 1976), Chap. 13.

[24]A. Einstein, *The Collected Papers of Albert Einstein, Vol. 2, The Swiss Years: Writings 1900–1909*, edited by J. Stachel *et al.* (Princeton U.P., Princeton, 1989), Doc. 61. For an English translation, see the companion volume, *The Collected Papers of Albert Einstein*, translated by A. Beck (Princeton U.P., Princeton, 1989), Doc. 61.

[25]See, for example, F. K. Richtmyer, E. H. Kennard, and J. N. Cooper, *Introduction to Modern Physics* (McGraw–Hill, New York, 1969), pp. 143–146; M. Born, *Atomic Physics* (Blackie, Glasgow, 1962), 7th ed., pp. 343–347.

[26]Of related interest is that the expansion of the universe causes the spectral distribution of the cosmic background radiation to vary, while maintaining its blackbody character. Such a variation is linked to the gravitational red shift and is not indicative of the radiation being able to thermalize. For a brief discussion, see W. Greiner, L. Neise, and H. Stöcker., Ref. 4, p. 333. For greater detail, see H. Höhl, "A contribution to the thermodynamics of the universe and to 3°K radiation," in *Perspectives in Quantum Theory*, edited by W. Yourgrau and A. van der Mewe (Dover, New York, 1971), Chap. 8.

[27]D. C. Spanner, *Introduction to Thermodynamics* (Academic, London, 1964), pp. 223–224.

[28]S. Tomonaga, *Quantum Mechanics* (North-Holland, Amsterdam, 1962), Vol. I, p. 18.

[29]C. V. Heer, Ref. 23, p. 234; P. G. Bergmann, *Basic Theories of Physics: Heat and Quanta* (Dover, New York, 1951), p. 143; E. A. Desloge, *Statistical Physics* (Holt, Rinehart and Winston, New York, 1966), p. 159.

[30]T. S. Kuhn, *Black-Body Theory and the Quantum Discontinuity 1894–1912* (Clarendon, Oxford, 1978), pp. 117–118.

[31]A. Einstein, Ref. 2, p. 391 of the English translation.

[32]In Ref. 2, Einstein refers to Δ as an increase in momentum and derives Eq. (20) of the present paper. In Ref. 1 Einstein refers to Δ as an increase in velocity, though the equation that he derives [Eq. (20) of the present paper] requires that Δ be viewed as the increase in momentum.

[33]A. Einstein, Ref. 6, p. 76 of the English translation.

[34]A. Einstein, Ref. 2, p. 386 of the English translation. For a brief account of the reception accorded Einstein's address at the Salzburg Congress of 1909, see A. Hermann, *The Genesis of Quantum Theory (1899–1913)* (MIT, Cambridge, MA, 1971), Chap. 3, Sec. 7; also the Editorial Note in A. Einstein, *The Collected Papers of Albert Einstein, Vol. 2, The Swiss Years: Writings 1900–1909*, edited by J. Stachel *et al.* (Princeton U.P., Princeton, 1989), pp. 134–148 (particularly pp. 147–148).

[35]Reference 14, p. 167.

[36]A. Einstein, "On the theory of light production and light absorption," Ann. Phys. (Leipzig) **20**, 199–206 (1906). Reprinted in A. Einstein, *The Collected Papers of Albert Einstein, Vol. 2, The Swiss Years: Writings 1900–1909*, edited by J. Stachel *et al.* (Princeton U.P., Princeton, 1989), Doc. 34. For an English translation, see the companion volume, *The Collected Papers of Albert Einstein*, translated by A. Beck (Princeton U.P., Princeton, 1989), Doc. 34 (see particularly p. 196). For a commentary, see M. Jammer, *The Conceptual Development of Quantum Mechanics* (McGraw–Hill, New York, 1966), pp. 26–27; T. Kuhn, Ref. 30, pp. 184–185.

[37]E. Wigner, as part of a discussion, in *Some Strangeness in the Proportion: A Centennial Symposium to Celebrate the Achievements of Albert Einstein*, edited by H. Woolf (Addison–Wesley, Reading, MA, 1980), p. 195 (also see the comments by Wigner on p. 194).

[38]For example, R. E. Sonntag and G. J. van Wylen, *Fundamentals of Statistical Thermodynamics* (Wiley, New York, 1966), pp. 177–183; S. Tomonaga, Ref. 28, pp. 72–75; H. S. Leff, "Teaching the photon gas in introductory physics," Am. J. Phys. **70**, 792–797 (2002).

[39]Reference 3, Chap. 2; M. V. Klein, *Optics* (Wiley, New York, 1970), Sec. 6.4; W. Martienssen and E. Spiller, "Coherence and fluctuations in light beams," Am. J. Phys. **32**, 919–926 (1964).

