A Theory of Lee Cyclogenesis

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(Manuscript received 19 September 1983, in final form 13 December 1983)

ABSTRACT

A theory of lee cyclogenesis is proposed, based on a linearized model of baroclinic wave generation by mountains in the presence of a background shear. The theory predicts that lee cyclogenesis will occur when the criterion for the existence of standing baroclinic lee waves is satisfied in the environment. For an infinite ridge, this requires that the component of wind across the ridge must reverse with height. The time scale for cyclone development, and the meaning of the ambiguous term "lee" are clarified by examining the group velocity of the baroclinic waves. Time dependent three-dimensional solutions are discussed along with their application to Alpine lee cyclogenesis.

1. Introduction

There are a number of special regions in the middle latitudes that experience an abnormally high frequency of cyclogenetic events. Perhaps the most remarkable of these (see the statistical summary of Radinovič and Lalič, 1959) is centered on the Gulf of Genoa just south of the French Alps. The problem of Alpine lee cyclogenesis has been approached using the methods of analysis of standard data (e.g., Buzzi and Tibaldi, 1978; McGinley, 1982), an international field program (Kuettner, 1982), numerical simulation (e.g., Bleck, 1977; Tibaldi *et al.*, 1980; and Mesinger and Strickler, 1982) and hydrodynamical theory (e.g., Illari *et al.*, 1981). In this paper I pursue the latter approach but with a new formulation of the problem.

It is probably productive to think of the lee cyclogenesis problem as a scattering problem in which an incoming synoptic scale baroclinic wave is distorted by the Alps. From this viewpoint, one would expect the orographically generated disturbance to depend on the nature of the incoming wave. Indeed, forecasters in the region have developed a classification of these systems based essentially on the amplitude of the approaching upper-level wave (Fett, 1981).

If the incoming wave has a sufficiently large scale, and sufficiently slow propagation and growth rates, then the Alps would seem to be in a slowly changing, nearly horizontally homogeneous environment. In this limiting case, the scattering problem reduces to a problem of how the Alps would generate a disturbance in a variety of uniform baroclinic basic states. Once this problem is solved, the aspects of the disturbance that have a reasonably small scale (even if it is not the mountain scale) and a reasonably quick response time, might be expected to be observed as the large-scale wave moves slowly through.

The question then is the following: What kind of a disturbance is created by a mountain located in a uniform environment of vertically sheared, geostrophic, baroclinic flow? It will be shown that under special circumstances—a certain angle between the thermal wind vector and a mountain ridge for example—a standing baroclinic wave is produced by the mountain. The time development of this wave is equivalent to the formation of a lee cyclone, where the word "lee" is given a definite meaning in terms of the group velocity of the standing wave.

In Section 2, the theory of quasi-geostrophic baroclinic waves is briefly reviewed. Section 3 describes the orographic forcing of a stationary baroclinic lee wave by a ridge. (An alternative "mountain wave" formulation of this problem is found in Appendix B.) The theory is generalized to include three dimensions and transience in Section 4.

In Section 5, the qualitative predictions of the theory will be compared with one case of Genoa cyclogenesis to put the theory into a meteorological context. Limitations on the theory are discussed in Section 6.

2. Stable baroclinic wave

The term "baroclinic wave" is so often used to imply a growing, baroclinically unstable wave that a careful definition of its use herein is required. We define a baroclinic wave as a surface-trapped wave in a baroclinic current whose restoring force is associated with temperature advection at the boundary. A concise description is given by Gill (1982), using the geostrophic Boussinesq potential vorticity equation on an f-plane

$$\frac{D_g}{Dt}\left(\nabla^2 P + \frac{f^2}{N^2} P_{zz}\right) = 0, \tag{2.1}$$

where P is the pressure, and using the surface temperature advection equation

$$\frac{D_g}{Dt}\theta + w\frac{\partial \theta}{\partial z} = 0, \quad (2.2)$$

where θ is the potential temperature. The formulas for the geostrophic wind components are

$$u_g = -\frac{1}{\rho_0 f}P_y, \quad (2.3a)$$

$$v_g = +\frac{1}{\rho_0 f}P_x. \quad (2.3b)$$

Now consider a basic state in which the velocity $U(z) = U(z)\mathbf{i} + V(z)\mathbf{j}$ is a linear function of height so that using (2.3), the potential vorticity in (2.1) vanishes everywhere. In this case, any perturbation to this state must obey

$$\nabla^2 p' + \frac{f^2}{N^2}p'_{zz} = 0. \quad (2.4)$$

The linearized form of (2.2) is

$$\frac{\partial \theta'}{\partial t} + U_0\frac{\partial \theta'}{\partial x} + V_0\frac{\partial \theta'}{\partial y} + u'_g\frac{\partial \bar\theta}{\partial x} + v'_g\frac{\partial \bar\theta}{\partial y} + w\frac{\partial \bar\theta}{\partial z} = 0. \quad (2.5)$$

If $p'(x, y, z, t)$ is taken of the form $p' = \hat p(z) \times e^{i(kx+ly+\sigma t)}$ then (2.4) requires that

$$\hat p(z) = Ae^{-z/H} + Be^{z/H}, \quad (2.6)$$

where

$$H(k, l) = \frac{f}{N}(k^2 + l^2)^{-1/2}. \quad (2.7)$$

The second term on the right in (2.6) is discarded by setting $B = 0$ as it violates a boundedness condition as $z \to \infty$ in the half-space. With the hydrostatic law,

$$\theta' = \frac{\theta_0}{g\rho_0}\frac{\partial p'}{\partial z},$$

and (2.6), (2.5) becomes

$$\left[(i\sigma + iU_0k + iV_0l)\left(\frac{-1}{H}\frac{\theta_0}{g\rho_0}\right) - \left(\frac{il}{\rho_0 f}\frac{dV}{dz} + \frac{ik}{\rho_0 f}\frac{dU}{dz}\right)\left(\frac{f\theta_0}{g}\right)\right]\hat p + \hat w\bar\theta_z = 0 \quad \text{at} \quad z = 0, \quad (2.8)$$

which reduces to the dispersion relation when $\hat w = 0$:

$$\sigma = -\left(U_0 + H\frac{dU}{dz}\right)k - \left(V_0 + H\frac{dV}{dz}\right)l$$
$$= -\bar U(H)\cdot\mathbf{k}. \quad (2.9)$$

