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Experiments of the EPR Type Involving *CP*-Violation Do not Allow Faster-than-Light Communication between Distant Observers (§).

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Abstract. – The proof that faster-than-light communication is not permitted by quantum mechanics derived some years ago by three of us is extended to cover the case of measurements which do not fit within the standard scheme based on sets of orthogonal projections. A detailed discussion of a recent proposal of superluminal transmission making resort to a *CP*-violating interaction is presented. It is shown that such a proposal cannot work.

The possibility of faster-than-light communication through the collapse of the wave function in experimental set-ups of the EPR type is, at a superficial analysis, very tempting and proposals of such a type of transmission appear from time to time in the literature. In the past there have been various suggestions [1] of practical or *gedanken* experiments of this type. However, it has to be remarked that a completely general and rigorous proof that superluminal communication is impossible if quantum postulates are consistently used has been given some years ago [2].

In spite of this fact, in a recent paper [3] it has been suggested once more that faster-than-light signalling should be possible by using a new EPR-type set-up, and this proposal has obtained some resonance as *undermining Einstein* [4]. According to the authors of ref. [3], the novelty of their argument derives from the fact that they consider a new situation, in which unstable particles decay weakly through a *CP*-violating interaction giving rise to a final state in which superpositions of nonorthogonal states are involved. This possibility, according to the authors, has not yet been properly taken into account and applicability of the theorem of ref. [2] «*for the case of the correlated pair consisting of weakly decaying particles, taking into account CP noninvariance, has been hitherto left unanalyzed*».

Due to the intrinsic interest of this type of investigations, we feel the necessity of discussing in detail the arguments of ref. [3]. This will give us also the opportunity of

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generalizing the results of ref. [2], obtaining in this way a deeper understanding of the subject.

We start by reconsidering the standard quantum description of measurement processes, as well as the derivation of the theorem proved in ref. [2].

Standard quantum theory associates to any observable Γ a family of projection operators $P(E)$, for any Borel set $E \in \boldsymbol{R}$. If one considers a nonselective measurement of Γ testing to which of the sets E_i ($\cup E_i = \boldsymbol{R}$, $E_i \cap E_j = 0$) the obtained results belong, the reduction of the wave packet takes place, *i.e.* the state ρ changes according to

$$\rho \to \tilde{\rho} = \sum_i P(E_i)\rho P(E_i) . \tag{1}$$

Using eq. (1) it is a simple matter to show that in the case of an EPR-type set-up in which one is dealing with a composite system $S = S_{\mathrm{L}} + S_{\mathrm{R}}$, measurements on one of the subsystems cannot influence the physics of the other subsystem. In fact, let us denote by $\rho_{\mathrm{L,R}}$ the statistical operator of system S before the measurement. Then, if we measure an observable Γ_{R} of S_{R}, eq. (1) gives

$$\rho_{\mathrm{L,R}} \to \tilde{\rho}_{\mathrm{L,R}} = \sum_i P_{\mathrm{R}}(E_i)\rho_{\mathrm{L,R}} P_{\mathrm{R}}(E_i) , \tag{2}$$

where the operators $P_{\mathrm{R}}(E)$ are projection operators of the Hilbert space H_{R} of S_{R}. To derive physical predictions about successive measurement on S_{L} in the absence of any knowledge of the results of the measurement on S_{R}, one has to use the statistical operator $\tilde{\rho}_{\mathrm{L}}$ of H_{L}, obtained by taking the partial trace of $\tilde{\rho}_{\mathrm{L,R}}$ on the Hilbert space H_{R}:

$$\tilde{\rho}_{\mathrm{L}} = \mathrm{Tr}_{\mathrm{R}}\,\tilde{\rho}_{\mathrm{L,R}} = \mathrm{Tr}_{\mathrm{R}} \sum_i P_{\mathrm{R}}(E_i)\rho_{\mathrm{L,R}} P_{\mathrm{R}}(E_i) = \mathrm{Tr}_{\mathrm{R}} \sum_i P_{\mathrm{R}}(E_i)\rho_{\mathrm{L,R}} = \mathrm{Tr}_{\mathrm{R}}\,\rho_{\mathrm{L,R}} = \rho_{\mathrm{L}} , \tag{3}$$

where ρ_{L} is the statistical operator one should use to describe system S_{L}, if the measurement on S_{R} had not been performed. In getting eq. (3) we have used the fact that the operators $P_{\mathrm{R}}(E_i)$ are idempotent and sum up to the identity operator of H_{R}. Equation (3) shows that all physical results of perspective measurements on S_{L} remain the same independently of having performed or not a measurement on S_{R}; this means that no action at a distance can be induced by the reduction of the wave packet.

