QUANTUM ATOM Frank Wilczek celebrates the mysterious electron **p.31**

GENOMICS Is personalized medicine squeezing out public health? **p.34**

ECONOMICS On the complex web of interactions that gives money meaning **p.35**

FILM Charting the exploitation of a fishery in the pristine Ross Sea **p.36**

UHLENBECK COLLECTION/AMERICAN INST. OF PHYSICS/SPL

Niels Bohr and his wife Margrethe around 1930.

The path to the quantum atom

John Heilbron describes the route that led Niels Bohr to quantize electron orbits a century ago.

In the autumn of 1911, the Danish physicist Niels Bohr set sail for a post-doctoral year in England inflamed with "all my stupid wild courage", as he expressed his state of mind in a letter to his fiancée, Margrethe Nørlund[1]. Bohr would need that courage on his route to his revolutionary quantum atom of 1913.

Bohr had reason to think himself designed for great things. He had won a gold medal from the Royal Danish Academy of Sciences in 1908, at the age of 23, for a theoretical and experimental study of water jets published by the Royal Society of London. His doctoral thesis on the electron theory of metals was so advanced that no one in Denmark could evaluate it fully.

Bohr went to the University of Cambridge, UK, to work with Joseph John (J. J.) Thomson, famous as the discoverer of the electron and recipient of the Nobel Prize in Physics for 1906. For Bohr, Thomson was "a genius who showed the way to everyone". But Thomson was too full of his own ideas to listen to those of a foreigner whose English ▶

THE QUANTUM ATOM
A *Nature* special issue
nature.com/bohr100

NIELS BOHR ARCHIVE, COPENHAGEN

Niels Bohr (left) with Albert Einstein in 1925.

he had to struggle to understand. "They say he would walk away from the king," Niels wrote to his brother Harald, "which means more in England than in Denmark"[1].

Even if Thomson had been interested, he would have had trouble perceiving that his postdoc was a mature mathematical physicist. Furthermore, Bohr's speciality was criticism. In his thesis work, he had discovered errors in Thomson's papers, which he tried to bring to the professor's attention. That was not the right gambit.

Thomson was preoccupied with developing consequences of the model atom he had proposed in 1903. Later inappropriately and derisively nicknamed a 'plum pudding', it consisted of concentric rings of electrons rotating through a resistanceless spherical space that acted as if it were positively charged. In this picture, Thomson elucidated the periodic properties of the elements, the formation of simple molecules, radioactivity, the scattering of X-rays and β-particles, and the ratio between the weight of an atom and the number of its electrons.

Bohr spent much of his time at Cambridge attending talks and reading widely. He extolled Thomson's lectures and found much to admire in the treatise *Aether and Matter* (1900), in which Joseph Larmor, the occupant of the chair of mathematics once held by Isaac Newton, developed a world system based on electrons conceived as permanent twists in the ether. "When I read something that is so good and grand as that," Bohr wrote to Margrethe[1], "then I feel such courage and desire to try whether I too could accomplish a tiny bit."

NUCLEAR MODEL

In February 1912, Bohr went to the Victoria University of Manchester, UK, to arrange to work on radioactivity in Ernest Rutherford's laboratory. He looked forward to it in his understated way: "My courage is ablaze, so wildly, so wildly"[1]. Rutherford satisfied his expectations: "a really first-rate man and extremely capable, in many ways more able than Thomson, even though perhaps he is not as gifted"[1].

Rutherford certainly surpassed Thomson as a research director. When Bohr arrived, several men in the laboratory were working on implications of the nuclear model of the atom that Rutherford had introduced in 1911. To explain the unexpected reflection of α-particles from thin metal foils, detected by his research students, Rutherford had found it necessary to collect all the positive charge in Thomson's spheres into a tiny kernel at the atom's centre.

Soon Bohr joined in via his natural route: criticism. In calculating the transfer of energy from an α-particle to atomic electrons, Rutherford's theorist Charles Galton Darwin had not taken into account the resonance that occurs when the time of passage of the particle past the atom coincides with the natural frequency at which the perturbed electrons oscillate.

In improving the calculations, Bohr discovered that some modes of oscillation of a ring of electrons in the plane of their orbit grow until they tear the atom apart. This mechanical instability could not be mended by deploying accepted physical concepts. Bohr's thesis work had familiarized him with more general examples of failure in theories of heat radiation and magnetism that allowed electrons all the freedom that statistical mechanics granted them. To his unique way of thinking, the nuclear model appealed to Bohr precisely because it expressed this failure so conspicuously.