The phase speed in the direction of the horizontal wave number vector (k) is

$$C_p = \bar U[z = H(k, l)]\cdot\frac{\mathbf{k}}{|\mathbf{k}|}, \quad (2.10)$$

which is the background wind at $z = H$, the so-called "steering level." The group velocity $\mathbf{C}_g \equiv -\partial\sigma/\partial\mathbf{k}$ is given by

$$\mathbf{C}_g = \mathbf{U}(H) - \frac{f}{N|\mathbf{k}|^3}\left(\mathbf{k}\cdot\frac{d\mathbf{U}}{dz}\right)\mathbf{k}. \quad (2.11)$$

To summarize, these baroclinic waves are dispersive waves with real frequencies which can propagate along the surface of the earth in the presence of a horizontal temperature gradient. By avoiding the rigid top lid (the Eady problem) and the β-effect (the Charney problem) the waves are stable in spite of the available potential energy. The choice of this particular problem is convenient for the investigation of lee cyclogenesis, as it allows a clear distinction between unstable growth (none in this problem) and orographic forcing.

3. Baroclinic wave generation by a ridge

Consider an infinite ridge lying parallel to the y-axis given, for example, by

$$h(x) = \frac{ha^2}{a^2 + x^2}. \quad (3.1)$$

A low-level wind blowing against the ridge will produce vertical velocities according to

$$w(x, z = 0) \approx U_0\frac{dh}{dx}, \quad (3.2)$$

assuming that the air goes over. For the steady state two dimensional problem, (2.8) reduces to

$$p(z = 0) = \frac{U_0\bar\theta_z(g\rho_0/\theta_0)\hat h}{(U_0/H + dU/dz)}, \quad (3.3)$$

where $\hat h(k)$ is the Fourier transform of $h(x)$,

$$\hat h(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} h(x)e^{-ikx}dx, \quad (3.4)$$

and in the case of (3.1),

$$h(k) = \frac{ha}{2}e^{-|k|x}. \quad (3.5)$$

The perturbation pressure field is then given by the inverse Fourier transform (using 2.6, 3.3 and 3.5)

$$p'(x, z) = U_0\bar\theta_z(g\rho_0/\theta_0)[2\mathrm{Re}] \times \underbrace{\int_0^{\infty}\frac{\hat h(k)e^{-z/H}e^{ikx}dk}{(U_0/H + dU/dz)}}_{\mathrm{I}}. \quad (3.6)$$

According to the Riemann–Lebesgue Lemma, the integral I will go rapidly to zero as $|x| \to \infty$ because of cancellations caused by the term e^{ikx}. This implies that the airflow will be disturbed only over the mountain. There is one exception to this and that is if the denominator of the integrand vanishes for some value of k. If we take $U_0 > 0$, this is possible if $dU/dz < 0$ and then that special wavenumber k^* is (from 2.7 with $l = 0$)

$$k^* = \frac{f}{NH^*}, \tag{3.7}$$

where H^* is the altitude at which the component of wind across the mountain vanishes. The vanishing of the wind $U(z)$ at some height is then the condition for obtaining a disturbance away from the mountain. Physically, this allows there to be one baroclinic wave with zero phase speed, and it is this wave that can be directly forced by the fixed mountain.

The integral "I" can be readily evaluated at large $|x|$ by evaluating the contributions near k^*. This is accomplished by expanding all functions of k in a Taylor series. The first term is sufficient for all but the denominator which gives

$$\left(\frac{U_0}{H} + \frac{dU}{dz}\right) \approx 0 + \left(-\frac{U_0}{H^2}\right)\frac{dH}{dk}\Delta k = \frac{U_0 N}{f}\Delta k, \tag{3.8}$$

where $\Delta k = k - k^*$. Then "I" in (3.6) becomes

$$\mathrm{I} = \frac{\hat{h}(k^*)e^{-z/H^*}e^{ik^*x}}{(U_0N/f)}\int_{-\infty}^{\infty}\frac{e^{i\Delta kx}d\Delta k}{\Delta k}. \tag{3.9}$$

The integral in (3.9) depends on where the contour is placed in the complex plane. Equation (2.11) shows that the group velocity for this standing wave is directed in the positive direction, in fact

$$C_{gx} = U_0, \tag{3.10}$$

so the contour is placed under the singularity, giving (from 3.9)

$$p'(x, z) = 0 \quad \text{for} \quad x < 0, \tag{3.11a}$$

$$p'(x, z) = -\bar{\theta}_z\left(\frac{g\rho_0}{\theta_0}\right)4\pi\frac{f}{N}\hat{h}(k^*)e^{-z/H^*}\sin k^*x \quad \text{for} \quad x > 0. \tag{3.11b}$$

Eq. (3.11b) describes a train of standing baroclinic "lee" waves behind (in the sense of C_g) the mountain range. Its phase is such that a low is present on the lee slope, producing a wave drag.

This wave has a true baroclinic structure. The low is associated hydrostatically with warm air and this warm air is produced by advection from the warm side, winning out over the midtroposphere ascent which is producing the low-level vorticity. There is no phase tilt as there would be in a growing Eady wave, except perhaps near the mountain.

The theory seems to be successful in a number of ways. It predicts the correct structure and a reasonable wavelength (if H = 5 km, $\lambda = 2\pi/k^*$ = 3000 km). The low is on the correct side of the mountain. This should not be considered trivial as there are waves, such as capillary waves, that put their "lee" waves upstream. Furthermore, the word "lee" is imprecise in this problem as the background flow is strongly sheared. The critical condition ($U(z)$ reversing) may be plausible as shown in Section 5. If these are wrong in detail, it may well be that this is because the idealizations, such as constant potential vorticity, quasi-geostrophic balance, and two-dimensional geometry are inaccurate rather than an error in the basic approach of looking for standing baroclinic waves.

