GUÍA 6

3) OBTENER LOS 4 PAIRING POSIBLES PARA OP. DE CAMPO

OPERADOR DE CAMPO DE DESTRUCCIÓN DE UN ELECTRÓN EN X

$$\psi(x) = \sum_{\epsilon_i \leq \epsilon_F} \underbrace{\varphi_i(x)\, b_i^+}_{\substack{\text{OP. DE} \\ \text{CREACIÓN HUECOS}}} + \sum_{\epsilon_i > \epsilon_F} \underbrace{\varphi_i(x)\, a_i}_{\substack{\text{OP. DE DESTRUCCIÓN} \\ \text{DE PARTÍCULAS}}}$$

$$\Rightarrow \boxed{\psi(x) = \underbrace{\psi_h^+(x)}_{\substack{\text{OP. CREACIÓN} \\ \text{HUECOS}}} + \underbrace{\psi_p(x)}_{\text{OP. DESTRUCCIÓN PARTÍCULAS}}}$$

OPERADOR DE CAMPO DE CREACIÓN DE UN ELECTRÓN EN X

$$\psi^+(x) = \sum_{\epsilon_i \leq \epsilon_F} \underbrace{\varphi_i^*(x)\, b_i}_{\substack{\text{OP. DE} \\ \text{DESTRUCCIÓN} \\ \text{DE HUECOS}}} + \sum_{\epsilon_i > \epsilon_F} \underbrace{\varphi_i^*(x)\, a_i^+}_{\substack{\text{OP. DE CREACIÓN} \\ \text{DE PARTÍCULAS}}}$$

$$\Rightarrow \boxed{\psi^+(x) = \underbrace{\psi_h(x)}_{\substack{\text{OP. DESTRUCCIÓN} \\ \text{HUECOS}}} + \underbrace{\psi_p^+(x)}_{\text{OP. CREACIÓN PARTÍCULAS}}}$$

PAIRING: $\underbrace{AB} = AB - N(AB)$

$\downarrow$

ORDEN NORMAL

CASOS POSIBLES:

1) $\underbrace{\psi(x)\psi(y)} = \psi(x)\psi(y) - N[\psi(x)\psi(y)]$

REESCRIBIENDO EN TÉRMINO DE LOS OPERADORES DE CREACIÓN Y DESTRUCCIÓN DE PARTÍCULAS Y HUECOS: $\psi(x) = \psi_h^+(x) + \psi_p(x)$

$$\psi(x)\psi(y) = \psi_h^+(x)\psi_h^+(y) + \psi_h^+(x)\psi_p(y) + \psi_p(x)\psi_h^+(y) + \psi_p(x)\psi_p(y)$$

$$N[\psi(x)\psi(y)] = N[\psi_h^+(x)\psi_h^+(y) + \psi_h^+(x)\psi_p(y) + \psi_p(x)\psi_h^+(y) + \psi_p(x)\psi_p(y)]$$

$$= \psi_h^+(x)\psi_h^+(y) + \psi_h^+(x)\psi_p(y) - \psi_h^+(y)\psi_p(x) + \psi_p(x)\psi_p(y)$$

$$\Rightarrow \underbrace{\psi(x)\psi(y)} = \cancel{\psi_h^+(x)\psi_h^+(y)} + \cancel{\psi_h^+(x)\psi_p(y)} + \psi_p(x)\psi_h^+(y) + \cancel{\psi_p(x)\psi_p(y)} - \cancel{\psi_h^+(x)\psi_h^+(y)} - \cancel{\psi_h^+(x)\psi_p(y)} + \psi_h^+(y)\psi_p(x) - \cancel{\psi_p(x)\psi_p(y)}$$

$$\Rightarrow \underbrace{\psi(x)\psi(y)} = \underbrace{\psi_p(x)\psi_n^+(y) + \psi_n^+(y)\psi_p(x)}$$

$$\{\psi_p(x), \psi_n^+(y)\}$$

$$\Rightarrow \boxed{\psi(x)\psi(y) = \{\psi_p(x), \psi_n^+(y)\}}$$

2) $\underbrace{\psi(x)\psi^+(y)} = \psi(x)\psi^+(y) - N[\psi(x)\psi^+(y)]$

$$\psi(x)\psi^+(y) = (\psi_n^+(x) + \psi_p(x))(\psi_n(y) + \psi_p^+(y))$$

$$= \psi_n^+(x)\psi_n(y) + \psi_n^+(x)\psi_p^+(y) + \psi_p(x)\psi_n(y) + \psi_p(x)\psi_p^+(y)$$