Let us come now to discuss the arguments of ref. [3]. The authors consider a composite system $S = S_{\mathrm{L}} + S_{\mathrm{R}}$ which, at time $t = 0$, is described by the state vector

$$|\Psi(0)\rangle = [|M^0\rangle_{\mathrm{L}}|\bar{M}^0\rangle_{\mathrm{R}} - |\bar{M}^0\rangle_{\mathrm{L}}|M^0\rangle_{\mathrm{R}}]/\sqrt{2} , \tag{4}$$

where $|M^0\rangle$ and $|\bar{M}^0\rangle$ are orthonormal states corresponding to systems which decay via a weak CP-violating interaction into decay products represented by nonorthogonal states $|\Phi_{\mathrm{l}}\rangle$ and $|\Phi_{\mathrm{s}}\rangle$. The state (4) evolves then naturally in a state (eq. (5) of ref. [3]) which can be written as

$$|\Psi(t)\rangle = |a(t)\rangle_{\mathrm{L}}|M^0\rangle_{\mathrm{R}} + |b(t)\rangle_{\mathrm{L}}|\bar{M}^0\rangle_{\mathrm{R}} + |c(t)\rangle_{\mathrm{L}}|\Phi_{\mathrm{l}}(t)\rangle_{\mathrm{R}} + |d(t)\rangle_{\mathrm{L}}|\Phi_{\mathrm{s}}(t)\rangle_{\mathrm{R}} , \tag{5}$$

where the states $|M^0\rangle$ and $|\bar{M}^0\rangle$ are orthogonal to $|\Phi_{\mathrm{l}}(t)\rangle$ and $|\Phi_{\mathrm{s}}(t)\rangle$, while, as already stated, $\langle\Phi_{\mathrm{l}}(t)|\Phi_{\mathrm{s}}(t)\rangle \neq 0$. The authors of ref. [3] argue then as follows:

1) A measurement is performed on the system R with three possible outcomes: undecayed M^0, $\bar{M}^0$; decay products $|\Phi_{\mathrm{l}}(t)\rangle_{\mathrm{R}}$; decay products $|\Phi_{\mathrm{s}}(t)\rangle_{\mathrm{R}}$.

2) According to the authors, due to the nonorthogonality of the states Φ, the collapse is only partial, and «*the exact treatment of this partial collapse within the framework of the standard quantum measurement theory is unclear*».

3) Owing to this fact, they assume that state (5) collapses to a mixture of its components in the three manifolds: *a*) spanned by $|M^0\rangle_R$ and $|\bar{M}^0\rangle_R$; *b*) of the state $|\Phi_l(t)\rangle_R$ and *c*) of the state $|\Phi_s(t)\rangle_R$.

4) The authors then prove that the physics of the L-system (in particular its content of $|M^0\rangle_L$ and $|\bar{M}^0\rangle_L$ states) as given by the so-introduced mixture is different from the one which is implied by state (5).

In order to understand whether the argument leading to the conclusion $\tilde{\rho}_L = \rho_L$ can be repeated or not in the present situation, one has to make precise the type of measurement described under 1).

We start by considering a physically natural possibility. Since the two states $|\Phi_l(t)\rangle$ and $|\Phi_s(t)\rangle$ have overlapping invariant mass distributions, an experimenter could define two nonoverlapping energy windows Δ_l and Δ_s, centred around the two peaks of the corresponding distributions and would say that he has detected l or s decay products according whether the energy is found to belong to Δ_l or to Δ_s, respectively. Standard quantum mechanics is perfectly able to describe such a situation using orthogonal projection operators on the four manifolds: the one spanned by the $|M^0\rangle$ and $|\bar{M}^0\rangle$ states, the one corresponding to the energy window Δ_l, the one corresponding to Δ_s and the manifold orthogonal to these last two.

Other possibilities in the framework of the standard formalism based on orthogonal projections to describe measurements could also be considered. In all cases, as shown by the previously proved theorem, the result $\tilde{\rho}_L = \rho_L$ remains valid.

