

f-3) Pozo finito:

$$\left[\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right] \psi_n(x) = E_n \psi_n(x)$$

$$\frac{\hbar^2}{2m} \frac{d^2 \psi_n}{dx^2} + (E_n - V(x)) \psi_n = 0$$

$$\frac{d^2 \psi_n}{dx^2} + k^2 \psi_n = 0 \rightarrow e^{\pm ikx}$$

I-III: $V=0 \rightarrow k^2 < 0 \rightarrow k = i\gamma \text{ con } \gamma > 0$
 $E_n < 0$

$$\frac{d^2 \psi_n}{dx^2} - \gamma^2 \psi_n = 0 \rightarrow e^{\pm \gamma x}$$

$$\psi_I(x) = A_I e^{\gamma x} + B_I e^{-\gamma x}$$

$$\psi_{II}(x) = A_{II} e^{ikx} + B_{II} e^{-ikx}$$

$$\psi_{III}(x) = A_{III} e^{\gamma x} + B_{III} e^{-\gamma x}$$

Con c.c.: $\psi_I(-\infty) = 0 \rightarrow B_I = 0$

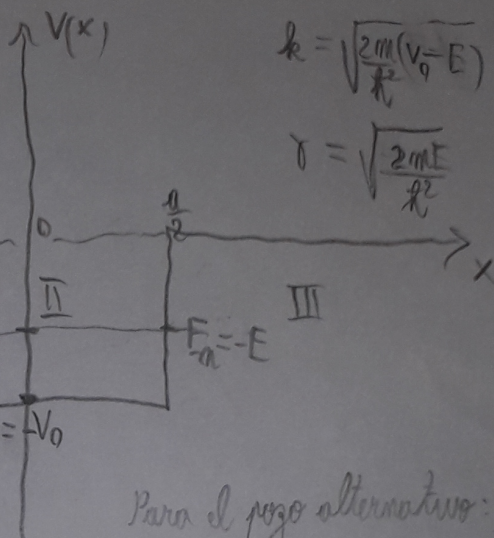
$$\psi_{III}(\infty) = 0 \rightarrow A_{III} = 0$$

$$\psi_I\left(\frac{a}{2}\right) = \psi_{II}\left(\frac{a}{2}\right) \Rightarrow A_I e^{\frac{\gamma a}{2}} = A_{II} e^{\frac{ika}{2}} + B_{II} e^{\frac{ika}{2}} \quad (1)$$

$$\psi_{II}\left(\frac{a}{2}\right) = \psi_{III}\left(\frac{a}{2}\right) \Rightarrow A_{II} e^{\frac{ika}{2}} + B_{II} e^{-\frac{ika}{2}} = B_{III} e^{-\frac{\gamma a}{2}} \quad (2)$$

$$\frac{d\psi_I}{dx}\left(\frac{a}{2}\right) = \frac{d\psi_{II}}{dx}\left(\frac{a}{2}\right) \Rightarrow A_I \gamma e^{\frac{\gamma a}{2}} = ik \left[A_{II} e^{\frac{ika}{2}} - B_{II} e^{\frac{ika}{2}} \right] \quad (3)$$

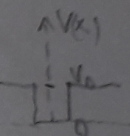
$$\frac{d\psi_{II}}{dx}\left(\frac{a}{2}\right) = \frac{d\psi_{III}}{dx}\left(\frac{a}{2}\right) \Rightarrow ik \left[A_{II} e^{\frac{ika}{2}} - B_{II} e^{-\frac{ika}{2}} \right] = -\gamma B_{III} e^{-\frac{\gamma a}{2}} \quad (4)$$



Para el pozo alternativo:

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\gamma = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$



f3(cont) Hacemos $ik(1) - (3) \rightarrow A_I e^{-\frac{\gamma a}{2}} (ik - \gamma) = B_{II} e^{\frac{ika}{2}} (2ik) \rightarrow$
 $\Rightarrow B_{II} = e^{-\frac{\gamma a}{2} - \frac{ika}{2}} \frac{(ik - \gamma) A_I}{2ik} \quad (*)$

$ik(2) - (4) \rightarrow 2ik B_{II} e^{-\frac{ika}{2}} = B_{III} e^{-\frac{\gamma a}{2}} (ik + \gamma) \rightarrow$
 $\Rightarrow B_{II} = e^{\frac{\gamma a}{2} + \frac{ika}{2}} \frac{(ik + \gamma) B_{III}}{2ik} \rightarrow \otimes \rightarrow$

$\Rightarrow B_{III} = e^{-\frac{\gamma a}{2} - \frac{ika}{2}} \frac{(ik - \gamma) A_I}{(ik + \gamma)} \quad \otimes$