The amplitude of the generated wave can be estimated as follows. Putting $N = 0.01\ \mathrm{s^{-1}}$, $f = 10^{-4}\ \mathrm{s^{-1}}$, $h(k^*) = (ha/2)e^{-k^*a} = (3 \times 10^3)(2.5 \times 10^5)(e^{-0.5})/2\ \mathrm{m^2}$ (from 3.5), $\rho_0 = 1\ \mathrm{kg\ m^{-3}}$ into the pressure amplitude formula (from 3.11b)

$$\rho_0 4\pi N f \hat{h}$$

gives p' = 2800 Pa = 28 mb. This large value illustrates the sensitivity of baroclinic waves to orographic forcing and suggests that in practice, nonlinear effects may become important.

Figure 1 shows the asymptotic steady state surface pressure field perturbation given by (3.11b). Also shown is the exact time-dependent solution found using methods in the next section. The agreement between the two illustrates how quickly in time, and downstream, the wave approaches its steady state asymptotic form.

4. The general three-dimensional time dependent problem

The above theory has one important disadvantage. It predicts a steady state train of lee waves rather than a cyclone developing in time. The steady state assumption in (3.6) was a mathematical convenience which had the result of emphasizing that while the baroclinic environment persists, the atmosphere is trying to establish a steady train of waves. In fact, the time development of the baroclinic wave is of central importance as we are trying to equate lee cyclogenesis with the growth of the first trough in the lee wave. The theoretical solution at longer times is of less interest as the baroclinic zone may have moved away from the mountain or some other growth or propagation mechanism may become dominant as the wave reaches larger amplitude. The time development of the first trough can be estimated as follows.

The wave train begins to form at the time the mountain is inserted in the flow or the time the correct baroclinic environment first exists over the mountain. The wave train then forms progressively from the mountain location, extending at a rate given by the group velocity of the standing wave. (This is quite easily observed in the laboratory tow tank experiments of buoyancy lee waves). In this case, with C_g given by (3.10), we can estimate that if the low level wind ap-

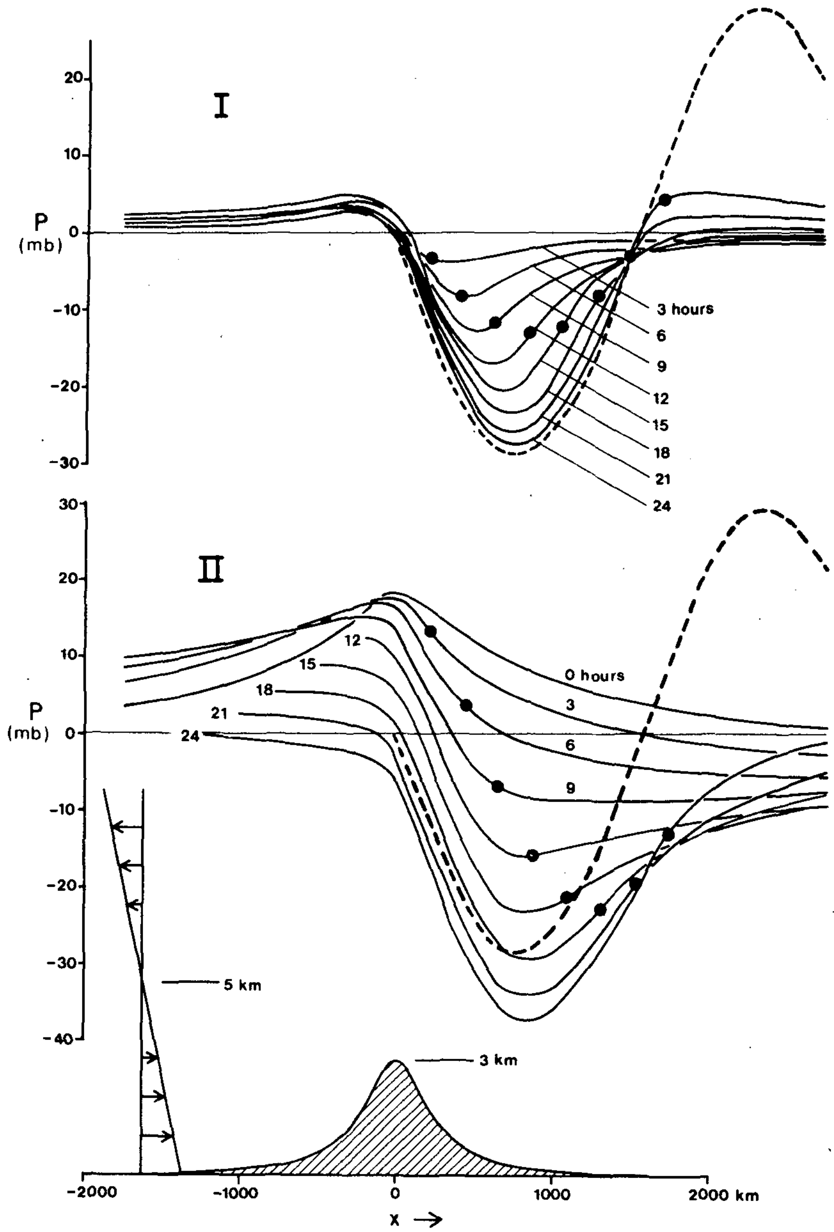

FIG. 1. Time development of surface pressure (mb) near a 3 km ridge in a sheared background flow according to linearized quasi-geostrophic theory [Eq. (4.2)]. Both initial conditions lead to rapid lee cyclogenesis with pressure tendencies of from 4 to 6 mb/(3 h)$^{-1}$. The fluid is trying to form the first trough of a standing baroclinic lee wave [shown dashed from Eq. (3.11b)]. The distance covered by the group velocity ($C_g = U_0 = 20$ m s^{-1}) is indicated by the dot on each curve. Initial condition I (undisturbed flow) produces the simple pattern of a wave growing away from its source, while condition II exhibits a rapid decay of the initial mountain anticyclone and a bit of overshoot in the lee trough amplitude.

proaches the mountain at 20 m s^{-1}, then it would take about 7 h for the first 500 km of the wave to be generated.