The model had further advantages. It made a clear distinction between radioactive and chemical phenomena, which in Bohr's view derived from the nucleus and the electronic structure, respectively. This inference was not as evident then as it is now. Even Rutherford had not yet grasped the distinction, and assigned the origin of β- and γ-rays to extra-nuclear electrons.

Most importantly, the nuclear model, combined with Rutherford's conception of the α-particle as a bare nucleus, almost thrust the concept of atomic number on physicists. They knew that the α-particle was a helium atom minus two electrons; its nucleus must therefore have a charge of two, implying that hydrogen's has a charge of one, lithium's a charge of three, and so on.

His confidence replenished by Rutherford's interest, Bohr drew up a memorandum in June or July 1912 to show how Max

Planck's idea that energy came in packets, or quanta, could extend the purview of the nuclear model to the problems that Thomson had considered, and to fix the size of atoms.

Although most of the memorandum was qualitative, in one essential point Bohr could be exact where Thomson could only estimate. Rutherford's scattering theory and experiments required that, for helium, the atomic weight (4) was twice the number of electrons (2). Thomson could only say, after extensive theoretical and experimental work on the scattering of X-rays and β-particles, that the number of electrons in an element was roughly three times its atomic weight.

After these first easy gains, Niels wrote to Harald, "Perhaps I have found out a little about the structure of atoms. If I should be right, it wouldn't be a suggestion of the nature of a possibility (i.e., impossibility as J. J. Thomson's theory) but perhaps a little bit of reality"[1].

Nonetheless, Bohr followed Thomson's lead in the other subjects he discussed with Rutherford: the periodic properties of the elements, determined by stability requirements imposed on their ring structures, and the binding of atoms into simple molecules, secured by exchanges of electrons.

To proceed with his calculations, Bohr laid down the ad-hoc postulate, conceived in analogy to Planck's radiation theory, that if the kinetic energy of each electron is proportional to the frequency of its orbit, it would neither radiate nor succumb to unstable oscillations, and he guessed that the constant of proportionality was a fraction of Planck's *h*.

BALMER'S NUMEROLOGY

Bohr's three-part paper on the constitution of atoms and molecules was published in the London-based *Philosophical Magazine* between July and November 1913. The second and third parts, which consider the periodic arrangements of the elements and molecular binding, record Bohr's debt to Thomson. Alone they would not have attracted attention or affected a revolution. What made Bohr's 'trilogy' memorable was its first part[2], on the spectrum of hydrogen, a subject he did not confront until February 1913.

A colleague asked him how he explained the formula for the frequencies of a series of spectral lines emitted by hydrogen, for which Johann Jakob Balmer had devised a simple arithmetical formula in 1885. Bohr replied that spectra were too complicated for his model, but had a look anyway. He saw immediately, so he later said, how to calculate the ratio of kinetic energy to orbital frequency for the model he had presented to Rutherford six months earlier (see 'Bohr's key to the microworld').

That early model had only a ground state, in which, by definition, the electrons have radiated away all the energy that nature allows them to dispose of. It could not explain why many frequencies are emitted. Bohr could see the bearing of the Balmer formula so quickly because, around New Year, he had extended his model in response to a remarkable series of papers by John William Nicholson, a mathematical physicist he had met in Cambridge.

Nicholson had matched the frequencies of many unattributed lines in solar and nebular spectra with the oscillations of electrons in a nuclear atom perpendicular to the plane of their ring. Unlike oscillations in the plane, perpendicular ones can be stable. By calculating the frequencies of rotation of his electrons from the spectra, he could compute their angular momenta. He found that, very closely, the angular momentum of each of his electrons was a small integral multiple of $h/2\pi$.

Nicholson's finding followed the lead of the *Conseil de Physique Solvay* of 1911, the conference at which Planck, Rutherford, Albert Einstein, Hendrik Antoon Lorentz and other luminaries considered problems in the theory of radiation. Discussion centred on Planck's concept of energy quanta: the simple harmonic oscillators by which he

THE BALMER FORMULA

Bohr's key to the microworld

The Balmer formula expresses the frequencies of some lines in the spectrum of hydrogen in simple algebra:

$$\nu_n = R(1/2^2 - 1/n^2)$$

where ν_n is the *n*th Balmer line and *R* is the universal Rydberg constant for frequency, named in honour of the Swedish spectroscopist Johannes Rydberg, who generalized Balmer's formula to apply to elements beyond hydrogen.