$ik(1) + (3) \rightarrow A_I e^{-\frac{\gamma a}{2}} (ik + \gamma) = 2ik A_{II} e^{-\frac{ika}{2}} \rightarrow$
 $\Rightarrow A_{II} = e^{\frac{\gamma a}{2} + \frac{ika}{2}} \frac{(ik + \gamma) A_I}{2ik} \quad \otimes$

$\gamma(2) + (4) \rightarrow (\gamma + ik) A_{II} e^{-\frac{ika}{2}} + (\gamma - ik) B_{II} e^{-\frac{ika}{2}} = 0$
 $\Rightarrow \frac{A_{II}}{B_{II}} = \frac{-(\gamma - ik) e^{-\frac{ika}{2}}}{(\gamma + ik)} \rightarrow \otimes \rightarrow 1 = \frac{(\gamma - ik)^2}{(\gamma + ik)^2} e^{-2ika} \rightarrow \frac{(\gamma - ik)^2}{(\gamma + ik)^2} = e^{2ika}$

$\rightarrow$ tenemos dos tipos de soluciones para los autovalores:

i) $\frac{\gamma - ik}{\gamma + ik} = -e^{ika} \Rightarrow \frac{\gamma}{k} = \tan \frac{ka}{2}$

$$ii) \frac{\gamma - ik}{\gamma + ik} = e^{ika} \Rightarrow -\frac{k}{\gamma} = \tan \frac{ka}{2} \quad (\text{como el pozo semi-infinito})$$

C.A) trucos para obtener expresiones i) y ii)

$$\left(\frac{i}{i} \right) \frac{\gamma - ik}{\gamma + ik} = \frac{-e^{i\theta}}{e^{-i\theta}} \quad \left(\begin{array}{l} \text{escribí en forma polar el n}^\circ \quad k \pm i\gamma = ne^{\pm i\theta} \\ \text{con } \tan \theta = \frac{\gamma}{k} \end{array} \right)$$

$$\frac{\gamma - ik}{\gamma + ik} = -e^{i2\theta} = -e^{ika} \rightarrow \theta = \frac{ka}{2} \rightarrow \tan \frac{ka}{2} = \frac{\gamma}{k}$$

$$\text{similar si } \frac{\gamma - ik}{\gamma + ik} = e^{ika} \rightarrow \frac{e^{i\alpha}}{e^{-i\alpha}} = e^{i2\alpha} = e^{ika} \rightarrow \alpha = \frac{ka}{2} \quad \text{con } \tan \alpha = -\frac{k}{\gamma}$$

• trucos para obtener las ecuaciones de autovalores en función de senos y cosenos:

$$\text{definimos } k_0 = \sqrt{\frac{2mV_0}{\hbar^2}} = \sqrt{k^2 + \gamma^2} \quad \text{con } \begin{cases} k = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} \\ \gamma = \sqrt{\frac{2mE}{\hbar^2}} \end{cases}$$

$$i) \frac{1}{\cos^2 \frac{ka}{2}} = 1 + \tan^2 \frac{ka}{2} = 1 + \left(\frac{\gamma}{k} \right)^2 = \frac{k^2 + \gamma^2}{k^2} = \frac{k_0^2}{k^2} \rightarrow$$

$$\rightarrow \left| \cos \frac{ka}{2} \right| = \frac{k}{k_0} \quad \text{y si } k_0 \rightarrow \infty \Rightarrow \cos \frac{ka}{2} = 0 \rightarrow k_m = \frac{(2m+1)\pi}{a}$$

(pozo infinito)

$$(\text{con pincho en } 0: k = \sqrt{\frac{2mE}{\hbar^2}}, \gamma = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} \rightarrow \infty)$$

$$\rightarrow E_m = \frac{\hbar^2 k_m^2}{2m}$$

$$ii) \left| \sin \frac{ka}{2} \right| = \frac{k}{k_0} \quad \text{y si } k_0 \rightarrow \infty \Rightarrow \sin \frac{ka}{2} = 0 \rightarrow k_m = \frac{2m\pi}{a}$$

Expreso la multivaluación del n -complejo a través de su fase

$$e^{2ika + 2m\pi} \quad \text{con } m = 0, 1, \dots$$

$$\sqrt[n]{e^{2ika + 2m\pi}} = \sqrt[n]{1} \cdot e^{\frac{2ika + 2m\pi}{n}} = \begin{cases} e^{ika} = e^{ika} \\ e^{ika + \pi} = -e^{ika} \\ \vdots \end{cases}$$