This group velocity argument is useful, but it is not much effort to go beyond and develop full time-dependent (and three-dimensional) solutions satisfying specified initial conditions. Starting with (2.5), a single ordinary differential equation in time is derived by taking the double Fourier transform and using (2.6) and (2.7)

$$\frac{\partial p}{\partial t} + \left(ikU_0 + ilV_0 + ikH\frac{dU}{dz} + ilH\frac{dV}{dz}\right)\hat{p} - \frac{Hg\rho_0\bar{\theta}_z}{\theta_0}(ikU_0 + ilV_0)\hat{h} = 0, \quad \text{at} \quad z = 0. \quad (4.1)$$

The general solution to (4.1) is

$$\hat{p}(t) = Ae^{-[B]t} + \frac{Hg\rho_0\bar{\theta}_z}{\theta_0}\frac{(ikU_0 + ilV_0)\hat{h}}{B}, \quad (4.2)$$

where

$$B = \left(ikU_0 + ilV_0 + ikH\frac{dU}{dz} + ilH\frac{dV}{dz}\right).$$

and A is a complex coefficient to be determined from the initial conditions.

It is a bit difficult to select initial conditions for (4.1) to correspond to real cases of cyclogenesis, so two different prototype conditions will be considered.

Condition I. Initially undisturbed flow

$$p(t = 0) = 0. \quad (4.3)$$

The reader should be aware in advance that this condition will produce a "starting" warm core cyclone in the barotropic case ($\bar{U}_z = \bar{V}_z = 0$) which will drift downstream. The dynamics and implications of this vortex are discussed in Appendix A.

Using (4.3), the constant

$$A = -\frac{Hg\rho_0\bar{\theta}_z}{\theta_0}\frac{(ikU_0 + ilV_0)\hat{h}}{B}, \quad (4.4)$$

so that

$$\hat{p}(t) = \frac{Hg\rho_0\bar{\theta}_z}{\theta_0}\frac{(ikU_0 + ilV_0)\hat{h}(1 - e^{Bt})}{B}. \quad (4.5)$$

Condition II. Initial steady barotropic flow over the mountain

$$\hat{p}(t = 0) = \rho_0 f N\frac{\hat{h}}{|\mathbf{k}|} \quad (4.6)$$

(Smith, 1979a). This condition eliminates the starting vortex but the initial "mountain anticyclone" is rather unrealistic.

Using (4.6) in (4.2) gives

$$A = \frac{Hg\rho_0\bar{\theta}_z\hat{h}}{\theta_0}\left(ikH\frac{dU}{dz} + ilH\frac{dV}{dz}\right)B^{-1}, \quad (4.7)$$

so that

$$\hat{p}(t) = \frac{Hg\rho_0\bar{\theta}_z\hat{h}}{\theta_0}\left[(ikU_0 + ilV_0) + \left(ikH\frac{dU}{dz} + ilH\frac{dV}{dz}\right)e^{-Bt}\right]B^{-1}. \quad (4.8)$$

For both conditions the quantity $p'(x, y, z, t)$ is recovered from

$$p' = \iint_{-\infty}^{\infty} \hat{p}e^{-z/H}e^{i(kx+ly)}dkdl. \quad (4.9)$$

These formulas can be used to investigate a wide variety of baroclinic environments and mountain geometries. As before, the qualitative nature of the developing flow is controlled by the zeros, if any, of B. The integral (4.9) can be analyzed by contour integration, asymptotic methods, or numerically using a fast Fourier transform (FFT). Fig. 1 shows the FFT results for a uniform ridge using initial conditions I and II respectively. The two cases begin very differently but rapidly converge as the first wave trough develops. At large time, both solutions approach the asymptotic result derived in Section 3.

Figure 2 illustrates the three-dimensional time-developing flow near an isolated mountain described in Appendix C. The wind hodograph of the environment has both speed and directional shear, turning from N at the surface to WSW aloft. Initial condition I was applied and after 18 h a lee cyclone has formed south of the mountain. This position lies within the range of group velocities of possible stationary baroclinic lee waves.

5. Application of the theory to Alpine lee cyclogenesis

There is still considerable uncertainty about the precise synoptic conditions that lead to Alpine lee cyclogenesis. One common situation is the eastward movement of an upper level trough towards the Alps from the Atlantic. As the trough approaches, low level cold air flows from the NW across France behind a cold front oriented SW–NE. The wind hodograph then shows NW winds at the surface turning to SW aloft. If the Alps is considered an E–W ridge, then the wind reversal criterion is met and a lee trough should begin to form in place, to the south of the Alps.

An example of this situation is shown in Fig. 3 and 4 from 0000 GMT on 5 March 1982. At 850 mb the NW flow is evident over France but the already developing cyclone, or possibly some other aspect of the mountain perturbation or synoptic scale pattern has reversed the winds in the lee. At 300 mb there are SW winds over the Alps associated with a trough axis to the west.

