Ejercicio 1 guía 7

1. Verifique el principio de incerteza con los siguientes casos (discuta cuidadosamente el ítem (c)): ¿Cómo expresaría el principio de incerteza en este caso?

   (a) $\psi(x) = A\exp(-a^2x^2)$

   (b) $\psi(x) = \begin{cases} A\exp(ax) & x \leq 0 \\ A\exp(-ax) & x \geq 0 \end{cases}$

   (c) $\psi(x) = \begin{cases} A\exp\left(ip_0x/\hbar\right) & -a/2 < x < a/2 \\ 0 & si\,no \end{cases}$

• REPASEMOS QUE ES LA FUNCIÓN DE ONDA Y COMO VAMOS DE ESPACIO DE POSICIONES A MOMENTOS.

$$\Delta x \Delta p \geq \hbar/2$$

$\psi(x)$ ⤳ FUNCIÓN DE ONDA $\psi_{(x)} \in \mathbb{C}$

$$\rho(x) = |\psi(x)|^2 = \psi(x)\,\psi^*(x) \qquad \rho(x) \in \mathbb{R}$$

⤳ $|\psi(x)|^2 dx\,dy\,dz$ ⤳ PROBABILIDAD

$$\int_{-\infty}^{+\infty} \rho(x)\,dx = \int_{-\infty}^{+\infty} dx\,\psi\,\psi^* = 1$$ ⤳ REQUIERE NORMALIZACIÓN

$$\psi(x) \longrightarrow \phi(p) \qquad |\phi|^2 = \phi\phi^*$$

$|\phi|^2\, dp_x dp_y dp_z$ = PROBABILIDAD $\underline{p}, \underline{p}+d\underline{p}$

$$\begin{cases} \phi(p,t) = \mathcal{F}[\psi(x,t)](p,t) = \dfrac{1}{\sqrt{2\pi\hbar}} \displaystyle\int_{-\infty}^{+\infty} \psi(x,t)\, e^{-i\frac{p}{\hbar}x}\, dx \quad \leftarrow k = p/\hbar \\ \psi(x,t) = \mathcal{F}^{-1}[\phi(p,t)](x,t) = \underbrace{\dfrac{1}{\sqrt{2\pi\hbar}}} \displaystyle\int_{-\infty}^{+\infty} \phi(p,t)\, e^{+i\frac{p}{\hbar}x}\, dp \end{cases}$$

$$2\pi\hbar = h$$

$$\psi(x) = A\, e^{-a^2x^2}$$

① NORMALIZAR

$$1 = \int_{-\infty}^{+\infty} \psi^*\psi\, dx = A^2 \int_{-\infty}^{+\infty} e^{-2a^2x^2} dx = A^2 \sqrt{\frac{\pi}{2}}\,\frac{1}{a}$$

$$A^2 = \sqrt{\frac{2}{\pi}}\, a$$

2 CALCULAR $\phi(p)$

$$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} dx \; \underbrace{A\, e^{-\left(a^2x^2 + \frac{ipx}{\hbar}\right)}}_{\psi(x)\, e^{-ipx/\hbar}} = ★$$

$$-a^2x^2 - \frac{iP}{\hbar}x - \frac{p^2}{4\hbar^2a^2} + \frac{p^2}{4\hbar^2a^2} = -\left[ax + \frac{iP}{2a\hbar}\right]^2 - \frac{p^2}{4\hbar^2a^2}$$

$$★ = \frac{A}{\sqrt{h}} \int_{-\infty}^{+\infty} dx \; e^{-a^2\left[x + \frac{iP}{2a^2\hbar}\right]^2} \cdot \underbrace{e^{-p^2/4\hbar^2a^2}}_{CTE} =$$

$$= \left(\frac{a}{h}\sqrt{\frac{2}{\pi}}\right)^{1/2} e^{-p^2/4\hbar^2a^2} \underbrace{\sqrt{\frac{\pi}{a^2}}}$$

$\phi(p)$

PARSEVAL $\quad \int |\mathcal{F}[f](k)|^2 dk = \int |f_{(x)}|^2 dx$

$\Delta x,\ \Delta p \qquad \Delta f^2 = \langle (f - \langle f\rangle)^2 \rangle = \langle f^2\rangle - \langle f\rangle^2$

$$\left[\ \langle f\rangle = \int dx\ \psi^*_{(x)}\ \hat{f}\ \psi_{(x)}\ \right]$$

TODO EL ESPACIO

3 CALCULAMOS VALORES MEDIOS

$$\langle x\rangle = \int \psi^*_{(x)}\, x\, \psi_{(x)}\, dx$$

$$\langle x\rangle = \int \underbrace{A^2 e^{-2a^2x^2}}_{PAR} \underbrace{x}_{IMPAR} dx = 0$$

$\langle p\rangle = 0 \qquad$ (POR LO MISMO)

$$\langle x^2\rangle = \int A^2 e^{-2a^2x^2} x^2 dx = A^2 \frac{1}{\ldots}\sqrt{\pi} = \frac{1}{\ldots}$$

$2 \quad \sqrt{8a^6} \qquad 4a^2$

$$\langle p^2 \rangle = \int dp \left(\frac{a}{\hbar} \sqrt{\frac{2}{\pi}} \right) e^{-\frac{p^2}{2a^2\hbar}} \sqrt{\frac{\pi}{a^2}}\, p^2 = a^2 \hbar^2$$

$$\Delta x \, \Delta p = \frac{1}{2a} \, a\hbar = \frac{\hbar}{2}$$