But one can also enrich the argument. In fact, the authors of ref. [3] state that a key point in the theorem derived in ref. [2] is that «*it relies on the condition that the measurement alluded to involves collapse of the pure state wave function to a mixture of mutually orthogonal states*». This is true, and we have used such states also in the previous discussion. So, it is clear that the authors have in mind measurements leading to statistical mixtures of nonorthogonal states. It is important to stress at this point that such kinds of measurements have been already considered and have been extensively discussed in the literature by various authors in a series of important papers [5]. Actually the quantum formalism has been enlarged long ago to take into account all those types of measurements which do not fit naturally within the scheme based on sets of orthogonal projections. The reasons for enlarging the scheme were both of practical nature (to describe nonideal measurements, approximate measurements or measurements in which the apparatuses have not efficiency one), and of mathematical and logical nature (to have a richer, completely consistent and rigorous scheme). In the new scheme the description of measurement processes based on orthogonal projections is substituted with the one based on the concept of operation. The key point is that one cannot invent arbitrary reductions without violating some essential requirements of consistency. We refer the reader to the analysis of this point given by Kraus in ref. [6].

We can then state that there already exists a perfectly defined generalization of the standard reduction process which covers practically any conceivable measurement, including the case of measurements which do not correspond to orthogonal projections.

As has been proved in ref. [6], the change induced on the statistical operator by any measurement process can be expressed in the form

$$\rho_{L,R} \to \tilde{\rho}_{L,R} = \sum_i A(i)\rho_{L,R}A^\dagger(i)\,, \tag{6}$$

where the $A(i)$'s are bounded operators on the Hilbert space of the measured system satisfying

$$\sum_i A^\dagger(i)A(i) = I . \tag{7}$$

Obviously, if the measurement involves only system R, the operators $A(i)$ act on the Hilbert space of this system alone. One can then apply to eq. (6) the same argument used for obtaining eq. (3) from eq. (2). We have

$$\tilde{\rho}_L = \mathrm{Tr}_R\,\tilde{\rho}_{L,R} = \mathrm{Tr}_R\left\{\sum_i A_R(i)\,\rho_{L,R}\,A_R^\dagger(i)\right\} = \\ = \mathrm{Tr}_R\left\{\sum_i A_R^\dagger(i)\,A_R(i)\,\rho_{L,R}\right\} = \mathrm{Tr}_R\,\rho_{L,R} = \rho_L . \tag{8}$$

Equation (8) shows again that even when, to describe measurements, use is made of the generalized formalism which takes into account the possibility of reduction on nonorthogonal manifolds, one cannot influence the physics of system L by measurements on system R.

Of course, a precise specification of the operators $A_R(i)$ requires a precise specification of the measurement which is being performed and an accurate analysis of it. With reference to the state (5), a possible model for the reduction process could be obtained by the following choice:

$$\begin{cases} A_R(1) = |M^0\rangle\langle M^0| + |\bar{M}^0\rangle\langle\bar{M}^0| , \\ A_R(2) = [1/d]\{[(1+d)/2]^{1/2}|\Phi_1(t)\rangle\langle\Phi_1(t)| - (x/2)[2/(1+d)]^{1/2}|\Phi_1(t)\rangle\langle\Phi_s(t)|\} , \\ A_R(3) = [1/d]\{[(1+d)/2]^{1/2}|\Phi_s(t)\rangle\langle\Phi_s(t)| - (x^*/2)[2/(1+d)]^{1/2}|\Phi_s(t)\rangle\langle\Phi_1(t)|\} , \end{cases} \tag{9}$$

where

$$x = \langle\Phi_1(t)|\Phi_s(t)\rangle \qquad \text{and} \qquad d = (1-|x|^2)^{1/2}, \tag{10}$$

all states in the above equations being referred to the R-system. It is easily checked that the requirement (7) is fulfilled by the above choice, I being now the identity in the manifold spanned by $|M^0\rangle$, $|\bar{M}^0\rangle$, $|\Phi_1(t)\rangle$, $|\Phi_s(t)\rangle$. It is seen that $A_R(2)$ and $A_R(3)$ reduce to the projections $|\Phi_1\rangle\langle\Phi_1|$ and $|\Phi_s\rangle\langle\Phi_s|$, respectively, in the limit in which $|\Phi_1\rangle$ and $|\Phi_s\rangle$ become orthogonal. If use is made of operators (9), no superluminal transmission can be obtained, as implied by eq. (8).