6. Discussion

Briefly stated, the present theory views the lee cyclogenesis process as the formation of the first trough of a baroclinic lee wave. The low begins to form near the mountain and deepens as it moves back toward the quarter-wave point of the standing wave. The time scale and the position of the cyclogenesis seem roughly correct. The necessary condition for lee cyclogenesis is that the environment must have a horizontal temperature gradient with sufficient strength and suitable orientation to support standing baroclinic waves. Unlike other known leeside pressure fall mechanisms such as a barotropic starting vortex or wave-induced downslope wind, this low is generated hydrostatically by warm air advection, winning out over mid-level ascent which produces the low-level vorticity. In spite of these

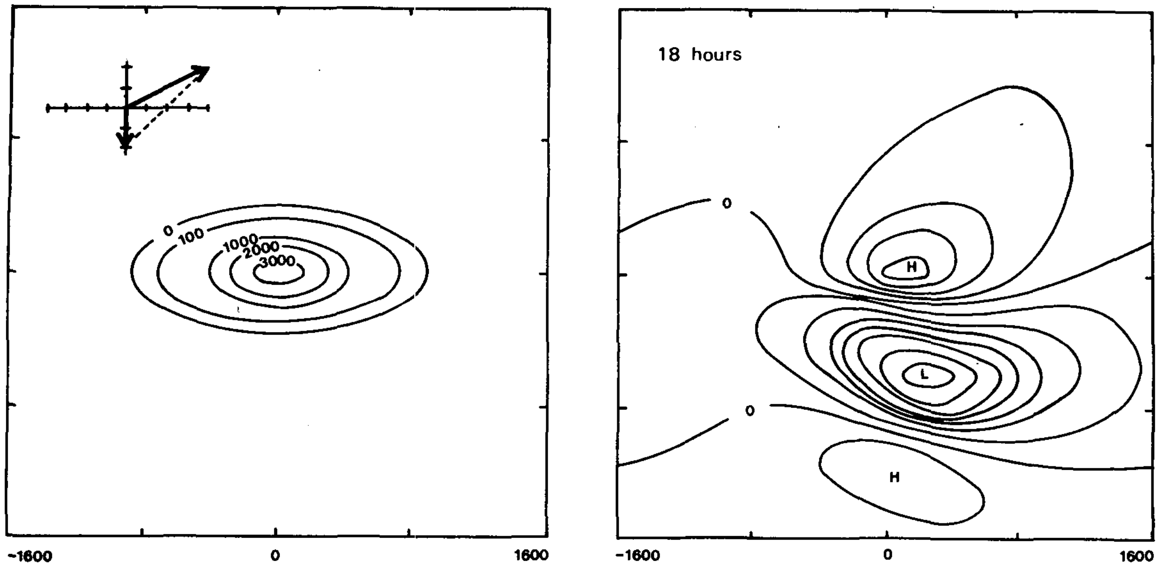

FIG. 2. A developing lee cyclone beside an isolated mountain in a baroclinic environment with strong vertical shear and cold advection. The left part shows the mountain shape and the wind hodograph. The right part shows the sea level perturbation pressure field, 18 h after starting the flow from an undisturbed state. At this time, the leeside pressure has dropped by 7.7 mb. The slight left-right asymmetry is due to the directional shear in the basic hodograph. The parameters used in this calculation are given in Appendix C. The contour interval is 1 mb.

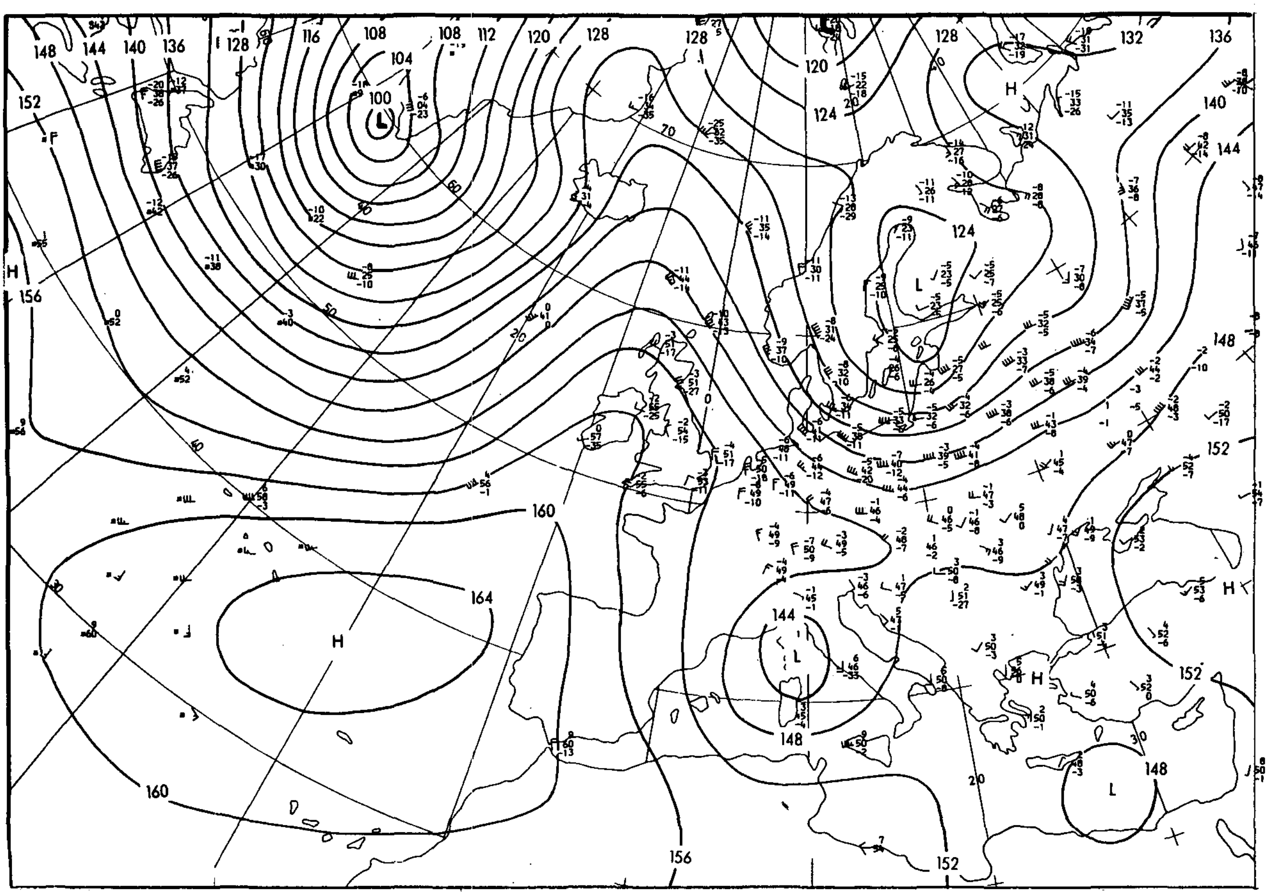

FIG. 3. The 850 mb chart over Europe on 5 March 1982 at 0000 GMT. North of the Alps, the flow is NW. South of the Alps, a Genoa cyclone has formed over the last six hours and is still deepening. Chart is from the German Weather Service (DWD).