Following Max Planck's radiation theory, Niels Bohr converted the equation into units of energy, by multiplying both sides by Planck's constant, *h*. This allowed him to identify the energy of the electron in its *n*th state with the second term, $-Rh/n^2$. The first term would then be the negative of the energy for the second state ($n=2$), and the formula could be read to mean that a Balmer line originates in a jump of a hydrogen electron from its *n*th to its second state.

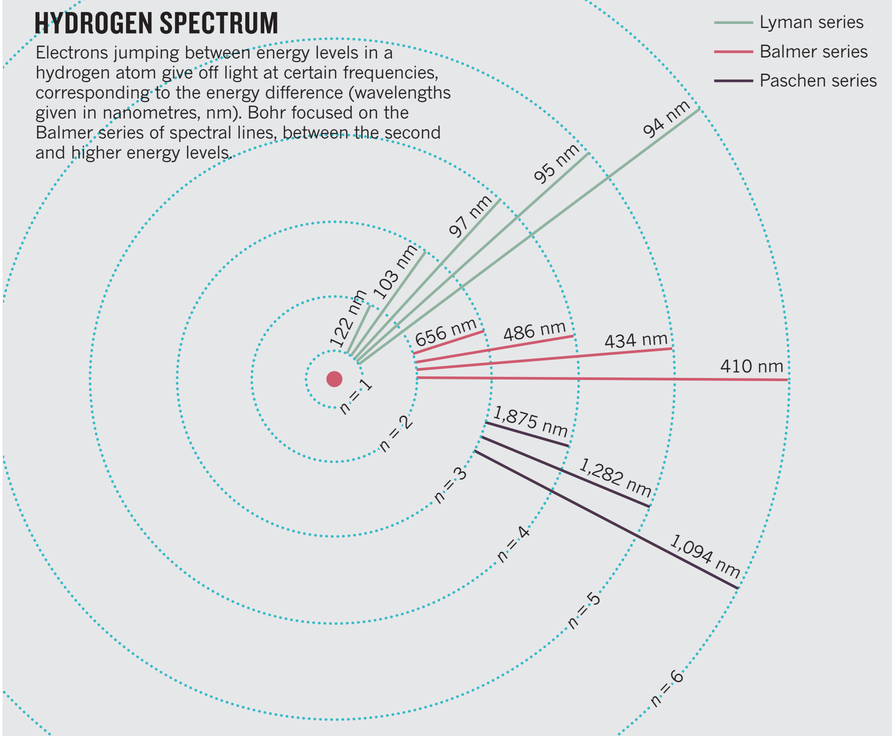
To calculate *R*, Bohr equated the energy of the *n*th state, $-Rh/n^2$, with the expression he already had for the kinetic energy T_n of an orbiting electron in his quantized model of the nuclear atom:

$$T_n = 2\pi^2 me^4/h^2n^2$$

where *e* and *m* are the charge and mass of the electron. Equating the two energies, Bohr had *R* in terms of the fundamental constants.

HYDROGEN SPECTRUM

Electrons jumping between energy levels in a hydrogen atom give off light at certain frequencies, corresponding to the energy difference (wavelengths given in nanometres, nm). Bohr focused on the Balmer series of spectral lines, between the second and higher energy levels.



represented material particles able to emit and absorb radiation possess energies only in integral multiples of their frequencies. An oscillator could emit or absorb radiation when its natural frequency equalled that of the radiation (v), and then only in energy increments ($E = hv$), or quanta.

Because Nicholson's matches were astonishingly exact, and his model, like Bohr's, was both nuclear and quantized, Bohr had to take it seriously. Still reluctant to investigate spectra, he compromised, imagining that a captured electron occupied a sequence of excited states, radiating energy from each in Nicholson's manner as it descended towards the nucleus and the ground state.

Bohr introduced a running integer (n) into his model to handle the ascending scale of electron energies. By making the kinetic energy of the nth orbit proportional to n times the orbital frequency, Bohr easily obtained Nicholson's result about angular momentum and the additional information that the constant of proportionality was $h/2$. Thus Bohr had an integer-based series in his mind when he glimpsed the Balmer formula.

Using the relation $E = hv$, Bohr transformed arithmetic into physics by multiplying Balmer's formula by Planck's constant, h. Making it into an energy equation allowed Bohr to identify the kinetic energies of the various states with corresponding terms in the altered formula. That enabled him to derive the parameter in the Balmer formula, known as the Rydberg constant in terms of Planck's constant and the charge and mass of the electron.

