1. Demostrar las reglas de anticonmutación (conmutación) que satisfacen los operadores de creación y aniquilación de fermiones (bosones).

• Estados en 2ª cuantización: $|m_1, m_2, \dots\rangle$ (B y F)
(con $u_i \to i$ - o $v_i \to i$ -) $|u_i, u_i, \dots, u_j, u_j, \dots, u_\ell, \dots\rangle$ (B) $|u_i, \dots, u_\ell, \dots\rangle$

• En esta descripción son importantes los op. de creación y destrucción:

$$\begin{cases} \hat{a}_{u_i} |m_1, m_2, \dots, m_i, \dots\rangle = \sqrt{m_i}\, |m_1, m_2, \dots, m_i - 1, \dots\rangle \\ a^{+}_{u_i} |m_1, m_2, \dots, m_i, \dots\rangle = \sqrt{m_i + 1}\, |m_1, m_2, \dots, m_i + 1, \dots\rangle \end{cases}$$

¿Qué propiedades satisfacen estos op. para B y F?

BOSONES) NOTACIÓN $a \to b$: $\{b_i, b_i^{+}\}$

$$\left.\begin{aligned} \hat{b}_i^{+} \hat{b}_j^{+} |m_1, m_2, \dots\rangle &= \sqrt{m_i + 1}\sqrt{m_j + 1}\, |m_1, \dots, m_i + 1, m_j + 1, \dots\rangle \\ \hat{b}_j^{+} \hat{b}_i^{+} |m_1, m_2, \dots\rangle &= \sqrt{m_j + 1}\sqrt{m_i + 1}\, |m_1, \dots, m_i + 1, m_j + 1, \dots\rangle \end{aligned}\right\} \Rightarrow \boxed{[\hat{b}_i^{+}, \hat{b}_j^{+}] = 0}$$

$$\left.\begin{aligned} \hat{b}_i \hat{b}_j |m_1, m_2, \dots\rangle &= \sqrt{m_i}\sqrt{m_j}\, |m_1, \dots, m_i - 1, m_j - 1, \dots\rangle \\ \hat{b}_j \hat{b}_i |m_1, m_2, \dots\rangle &= \sqrt{m_j}\sqrt{m_i}\, |m_1, \dots, m_i - 1, m_j - 1, \dots\rangle \end{aligned}\right\} \Rightarrow \boxed{[\hat{b}_i, \hat{b}_j] = 0}$$

o bien: $\hat{b}_i \hat{b}_j = \hat{b}_j \hat{b}_i \Rightarrow (\hat{b}_i \hat{b}_j)^{+} = \hat{b}_j^{+} \hat{b}_i^{+} = (\hat{b}_j \hat{b}_i)^{+} = b_i^{+} b_j^{+}$.

$i \neq j$

$$b_i b_j^+ |n_1, \dots, n_i, \dots, n_j, \dots\rangle = \sqrt{n_i}\sqrt{n_j+1}\,|n_1, \dots, n_i - 1, \dots, n_j+1, \dots\rangle$$

$$b_j^+ b_i |n_1, \dots, n_i, \dots, n_j, \dots\rangle = \sqrt{n_j+1}\sqrt{n_i}\,|n_1, \dots, n_i - 1, \dots, n_j+1, \dots\rangle$$

$\Big\}\ [b_i, b_j^+] = 0$ si $i \neq j$.

$$i = j \Rightarrow \quad b_i b_i^+ |n_1, \dots, n_i, \dots\rangle = \overbrace{\sqrt{n_i+1}\sqrt{n_i+1}}^{(n_i+1)}\,|n_1, \dots, n_i, \dots\rangle$$

$$\underbrace{b_i^+ b_i}_{\hat{n}_i} |n_1, \dots, n_i, \dots\rangle = n_i\,|n_1, \dots, n_i, \dots\rangle$$

$\Big\}\ [b_i, b_i^+] = 1$

$$\therefore \boxed{[b_i, b_j^+] = \delta_{ij}}$$

FERMIONES $\hat{o}_i \rightarrow \{\hat{c}_i, \hat{c}_i^+\}$.

- Suponemos un estado tal que $n_i = n_j = 0$

$$\hat{c}_i^+ \hat{c}_j^+ |n_k, \dots\rangle = \hat{c}_i^+ |1_j, n_k, \dots\rangle = |1_i, 1_j, n_k, \dots\rangle$$

$$\hat{c}_j^+ \hat{c}_i^+ |n_k, \dots\rangle = |1_j, 1_i, n_k, \dots\rangle = -|1_i, 1_j, n_k, \dots\rangle$$

$$\Big\} \Rightarrow \boxed{\{\hat{c}_i^+, \hat{c}_j^+\} = 0}$$

- $(\hat{c}_i^+ \hat{c}_j^+)^+ = -(\hat{c}_j^+ \hat{c}_i^+)^+ \Rightarrow \hat{c}_j \hat{c}_i = -\hat{c}_i \hat{c}_j \Rightarrow \boxed{\{\hat{c}_i, \hat{c}_j\} = 0}$

• Si $n_i = 1$ y $n_j = 0$, para $i \neq j$

$$\hat{c}_j^\dagger \hat{c}_i |n_i, n_k, \dots\rangle = |n_j, n_k, \dots\rangle$$

$$\hat{c}_i \hat{c}_j^\dagger \underbrace{|n_i, n_k, \dots\rangle}_{c_i^\dagger |n_k, \dots\rangle} = \hat{c}_i \underbrace{\hat{c}_j^\dagger \hat{c}_i^\dagger}_{-c_i^\dagger c_j^\dagger} |n_k, \dots\rangle = -\hat{c}_i \hat{c}_i^\dagger |n_j, n_k, \dots\rangle = -|n_j, n_k, \dots\rangle$$

$$\{\hat{c}_i, \hat{c}_j^\dagger\} = 0$$

$i = j$, $n_i = 0$

$$\left.\begin{aligned} \hat{c}_i \hat{c}_i^\dagger |n_k, \dots\rangle &= |n_k, \dots\rangle \\ \hat{c}_i^\dagger \hat{c}_i |n_k, \dots\rangle &= 0 \end{aligned}\right\} \{\hat{c}_i, \hat{c}_i^\dagger\} = 1$$

$$\therefore \boxed{\{\hat{c}_i, \hat{c}_j^\dagger\} = \delta_{ij}}$$