$\boxed{\text{G6 E1}}$ Escribir la ecuación de movimiento de la función de Green de n-partículas usando la ec. (16.10) de Gross, Runge y Heinonen para los casos $n=1,2,3$

$$\left[i\frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} \right] G_n(x_1t_1,\dots,x_nt_n; x_1't_1',\dots,x_n't_n') = \sum_{j=1}^{n} (-1)^{n-j} \delta_{x_1,x_j'}\, \delta(t_1-t_j')\, G_{n-1}(x_2t_2,\dots,x_nt_n; x_1't_1',\dots,x_{j-1}'t_{j-1}', x_{j+1}'t_{j+1}',\dots,x_n't_n')$$

$$- i\int dy\; v(x_1,y)\, G_{n+1}(x_1t_1,\dots,x_nt_n, yt_1; yt_1^+, x_1't_1',\dots,x_n't_n')$$

A destacar:

•) Con este método armamos un sist. de ec. acopladas para las distintas f.d. Green.

•) De igual forma que no podemos calcular exactamente la f.d.o., tampoco la de Green. Pero incluso a ordenes pequeños donde la función de Green tendrá poca información pero relevante (ver Capítulo 16 de Gross)

•) No tomamos en cuenta potencial externo $u(x)$

$$\left[i\frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} \right] G_n(x_1t_1, \dots, x_nt_n ; x_1't_1', \dots, x_n't_n') = \sum_{j=1}^{n} (-1)^{n-j} \delta_{x_1,x_j'} \delta(t_1 - t_j') \, G_{n-1}(x_2t_2, \dots, x_nt_n ; x_1't_1', \dots, x_{j-1}'t_{j-1}', x_{j+1}'t_{j+1}', \dots, x_n't_n')$$

$$- i \int dy \; v(x_1,y) \, G_{n+1}(x_1t_1, \dots, x_nt_n, yt_1 ; yt_1^+, x_1't_1', \dots, x_n't_n')$$

$n=1$ $\quad (x_1,t_1) \to (x,t)$

$$\left[i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m} \right] G_1(xt ; x't') = 1 .\, \delta_{x,x'} \delta(t - t') - i \int dy \; v(x,y) \, G_2(xt, yt ; yt^+, x't')$$

$n=2$

$$\left[i\frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} \right] G_2(x_1t_1, x_2t_2 ; x_1't_1', x_2't_2') = (-1) \delta_{x_1x_1'} \delta(t_1 - t_1') \, G_1(x_2t_2 ; x_2't_2') + 1 \, \delta_{x_1x_2'} \delta(t_1 - t_2') \, G_1(x_2t_2 ; x_1't_1')$$

$$- i \int dy \; v(x_1 y) \, G_3(x_1t_1, x_2t_2, yt_1 ; yt_1^+, x_1't_1', x_2't_2')$$

$$\left[i\frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} \right] G_n(x_1t_1, \dots, x_nt_n; x_1't_1', \dots, x_n't_n') = \sum_{j=1}^{n} (-1)^{n-j} \delta_{x_1,x_j'} \, \delta(t_1-t_j') \, G_{n-1}(x_2t_2, \dots, x_nt_n; x_1't_1', \dots, x_{j-1}'t_{j-1}', x_{j+1}'t_{j+1}', \dots, x_n't_n')$$

$$- i \int dy \; v(x_1,y) \, G_{n+1}(x_1t_1, \dots, x_nt_n, yt_1; yt_1^+, x_1't_1', \dots, x_n't_n')$$

$n=3$

$$\left[i\frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} \right] G_3(x_1t_1, x_2t_2, x_3t_3; x_1't_1', x_2't_2', x_3't_3') = 1 \, \delta_{x_1,x_1'} \, \delta(t_1-t_1') \, G_2(x_2t_2, x_3t_3 \; ; x_2't_2', x_3't_3')$$

$$+ (-1) \, \delta_{x_1,x_2'} \, \delta(t_1-t_2') \, G_2(x_2t_2, x_3t_3 \; ; x_1't_1', x_3't_3')$$

$$+ 1 \, \delta_{x_1,x_3'} \, \delta(t_1-t_3') \, G_2(x_2t_2, x_3t_3 \; ; x_1't_1', x_2't_2')$$

$$- i \int dy \; v(x_1,y) \, G_4(x_1t_1, x_2t_2, x_3t_3, yt_1 \; ; yt_1^+, x_1't_1', x_2't_2', x_3't_3')$$

$\boxed{66\ E2}$ Demostrar que:

$$\hat{\psi}_p(xt)\,\hat{\psi}_p^{\dagger}(yt') = \sum_{\epsilon_j>\epsilon_F} \varphi_j(x)\,\varphi_j^{*}(y)\, e^{i\epsilon_j(t-t')}$$

para empezar recordemos que para partículas $\hat{\Psi}_p(xt) = \sum_{\epsilon_i>\epsilon_F} \varphi_i(x)\, c_i(t)$ con $c_i(t) = c_i\, e^{-i\epsilon_i t}$

$$\Rightarrow \hat{\psi}_p(xt)\,\hat{\psi}_p^{\dagger}(yt') = \left[\sum_{\epsilon_i>\epsilon_F} \varphi_i(x)\, c_i(t)\right]\left[\sum_{\epsilon_j>\epsilon_F} \varphi_j^{*}(y)\, c_j^{\dagger}(t')\right]$$

$$= \sum_{\epsilon_i,\epsilon_j>\epsilon_F} \varphi_i(x)\,\varphi_j^{*}(y)\; c_i(t)\, c_j^{\dagger}(t')$$

y si recordamos

$$c_i(t)\, c_j^{\dagger}(t') = e^{i\epsilon_j(t-t')}\,\delta_{ij}$$

$$\Rightarrow \hat{\psi}_p(xt)\,\hat{\psi}_p^{\dagger}(yt') = \sum_{\epsilon_i\epsilon_j} \varphi_i(x)\,\varphi_j^{*}(y)\, e^{i\epsilon_j(t-t')}\,\delta_{ij} = \boxed{\sum_{\epsilon_j>\epsilon_F} \varphi_j(x)\,\varphi_j^{*}(y)\; e^{i\epsilon_j(t-t')}}$$