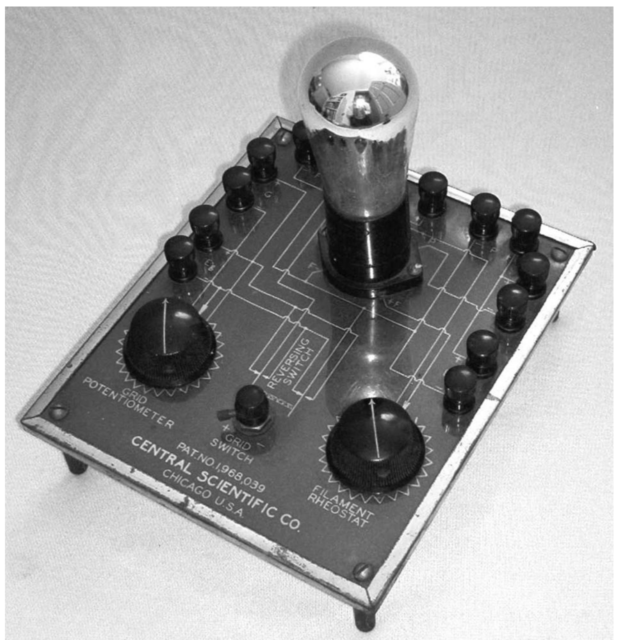
[40]For a broad account of radiation from solids, see G. A. W. Rutgers, "Temperature radiation of solids," in *Handbuch der Physik*, edited by S. Flügge (Springer, Berlin, 1958), Vol. XXVI, pp. 129–170; H. Blau and H. Fischer (eds.), *Radiative Transfer from Solid Materials* (Macmillan, New York, 1962).

[41]For a useful discussion of detailed balance, see R. W. Ditchburn, *Light* (Blackie, London, 1958), pp. 570–571; J. K. Roberts, *Heat and Thermodynamics* (Blackie, London, 1940), 3rd ed., pp. 385–386.

[42]See, for example, J. Cooper, "Plasma spectroscopy," Rep. Prog. Phys. **29**, 35–130 (1966) (particularly Sec. 13). For radiative transfer through a weakly absorbing, solid material, see E. U. Condon, "Radiative transfer in hot glass," J. Quant. Spectrosc. Radiat. Transf. **8**, 369–385 (1968) [note that Eq. (11) should read $\epsilon_\nu = \alpha_\nu B_\nu(T)$].

[43]See E. U. Condon, Ref. 42, p. 374; E. Mach, *Principles of the Theory of Heat* (Reidel, Dordrecht, 1986), pp. 139–140.

[44]R. Loudon suggests that "the level populations N_1 and N_2 also exhibit fluctuations owing to the emission and absorption of photons by the atoms." R. Loudon, *The Quantum Theory of Light* (Clarendon, Oxford, 1973), p. 18.
[45]See F. Seitz, *The Modern Theory of Solids* (McGraw–Hill, New York, 1940), Sec. 96; C. Kittel, *Introduction to Solid State Physics* (Wiley, New York, 1986), 6th ed., pp. 296–306; C. T. Walker and G. A. Slack, "Who named the-ON's?" Am. J. Phys. **38**, 1380–1389 (1970), pp. 1384–1385.
[46]A. Pais, Ref. 7, pp. 73–74.
[47]Y. Rocard and C. R. S. Manders, Ref. 4, p. 480, refer to the number of particles (actually the density) being held constant. Also see W. Greiner, L. Neise, and H. Stöcker, Ref. 4, p. 194.
[48]Reference 6. The English translation is quoted from Ref. 16, p. 69.
[49]For a commentary on Einstein's calculation, see B. H. Lavenda, *Statistical Physics; A Probabilistic Approach* (Wiley, New York, 1991), pp. 84–85; A. Pais, Ref. 7, pp. 69–70; Ref. 16, p. 69; Ref. 30, pp. 178–179.
[50]Reference 6, p. 77 of the English translation.
[51]Editorial Note in A. Einstein, *The Collected Papers of Albert Einstein, Vol. 2, The Swiss Years: Writings 1900–1909*, edited by J. Stachel *et al.* (Princeton U.P., Princeton, 1989), p. 138.



Triode Demonstrator. By the time this apparatus appeared in the 1936 Central Scientific Company, the triode vacuum tube was being studied by introductory physics students. Rheostats control the current through the filament and the potential on the grid, and there are connections for the plate power supply, the plate voltmeter, the plate current and the grid potential. The tube was a standard four-pin triode of the day, a type 201A with two pins for the filament, and pins for the grid and plate. Cenco patented this design, which has the wiring diagram laid out on top to aid the student. The cost was $14.50. This apparatus is in the Greenslade Collection. (Photograph and notes by Thomas B. Greenslade, Jr., Kenyon College)