$$\Rightarrow \underbrace{\psi(x)\psi^+(y)} = \psi_p(x)\psi_p^+(y) + \psi_p^+(y)\psi_p(x)$$

$$= \{\psi_p(x), \psi_p^+(y)\} = \delta(x-y)$$

3) $\underbrace{\psi^+(x)\psi(y)} = \psi_n(x)\psi_n^+(y) + \psi_n^+(y)\psi_n(x)$

$$= \{\psi_n(x), \psi_n^+(y)\} = \delta(x-y)$$

4) $\underbrace{\psi^+(x)\psi^+(y)} = \cancel{\{\psi_n(x), \psi_p^+(y)\}} \; \psi_n(x)\psi_p^+(y) + \psi_p^+(y)\psi_n(x)$

$$= \{\psi_n(x), \psi_p^+(y)\}$$

4) VERIFICAR EL TEOREMA DE WICK PARA PRODUCTOS DE OPERADORES, CON 2 Y 3 OPERADORES

TEOREMA DE WICK (SIN ORDENAMIENTO TEMPORAL) PARA PRODUCTOS:

$$A_1 A_2 \dots A_m = N(A_1 A_2 \dots A_m) + N(\underbrace{A_1 A_2}_{} A_3 \dots A_m)$$

$$+ N(\underbrace{A_1 A_2 A_3}_{} \dots A_m) + \text{TODOS LOS CASOS 1 PAIRING}$$

$$+ N(\underbrace{A_1 A_2}_{} \underbrace{A_3 A_4}_{} \dots A_m) + \text{TODOS LOS CASOS 2 PAIRING}$$

$$+ \text{TÉRMINOS COMPLETAMENTE APAREADOS}$$

<u>M = 2</u>

$$A_1 A_2 = N(A_1 A_2) + N(\underbrace{A_1 A_2}_{})$$

VERIFICACIÓN (NO DEMOSTRACIÓN)

$A_1 = \ell_1^+$

$A_2 = \ell_2$

$$\Rightarrow \ell_1^+ \ell_2 = N(\ell_1^+ \ell_2) + N(\ell_1^+ \ell_2 - N(\ell_1^+ \ell_2))$$

$$= \ell_1^+ \ell_2 + N(\cancel{\ell_1^+ \ell_2} - \cancel{\ell_1^+ \ell_2})$$

$$= \ell_1^+ \ell_2$$

$$\Rightarrow \boxed{\ell_1^+ \ell_2 = \ell_1^+ \ell_2}$$

$A_1 : \ell_1$

$A_2 : \ell_2^{+}$

$\Rightarrow \ell_1 \ell_2^{+} = N(\ell_1 \ell_2^{+}) + N(\ell_1 \ell_2^{+} - N(\ell_1 \ell_2^{+}))$

$\qquad = -\ell_2^{+} \ell_1 + N(\ell_1 \ell_2^{+} + \ell_2^{+} \ell_1)$

Si $\ell_1 = \ell_2 \Rightarrow \ell_1 \ell_2^{+} + \ell_2^{+} \ell_1 = \ell_1 \ell_1^{+} + \ell_1^{+} \ell_1 = \{\ell_1, \ell_1^{+}\} = 1$

$\Rightarrow \ell_1 \ell_2^{+} = -\ell_2^{+} \ell_1 + N(1)$

$\quad \ell_1 \ell_2^{+} = -\ell_2^{+} \ell_1 + 1$

$\Rightarrow \{\ell_1, \ell_2^{+}\} = 1$ si $\ell_1 = \ell_2$

Si $\ell_1 \neq \ell_2$

$\Rightarrow \ell_1 \ell_2^{+} = -\ell_2^{+} \ell_1 + N(\ell_1 \ell_2^{+} + \ell_2^{+} \ell_1)$

$\qquad = \cancel{-\ell_2^{+} \ell_1 + \ell_2 \ell_2^{+} + \ell_2^{+} \ell_1}$

$\Rightarrow \ell_1 . \ell_2^{+} = -\ell_2^{+} \ell_1 - \cancel{\ell_2^{+} \ell_1} + \cancel{\ell_2^{+} \ell_1}$