The successful computation of the Rydberg constant demanded serious sacrifices from physicists. It made a Balmer line originate in a jump of an electron to the second orbit from a higher one, and put the explanation of such jumps beyond the reach of physics. Rutherford spotted this immediately: in order to 'vibrate' at the appropriate frequency, an electron would have to know where it would stop before it leapt. He was unwilling to concede foreknowledge to electrons or admit frequencies without vibrations.

Bohr replied that physicists must "renounce" — a word he came to use frequently — the possibility of exact descriptions of certain processes in the microworld.

Einstein perceived a greater loss. Planck had equated the frequencies of radiated light and mechanical oscillation. This was possible because the frequency of a simple harmonic oscillator is the same regardless of its energy. The oscillations of the radiator directly excited the 'ether', or the radiation field. But Bohr's jumps involved two orbits of different periods. The frequency of light emitted did not correspond with the motions of the electron supposed

To develop his model, Bohr followed an analogy to the radiation theory of Max Planck (right).

NIELS BOHR ARCHIVE, COPENHAGEN

to produce it, contrary to the concepts by which physicists usually dealt with radiation.

Bohr's sense of responsibility directed him to attempt to anchor the basic postulate of his quantum atom — that the ratio of kinetic energy to orbital frequency in the nth state is proportional to $nh/2$ — in deeper foundations. He did not find the job easy. The first instalment of the trilogy contains four distinct, and largely contradictory, attempts.

Two of them develop the analogy to Planck's radiation theory that provided the form of Bohr's postulate. The third foundation is altogether different. It requires that in jumps between very large neighbouring orbits, where the electron is almost free from the nucleus, the radiation frequency is asymptotically equal to the frequency of the orbits, which are asymptotically equal to one another. This anticipates Bohr's correspondence principle, according to which, at an appropriate limit, calculations of a physical quantity must give the same numerical result in ordinary physics and in quantum theory.

By the end of 1913, Bohr had given up the Planck pedigree as "misleading" (the nuclear atom is not a simple harmonic oscillator) and adopted the correspondence principle as his preferred foundation. He also retained the fourth formulation, the only one now remembered: the quantization of the angular momentum (which follows from the basic postulate by replacing the ratio of kinetic energy to orbital frequency by its mechanical equivalent, π times angular momentum). As a condition on the orbit, the fourth foundation differs conceptually from the other three, which relate the orbit to the radiation emitted by an electron undergoing a quantum jump.

OPEN TO AMBIGUITY

Bohr's ability to entertain several conflicting ideas, and his courage in demanding sacrifices of physicists like Einstein, Planck and Lorentz, are breathtaking. We know that he did not lack confidence. Blazing courage is one thing, but tolerance of ambiguity is quite another.

Correspondence with his immediate family, especially Margrethe, suggests sources for this tolerance. Well before he became entangled in the quantum atom, Bohr had developed a doctrine of multiple partial truths, each of which contained some bit of reality, and all of which together might exhaust it. "There exist so many different truths," he wrote to Margrethe. "I can almost call it my religion, that I think that everything that is of value is true."[1]

Bohr's seminal analysis of the Balmer spectrum expressed the partial truth of Planck's radiation theory and the partial truth of classical physics. Bohr may have owed his notion of partial truth at least in part to ideas he found in the writings of his professor of philosophy, Harald Høffding, and in William James's *Pragmatism*, published in 1907, which Bohr may have known from Høffding.

Family letters hitherto unavailable, which will be published in part next month by Finn Aaserud and myself in *Love, Literature, and the Quantum Atom*[1], open new directions in which to explore this connection, about which historians and philosophers have speculated on the basis of the later and slighter evidence of Bohr's principle of complementarity.

Between periods in which his courage blazed and his blood boiled, Bohr was subject to the sorts of self-doubt that ordinary people have. As their correspondence shows, Margrethe played an important, perhaps an essential, part in smoothing out Niels' mood swings and reassuring him that he was the great man his Danish support system took him to be.

In many letters he asks her to help him pay his debts, by which he meant the obligations he felt he owed for his great gifts, for the encouragement he had received to develop them and, perhaps, for the wider perspectives he gained in England. He could discharge these debts only by great deeds. He made a huge down payment to these imaginary creditors — including Thomson and Rutherford — with his revolutionary quantum atom of 1913. ■

John L. Heilbron *is emeritus professor of history at the University of California, Berkeley, USA.*
e-mail: john@heilbron.eclipse.co.uk

1. Aaserud, F. & Heilbron, J. L. *Love, Literature, and the Quantum Atom: Niels Bohr's 1913 Trilogy Revisited* (Oxford University Press, 2013).
2. Bohr, N. *Philos. Mag.* **26,** 1–25 (1913).