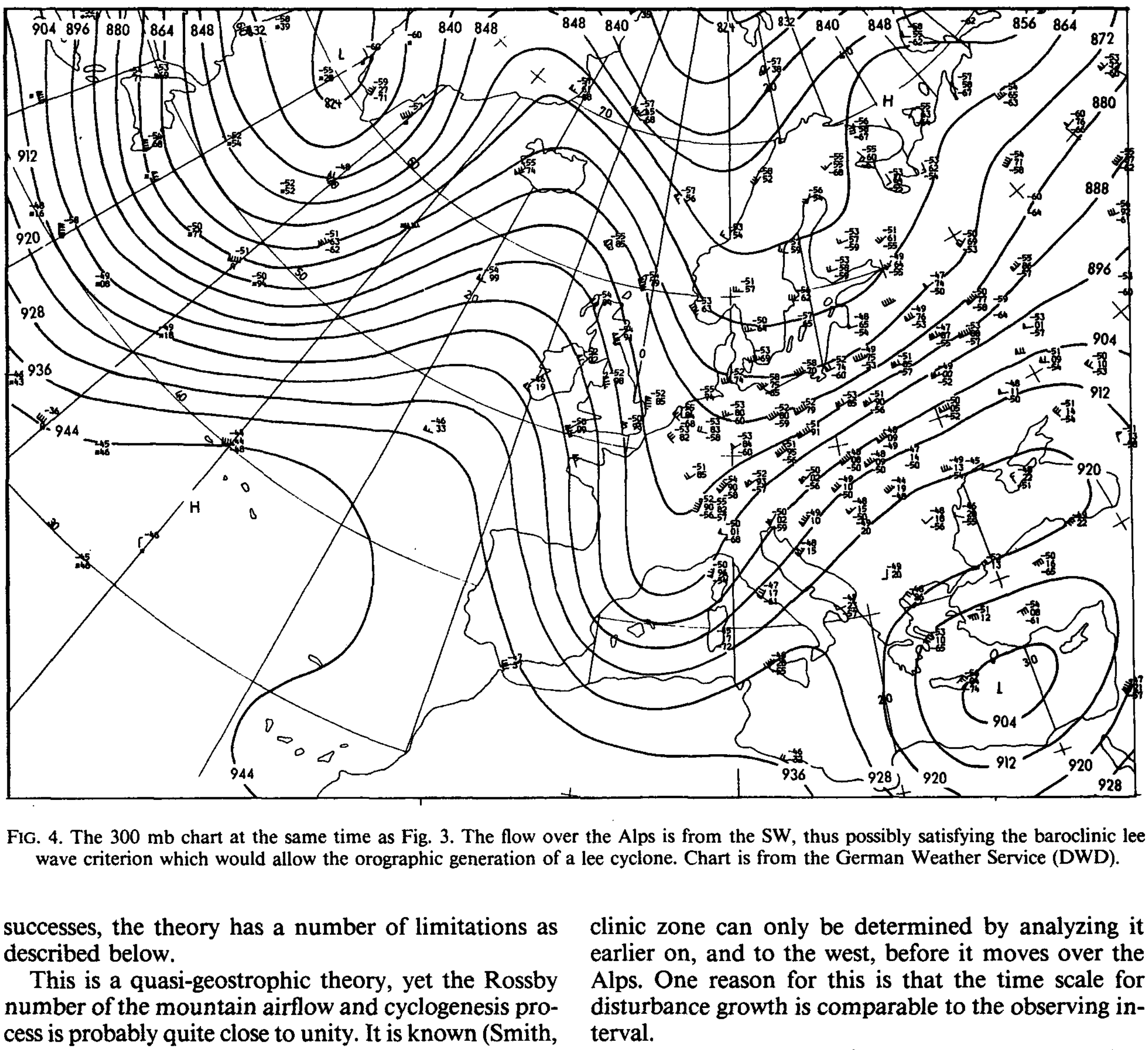

FIG. 4. The 300 mb chart at the same time as Fig. 3. The flow over the Alps is from the SW, thus possibly satisfying the baroclinic lee wave criterion which would allow the orographic generation of a lee cyclone. Chart is from the German Weather Service (DWD).

successes, the theory has a number of limitations as described below.

This is a quasi-geostrophic theory, yet the Rossby number of the mountain airflow and cyclogenesis process is probably quite close to unity. It is known (Smith, 1979a,b; 1982) that within linear steady state theory, the quasi-geostrophic approximation eliminates realistic low-level blocking and deflection by the mountains. This blocking may be as effective as orographic lifting at creating a lee cyclone. If conditions are right, any disturbance to the low-level temperature field, whether by lifting or by altered temperature advection could create a lee cyclone (Smith, 1984).

The specification of the lower boundary condition (as in 3.2) treats the mountains as smooth, rather than as a pinnacled and dissected irregular surface. The variety of scales and processes within the mountain valleys are neglected.

The theory is based on the linearized equations even though the local mountain disturbance and the lee cyclone have finite amplitude. The difficulty with this is immediately evident when trying to distinguish the environment from the perturbation using real data. For example, the structure of the undisturbed baroclinic zone can only be determined by analyzing it earlier on, and to the west, before it moves over the Alps. One reason for this is that the time scale for disturbance growth is comparable to the observing interval.

