Ejercicio 16 guía 9

16. Demostrar que para un átomo en un autoestado descripto por n, ℓ, s, j y m_j vale que ($\ell > 0$):

$$\left\langle \hat{S} \cdot \hat{L} \right\rangle = \begin{cases} \frac{1}{2}\ell\hbar^2 & j = \ell + \frac{1}{2} \\ -\frac{1}{2}\left(\ell + 1\right)\hbar^2 & j = \ell - \frac{1}{2} \end{cases}$$

(Nota: al término $\left\langle \hat{S} \cdot \hat{L} \right\rangle$ se lo conoce como acoplamiento spin-órbita). ¿Qué pasa si $\ell = 0$?

$\hat{H} = \hat{H}_0 \rightsquigarrow \psi_{n,l,s,j,m_j}$

- Accoppiamento spin-orbita $\lambda_{(r)} S \cdot L$

$\tilde{H} = \hat{H}_0 + \hat{H}_1 \rightsquigarrow$ perturbazione H_1 αH_1 α ottico

$\tilde{E} = E_0 + \Delta E \qquad \langle \hat{H}_1 \rangle$

$$\frac{\hat{H}_{SO}}{H_0} \sim \left(\frac{1}{137}\right)^2 \sim 10^{-4}$$

$\langle \hat{S} \cdot \hat{L} \rangle$

!

$|\psi\rangle = |n, l, s, m, m_s\rangle$ ó $|n, l, s, j, m_j\rangle$

$|KET\rangle \to \psi$

$\langle BRA| \to \psi^*$

$\int \psi^* \psi \, dx = \langle BRA | KET \rangle$

$\langle \psi | f | \psi \rangle = \int \psi^* f \psi \, dx = \langle f \rangle$

$\underline{J} = \underline{L} + \underline{S} \qquad J^2 = L^2 + S^2 + \underbrace{2 \underline{L} \cdot \underline{S}}$

$$\underline{L} \cdot \underline{S} = \frac{J^2 - L^2 - S^2}{2}$$

$|\psi\rangle = |n, l, s, j, m_j\rangle$

$\langle \psi | J^2 | \psi \rangle = \hbar^2 j(j+1) \underbrace{\langle \psi | \psi \rangle}_{1} \qquad \int \psi^* \psi \, dx = 1$

$\langle \psi | L^2 | \psi \rangle = \hbar^2 l(l+1)$

$\langle \psi | S^2 | \psi \rangle = \hbar^2 s(s+1)$

$$\langle L \cdot S \rangle = \frac{\hbar^2}{2} \left[j(j+1) - l(l+1) - s(s+1) \right]$$

$$= \frac{\hbar^2}{2} \left[j(j+1) - l(l+1) - 3/4 \right]$$

$s = 1/2$

$$j = \ell + 1/2 \longrightarrow \langle \underline{L}\cdot\underline{S}\rangle = \frac{\ell\hbar^2}{2} \rightsquigarrow \uparrow$$

$$j = \ell - 1/2 \longrightarrow \langle \underline{L}\cdot\underline{S}\rangle = -\frac{1}{2}(\ell+1)\hbar^2 \rightsquigarrow \downarrow$$