It is then useful to present a more detailed analysis which makes more clear the origin of the curious result of ref. [3]. To this purpose one can consider a normalized state of the type

$$|\Psi\rangle = c_1|\alpha\rangle_L|\beta\rangle_R + c_2|\beta\rangle_L|\alpha\rangle_R\} , \tag{11}$$

where the states $|\alpha\rangle$ and $|\beta\rangle$ are normalized but not orthogonal. According to quantum mechanics, such a state is perfectly admissible for any composite system. The assumption made by the authors of ref. [3] corresponds to introducing a reduction mechanism leading from state (11) to a statistical mixture of the two states which appear in it. Let us consider the most general mixture of this type

$$\tilde{\rho}_{L,R} = p_1|\alpha\rangle_{L\,L}\langle\alpha|\otimes|\beta\rangle_{R\,R}\langle\beta| + p_2|\beta\rangle_{L\,L}\langle\beta|\otimes|\alpha\rangle_{R\,R}\langle\alpha| \tag{12}$$

with p_1 and p_2 two nonnegative numbers summing up to 1. Using eq. (12) it is immediately shown that

$$\mathrm{Tr_R}|\Psi\rangle\langle\Psi| \neq \mathrm{Tr_R}\,\tilde{\rho}_{\mathrm{L,R}}\,, \tag{13}$$

for any choice of p_1 and p_2. However, due to the preceding result, one can argue that such a reduction mechanism must contradict some of the requirements used to derive eq. (8).

To clarify this point we shall show, by means of a simple example, that a correct reduction mechanism leading to eq. (12) can actually be considered within the generalized description of measurement, but that, in such a case, the measured quantity necessarily refers to the whole system $S_\mathrm{L} + S_\mathrm{R}$.

Suppose to have a measuring procedure inducing the following change in the state of a system:

$$\rho_{\mathrm{L,R}} \to \tilde{\rho}_{\mathrm{L,R}} = A(1)\rho A^\dagger(1) + A(2)\rho A^\dagger(2)\,, \tag{14}$$

where

$$A(1) = |\alpha\rangle_\mathrm{L}|\beta\rangle_\mathrm{R}\,\langle\Xi_{\mathrm{L,R}}|\,, \qquad A(2) = |\beta\rangle_\mathrm{L}\,|\alpha\rangle_\mathrm{R}\,\langle\Xi^\perp_{\mathrm{L,R}}|\,, \tag{15}$$

being $|\Xi_{\mathrm{L,R}}\rangle$ and $|\Xi^\perp_{\mathrm{L,R}}\rangle$ any pair of orthogonal states spanning the same linear manifold as $|\alpha\rangle_\mathrm{L}|\beta\rangle_\mathrm{R}$ and $|\beta\rangle_\mathrm{L}|\alpha\rangle_\mathrm{R}$. We note that

$$A^\dagger(1)A(1) + A^\dagger(2)A(2) = I\,, \tag{16}$$

I being the identity operator in the two-dimensional considered space. From eqs. (14) and (15) one gets immediately

$$\tilde{\rho}_{\mathrm{L,R}} = \langle\Xi_{\mathrm{L,R}}|\rho|\Xi_{\mathrm{L,R}}\rangle\,|\alpha\rangle_\mathrm{L}|\beta\rangle_\mathrm{R}\,{}_\mathrm{L}\langle\alpha|_\mathrm{R}\langle\beta| + \langle\Xi^\perp_{\mathrm{L,R}}|\rho|\Xi^\perp_{\mathrm{L,R}}\rangle\,|\beta\rangle_\mathrm{L}|\alpha\rangle_\mathrm{R}\,{}_\mathrm{L}\langle\beta|\,{}_\mathrm{R}\langle\alpha|\,. \tag{17}$$

Equation (17), which fits perfectly the general scheme, is of the form of eq. (12). As one can see, however, the operators $A(i)$ appearing in it act on the whole Hilbert space $H_\mathrm{L}\otimes H_\mathrm{R}$ and not on H_R alone. If one takes into account this fact, one is compelled to conclude that the assumption made by the authors of ref. [3] corresponds to a physical set-up in which one is using a measuring apparatus which interacts with and affects simultaneously both systems L and R. The apparent nonlocal effect is, therefore, due neither to the quantum theory of measurement, nor to the peculiar correlation of the two states, nor to the occurrence of CP-violation, but exclusively to the fact that a precise physical action is simultaneously performed on the two separated subsystems, *i.e.* to the fact that one has two separated apparatuses which interact at the same instant of time with the two subsystems.

The conclusion is that the origin of the curious result that the authors claim could occur is perfectly clear from a physical point of view. No action at a distance is involved, and the physics of the whole process is in complete agreement with relativistic requirements.

Additional Remark. – After this paper had been submitted for publication four papers criticizing the claim of ref. [3] have appeared [7, 8]. Those listed under [8] do not consider the extended formalism for measurement processes, so that their arguments are less general than the one presented here. The treatment of ref. [7] is very close, in its basic points, to the one given here. However, since we present explicit examples of reduction mechanisms leading to mixtures of nonorthogonal states, our analysis makes more clear, from a physical point of view, the mistake made in ref. [3].

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