The theory made use of the enormous mathematical simplification that follows from the assumption of a uniform potential vorticity environment. This implies linear shear and horizontally uniform winds at each altitude, as well as horizontally uniform temperature gradients. Thus, we cannot describe mathematically our physical picture of a baroclinic zone of finite width moving over the mountains, and thus initiating the lee cyclogenesis. Neither can we describe the role of jet streams or jet streaks in the cyclogenesis process, except insofar as they delineate a broad baroclinic environment in which the mountain can act.

In order to isolate the orographic forcing of baroclinic waves, the theory considered a stable basic state. It may be, however, that unstable growth is involved in lee cyclogenesis, particularly in the later stages. This may be due to the tropopause, the β-effect, non-uniform potential vorticity, or caused by nonlinear effects after the lee cyclone has reached a certain amplitude.

The theory reveals a rather definite necessary condition for lee cyclogenesis (i.e., the wind component reversal). Observational data may not support this in all cases. Perhaps other mechanisms exist for lee cyclogenesis. On the other hand, more general treatments of baroclinic lee-wave generation may show that the current condition is not absolute. This of course would make it somewhat more difficult to disprove the theory.

The work of extending the theory, and testing it against data from the Alps and elsewhere, is left for the future. Appendix B indicates how the effects of ageostrophy and wind profile curvature could be evaluated. Numerical models of a simple type could be used to verify the current results and check on the effects of incoming wave structure and response nonlinearity. More complicated models could determine whether the mechanism described here can operate in the presence of boundary layers, inversions, and clouds. Data from cases of lee cyclogenesis could be examined to see if the current necessary condition for lee cyclogenesis is met in the environment. This promises to be difficult both because the theoretical condition in three dimensions is not simple, and because it is not easy to separate "environment" from "disturbance" in real data. Comparison of the mechanism of temperature change, vorticity production, and energy conversion might help to test the validity of the theory.

Acknowledgments. This theory was begun in preparation for and during the International School of Mediterranean Meteorology in May 1982 in Erice, Sicily. I am indebted to two groups for the evolution of this theory. The students in my course "Mesoscale Dynamics" at Yale provided a critical audience for the review of the theoretical elements of this subject, and the Alpex scientists R. Bleck, J. McGinley, J. Paegle, and A. Buzzi helped instruct me concerning the observations of lee cyclogenesis. This material is based upon work supported jointly by the National Science Foundation and the National Oceanic and Atmospheric Administration under Grant ATM 8023348.

APPENDIX A

The Starting Vortex

When there is initially no disturbance over the mountain (i.e., no relative vorticity) and lee side descent begins, quasi-geostrophic theory predicts the formation of a starting vortex. To illustrate the starting vortex it is possible to write down the solution to (2.4) with (2.5) and (3.2) or initial condition I and a mountain:

$$h(x, y) = \frac{ha^3}{(x^2 + y^2 + a^2)^{3/2}} \tag{A1}$$

in a uniform basic state, $\mathbf{U}(x, y, z, t) = (U, O)$. This is (from Smith 1979a)

$$\begin{aligned} p'(x, y, z, t) &= \rho N f h a^2\left[x^2 + y^2 + \frac{N^2}{f^2}\left(z + \frac{f}{N}a\right)^2\right]^{-1/2} \\ &\quad - \rho N f h a^2\left[(x - Ut)^2 + y^2 + \frac{N^2}{f^2}\left(z + \frac{f}{N}a\right)^2\right]^{-1/2}. \end{aligned} \tag{A2}$$

At $t = 0$ the first term (the mountain anticyclone) cancels the second term (the starting vortex). At later times the warm core starting vortex drifts downstream at the speed U.

In a sense, this solution constitutes a theory of lee cyclogenesis although it is so dependent on the initial conditions that it is not very satisfying. Also, the warmth of the starting vortex is caused by descent rather than warm air advection as in the present model.

When condition I is used in Section 4, one might expect some tendency to form a starting vortex even though the background state is strongly sheared. In fact, this tendency seems almost completely absent. This is evidenced by the fact that both initial conditions I and II produce lee-side lows at about the same rate. Condition II of course has no tendency to form a starting vortex as there is an anticyclonic vortex above the mountain at $t = 0$. Furthermore, the vortex tube shrinking due to windward side ascent must be nearly eliminated by the shear as there is no evidence of a mountain anticyclone at later times (Fig. 1).

It seems then that the starting vortex mechanism is not directly related to the present results.

APPENDIX B

Direct Calculation of the Vertical Velocity Field Using Two-Dimensional Lee-Wave Theory

In Section 3, the lee-wave pressure field was computed using quasi-geostrophic theory and the surface advection equation. The result, from (3.11b), was of the form

$$p = Ce^{-z/H^*} \sin k^* x. \tag{B1}$$

This can be used to compute the ageostrophic wind and vertical winds as follows. According to quasi-geostrophic theory

$$u_a = \frac{-1}{f}\frac{D_g(v_g)}{Dt}, \tag{B2a}$$

$$v_a = \frac{1}{f}\frac{D_g(u_g)}{Dt}, \tag{B2b}$$

which for this linearized, steady, two-dimension problem becomes

$$u_a = -\frac{U}{f}\frac{\partial v_g}{\partial x}, \tag{B3a}$$

$$v_a = 0. \quad \text{(B3b)}$$

Using (B3a) in (B1) gives

$$u_a = \frac{C}{\rho_0 f^2} e^{-z/H^*} U_0 \left(1 - \frac{z}{H^*}\right) k^{*2} \sin k^* x. \quad \text{(B4)}$$

The vertical velocity field associated with this wave can be determined from the continuity equation in the form

$$w(x, z) = -\int_0^z \frac{\partial u_a}{\partial x} dz', \quad \text{(B5)}$$

which gives

$$w(x, z) = \frac{C}{\rho_0 f^2} k^{*3} U_0 z e^{-z/H^*} \cos k^* x. \quad \text{(B6)}$$

The method of Section 3 has the advantage of emphasizing the importance of low-level temperature advection but it is not the only way or even the best way to do the calculation. In the following few lines, the baroclinic lee wave amplitude is rederived using a method that is more akin to classical mountain wave theory. This method gives the field of vertical velocity (B6) directly, instead of deriving it from the geostrophic pressure field.