$\Rightarrow \{\ell_1, \ell_2^{+}\} = 0$ si $\ell_1 \neq \ell_2$

JUNTANDO LOS DOS CASOS $\{\ell_1, \ell_2^+\} = S_{1,2}$

$M = 3$

$$A_1 A_2 A_3 = N(A_1 A_2 A_3) + N(\underbrace{A_1 A_2}_{} A_3) + N(\underbrace{A_1 A_2 A_3}_{})$$

$$N(\underbrace{A_1 A_2}_{} A_3) = N(\underbrace{A_1 A_2}_{} \underbrace{A_3 \mathbb{1}}_{}) = \underbrace{A_1 A_2}_{} A_3$$

$$N(\underbrace{A_1 A_2 A_3}_{}) = -\underbrace{A_1 A_3}_{} A_2$$

$$\Rightarrow A_1 A_2 A_3 = N(A_1 A_2 A_3) + \underbrace{A_1 A_2}_{} A_3 - \underbrace{A_1 A_3}_{} A_2$$

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$A_1 = \ell_1^+$

$A_2 = \ell_2$

$A_3 = \ell_3$

$$\Rightarrow \ell_1^+ \ell_2 \ell_3 = N(\ell_1^+ \ell_2 \ell_3) + \underbrace{\ell_1^+ \ell_2}_{} \ell_3 - \underbrace{\ell_1^+ \ell_3}_{} \ell_2$$

$$\underbrace{\ell_1^+ \ell_2}_{} = \ell_1^+ \ell_2 - N(\ell_1^+ \ell_2) = 0$$

$$\underbrace{\ell_1^+ \ell_3}_{} = \ell_1^+ \ell_3 - N(\ell_1^+ \ell_3) = 0$$

$$\Rightarrow \ell_1^+ \ell_2 \ell_3 = N(\ell_1^+ \ell_2 \ell_3) = \ell_1^+ \ell_2 \ell_3 \quad ✓$$

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$A_1 = \ell_1$

$A_2 = \ell_2^+$

$A_3 = \ell_3$

$$\Rightarrow \ell_1 \ell_2^+ \ell_3 = N(\ell_1 \ell_2^+ \ell_3) + \underbrace{\ell_1 \ell_2^+}_{} \ell_3 - \underbrace{\ell_1 \ell_3}_{} \ell_2^+$$

$$\underbrace{\ell_1 \ell_2^+}_{} = \ell_1 \ell_2^+ - N(\ell_1 \ell_2^+)$$

$$= \ell_1 \ell_2^+ + \ell_2^+ \ell_1$$

$$\underbrace{\ell_1 \ell_3}_{} = \ell_1 \ell_3 - N(\ell_1 \ell_3) = 0$$

$$\Rightarrow \ell_1 \ell_2^+ \ell_3 = -\cancel{\ell_2^+ \ell_1} \ell_3 + \ell_1 \ell_2^+ \ell_3 + \cancel{\ell_2^+ \ell_1} \ell_3$$

$$\Rightarrow \ell_1 \ell_2^+ \ell_3 = \ell_1 \ell_2^+ \ell_3 \quad \checkmark$$

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$A_1 = \ell_1$

$A_2 = \ell_2$

$A_3 = \ell_3^+$

$$\Rightarrow \ell_1 \ell_2 \ell_3^+ = N(\ell_1 \ell_2 \ell_3^+) + \underbrace{\ell_1 \ell_2}_{} \ell_3^+ - \underbrace{\ell_1 \ell_3^+}_{} \ell_2$$

$$\underbrace{\ell_1 \ell_2}_{} = 0$$

$$\ell_1 \ell_3^+ = \ell_1 \ell_3^+ - N(\ell_1 \ell_3^+)$$

$$= \ell_1 \ell_3^+ + \ell_3^+ \ell_1$$

$$\Rightarrow \ell_1 \ell_2 \ell_3^+ = \cancel{\ell_3^+ \ell_1} \ell_2 - \ell_1 \ell_3^+ \ell_2 - \cancel{\ell_3^+ \ell_1} \ell_2$$

$$\Rightarrow \ell_1 \ell_2 \ell_3^+ = -\underbrace{\ell_1 \ell_3^+}\ell_2$$

$$= -\ell_2 \ell_3^+$$

$$\Rightarrow \ell_1 \ell_2 \ell_3^+ = \ell_1 \ell_2 \ell_3^+ \quad \checkmark$$

Y ASI SUCESIVAMENTE...