Modelo de Ising

Red de spines $s = \pm 1$ en un arreglo $L \times L$.

$$\mathcal{H} = -J^* \sum_{\langle i,j \rangle} s_i \, s_j - B^* \sum_i s_i \tag{1}$$

donde $J^* = J/KT$ y $B^* = B/KT$.

El caso $J \neq 0$

Cada spin $s_i = \pm 1$ tiene cuatro vecinos $s_j = \pm 1$. Cada inversión de spin varía la energía en

$$\mathcal{H}_{k+1} - \mathcal{H}_k = -J^* \sum_{j=1}^{4} [s_i^{k+1} - s_i^k] \, s_j - B^* \, [s_i^{k+1} - s_i^k] \qquad (2)$$

donde $s_i^{k+1} - s_i^k$ puede tomar únicamente los valores -2,0,+2.

Posibles valores de energía ($J \neq 0$)

$$\mathcal{H}_{k+1} - \mathcal{H}_k = \left[s_i^{k+1} - s_i^k\right]\left[-J^*\left(s_1 + s_2 + s_3 + s_4\right) - B^*\right] \qquad (3)$$

$$s_i^{k+1} - s_i^k = \begin{cases} -2 \\ 0 \\ +2 \end{cases} , \quad s_1 + s_2 + s_3 + s_4 = \begin{cases} -4 \\ -2 \\ 0 \\ +2 \\ +4 \end{cases} \qquad (4)$$

La diferencia $s_i^{k+1} - s_i^k = 0$ no interesa y la eliminamos. La cantidad total de combinaciones es (en principio) $2 \times 5 = 10$

Tabla de energías

Armo una tabla de los posibles $\Delta\mathcal{H}$.

$s_i^{k+1} - s_i^k$	$s_1 + s_2 + s_3 + s_4$	$\Delta\mathcal{H}$
-2	+4	$-2\left(-4 * J^* - B^*\right)$
-2	+2	$-2\left(-2 * J^* - B^*\right)$
-2	0	$-2\left(+0 * J^* - B^*\right)$
-2	-2	$-2\left(+2 * J^* - B^*\right)$
-2	-4	$-2\left(+4 * J^* - B^*\right)$
+2	+4	$+2\left(-4 * J^* - B^*\right)$
+2	+2	$+2\left(-2 * J^* - B^*\right)$
+2	0	$+2\left(+0 * J^* - B^*\right)$
+2	-2	$+2\left(+2 * J^* - B^*\right)$
+2	-4	$+2\left(+4 * J^* - B^*\right)$

Tabla de energías reducida

$(s_i^{k+1} - s_i^k) \cdot (s_1 + s_2 + s_3 + s_4)$	$\omega = \exp(-\Delta\mathcal{H})$
$(-2) \times (+4) = -8$	$\exp(-8 * J^* - 2B^*)$
$(-2) \times (+2) = -4$	$\exp(-4 * J^* - 2B^*)$
$(-2) \times (+0) = +0$	$\exp(-0 * J^* - 2B^*)$
$(-2) \times (-2) = +4$	$\exp(+4 * J^* - 2B^*)$
$(-2) \times (-4) = +8$	$\exp(+8 * J^* - 2B^*)$
$(+2) \times (+4) = +8$	$\exp(+8 * J^* + 2B^*)$
$(+2) \times (+2) = +4$	$\exp(+4 * J^* + 2B^*)$
$(+2) \times (+0) = +0$	$\exp(-0 * J^* + 2B^*)$
$(+2) \times (-2) = -4$	$\exp(-4 * J^* + 2B^*)$
$(+2) \times (-4) = -8$	$\exp(-8 * J^* + 2B^*)$

Tabla de energías reducida

Una forma fácil de indexar esto en un vector es hacer

$$h = i + j \tag{5}$$

donde

$$\begin{cases} j = 2 + (s_1 + s_2 + s_3 + s_4)/4 \\ i = [1 + (s_i^{k+1} - s_i^k)/2]/2 * 5 \end{cases} \tag{6}$$

Indexación de la Tabla

$(s_i^{k+1} - s_i^k) \cdot (s_1 + s_2 + s_3 + s_4)$	h	$\omega = \exp(-\Delta \mathcal{H})$
$(-2) \times (+4) = -8$	0	$\exp(-8 * J^* - 2B^*)$
$(-2) \times (+2) = -4$	1	$\exp(-4 * J^* - 2B^*)$
$(-2) \times (+0) = +0$	2	$\exp(-0 * J^* - 2B^*)$
$(-2) \times (-2) = +4$	3	$\exp(+4 * J^* - 2B^*)$
$(-2) \times (-4) = +8$	4	$\exp(+8 * J^* - 2B^*)$
$(+2) \times (+4) = +8$	10	$\exp(+8 * J^* + 2B^*)$
$(+2) \times (+2) = +4$	9	$\exp(+4 * J^* + 2B^*)$
$(+2) \times (+0) = +0$	8	$\exp(-0 * J^* + 2B^*)$
$(+2) \times (-2) = -4$	7	$\exp(-4 * J^* + 2B^*)$
$(+2) \times (-4) = -8$	6	$\exp(-8 * J^* + 2B^*)$