The equation governing vertical velocity in two-dimensional steady state hydrostatic flow is given by Jones (1967), Eliassen (1968), Smith (1979b), as

$$\left(\frac{f^2}{U^2k^2} - 1\right)\hat{w}_{zz} - \frac{2f^2U_z}{k^2U^3}\hat{w}_z + \left(\frac{U_{zz}}{U} - \frac{N^2}{U^2}\right)\hat{w} = 0. \quad \text{(B7)}$$

This equation is valid for any Rossby number (Uk/f). The second term takes into account the direct effect of vertical shear and the effect of horizontal temperature advection. The critical points in this equation ($U = 0$, $|Uk| = f$) prevent an analytical solution for Rossby number of order (1), (see Jones, 1967, and Eliassen, 1968) but the equation is very much simpler if $R_0 \ll 1$ and $U_{zz} = 0$. Then

$$\hat{w}_{zz} - 2\frac{U_z}{U}\hat{w}_z - \frac{k^2N^2}{f^2}\hat{w} = 0. \quad \text{(B8)}$$

If solutions to this equation can be found satisfying the decay or radiation condition aloft, then the vertical velocity field can be expressed as:

$$w(x, z) = [2 \cdot \mathrm{Re}] \int_0^\infty ikU_0\hat{h}(k)\frac{\hat{w}(k, z)}{w(k, 0)} e^{ikx} dk. \quad \text{(B9)}$$

Reasoning as before, this perturbation will decay downstream unless $w(k, 0)$ has one or more zeroes. In fact, with $U(z) = U_0(1 - z/H^*)$, there is one such $k = k^* = f/H^*N$ corresponding to the eigenfunction

$$\hat{w}(k^*, z) = Dze^{-z/H^*}, \quad \text{(B10)}$$

where H^* is the height of wind reversal. Lee wave theory then gives, as $x \rightarrow \infty$,

$$w(x, z) = -\frac{4\pi U_0 k^* \hat{h}(k^*)\hat{w}(k^*, z)\cos k^* x}{\left.\dfrac{\partial w(0)}{\partial k}\right|_{k^*}}. \quad \text{(B11)}$$

The general solution to (B8) is readily found by introducing the new independent variable[1]

$$\hat{z} = \frac{Nk}{f}(z - H), \quad \text{(B12)}$$

so (B8) becomes

$$\hat{w}_{\hat{z}\hat{z}} - \frac{2}{\hat{z}}\hat{w}_{\hat{z}} - \hat{w} = 0, \quad \text{(B13)}$$

which is an equation known to be satisfied by modified spherical Bessel functions of order zero (Abramowitz and Stegun, 1965, section 10.2). This solution has a zero at $\hat{z} = -1$, which verifies the result (3.7), and the structure in (B10).

Since we already know the eigenvalue and eigenfunction, the primary reason for using the transformation (B12) is that it makes it possible to express the important amplitude factor in the denominator of (B11) in terms of the eigenfunction alone, $\hat{w}$ without needing to know how $w(k, 0)$ behaves near k^*. Taking derivatives with respect to k and z gives

$$\frac{\left.\partial\hat{w}/\partial k\right|_{z=0}}{\left.\partial\hat{\omega}/\partial z\right|_{k=k^*}} = -\frac{NH^{*2}}{f}. \quad \text{(B14)}$$

Combining (B10), (B11), and (B14) gives

$$w(x, z) = +4\pi\left(\frac{f^2}{N^2H^{*3}}\right)\hat{h}(k^*)ze^{-z/H^*}\cos k^* x, \quad \text{(B15)}$$

which agrees with (B6) and (3.11b).

This calculation is useful for a number of reasons. It provides a check on the temperature advection method, while emphasizing a different aspect of the dynamics. It gives the ageostrophic vertical velocity directly and it shows the connection to 2-D mountain wave theory. Furthermore, equation (B7) provides the basis for the study of the effects of finite Rossby number and mean profile curvature.

APPENDIX C

Parameters Used in the Construction of Fig. 2

Figure 2 is constructed from the formula (4.5) using a Fast Fourier Transform. The parameters are as follows:

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[1] This transformation was suggested by Roger Hughes of Yale University. An alternative method for finding $\partial w/\partial k|_0$ involving integration by parts [Smith 1979a, Eq. (2.90)] is not useful here as the contribution from the second term in (B8) is difficult to estimate.

Environment

$$f = 10^{-4}\ \mathrm{s}^{-1}$$

$$N = 10^{-2}\ \mathrm{s}^{-1}$$

$$U_0 = 0$$

$$V_0 = -15\ \mathrm{m\ s}^{-1}$$

$$U_z = (20/5000)\ \mathrm{s}^{-1}$$

$$V_z = (20/5000)\ \mathrm{s}^{-1}$$

(For the northward directed standing wave, the steering level is $H^* = 3750$ m.)

Mountain

$$h(x, y) = \frac{h}{[(x/a_x)^2 + (y/a_y)^2 + 1]^3}$$

$$h = 3000\ \mathrm{m}$$

$$a_x = 600\ \mathrm{km}$$

$$a_y = 250\ \mathrm{km}$$

Initial Condition

Undisturbed flow $p'(x, y, z, t = 0) = 0$

Time

$$t = 18\ \mathrm{h}$$

Numerical Calculation

Array size 16×16

Grid size 200×200 km

Domain $-1600 < x < 1600$ km

$-1600 < y < 1600$ km

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