18.

(a) A partir de la ecuación de Schrödinger, y teniendo en cuenta que $\hat{H}$ es hermítico, demostrar que:

$$\frac{d\langle A\rangle}{dt} = \frac{1}{i\hbar}\left\langle\left[\hat{A},\hat{H}\right]\right\rangle + \left\langle\frac{\partial \hat{A}}{\partial t}\right\rangle$$

(b) Con el resultado del punto anterior hallar $\langle\dot{x}\rangle$ y $\langle\dot{p}\rangle$ (teorema de Ehrenfest). Interpretar físicamente el resultado.

$$\frac{d}{dt}\langle A\rangle = \frac{d}{dt}\left(\int \Psi^* A \Psi \, dx\right) =$$

$$= \int \left(\underbrace{\frac{\partial}{\partial t}\Psi^*}_{= \frac{i}{\hbar}\Psi^* H} A\Psi + \underbrace{\Psi^* \frac{\partial A}{\partial t}\Psi}_{\left\langle \frac{\partial A}{\partial t}\right\rangle} + \Psi^* A \underbrace{\frac{\partial \Psi}{\partial t}}_{= -\frac{i}{\hbar} H\Psi} \right) dx =$$

$$= \frac{i}{\hbar}\int \left(\Psi^* H A \Psi - \Psi^* A H \Psi\right) dx + \left\langle \frac{\partial A}{\partial t}\right\rangle =$$

$$= \frac{i}{\hbar}\left\langle [H,A]\right\rangle + \left\langle \frac{\partial A}{\partial t}\right\rangle$$

$$\frac{d\langle x\rangle}{dt} = \frac{1}{i\hbar}\left\langle [x,H]\right\rangle + \overbrace{\left\langle \frac{\partial x}{\partial t}\right\rangle}^{=0} =$$

$$= \frac{\langle p\rangle}{m}$$

$$\frac{d\langle p\rangle}{dt} = \frac{1}{i\hbar}\langle[p,H]\rangle + \overbrace{\left\langle \frac{\partial p}{\partial t}\right\rangle}^{=0} =$$

$$= \frac{1}{i\hbar}\langle[p,V(x)]\rangle =$$

$$H = \frac{p^2}{2m} + V(x)$$

$$= -\int \psi^* \underbrace{\frac{\partial}{\partial x}\left(V\psi\right)}_{\frac{\partial V}{\partial x}\psi + V\frac{\partial \psi}{\partial x}} d_x + \int \psi^* V \frac{\partial \psi}{\partial x} dx =$$

$$= -\left\langle \frac{\partial V}{\partial x}\right\rangle$$