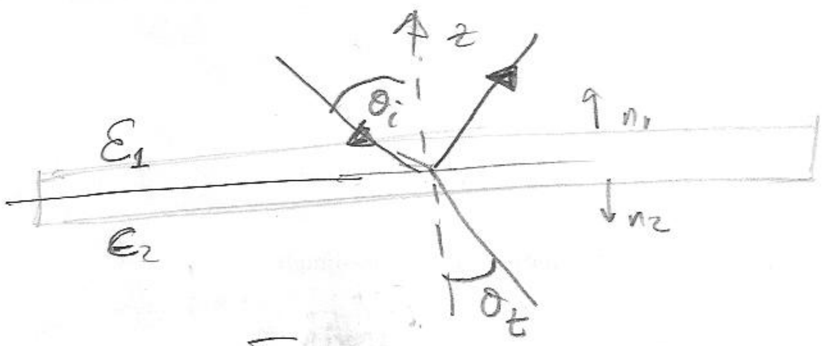
(5)



$$F_{\text{fuerzo sobre material y carg}} + \frac{d\bar{P}_{eM}}{dt} = \oint \overleftrightarrow{T} \cdot \hat{n}\, ds$$

$$\bar{P}_{em} = \frac{S}{c^2} = \bar{g}$$

$$\langle \bar{F} \rangle = \langle \oint \overleftrightarrow{T} \cdot \hat{n}\, dS \rangle - \langle \int \frac{\partial g}{\partial t} \rangle$$

$$\langle \int \frac{\partial g}{\partial t}\, dV \rangle = \frac{1}{\tau} \int_0^{\tau} dt \int_V \frac{\partial g}{\partial t}\, dV = \frac{1}{\tau} \int_V \left[g(\tau) - g(0) \right]$$

$\underline{\tau \to \infty}$ $g(\tau)$ oscila

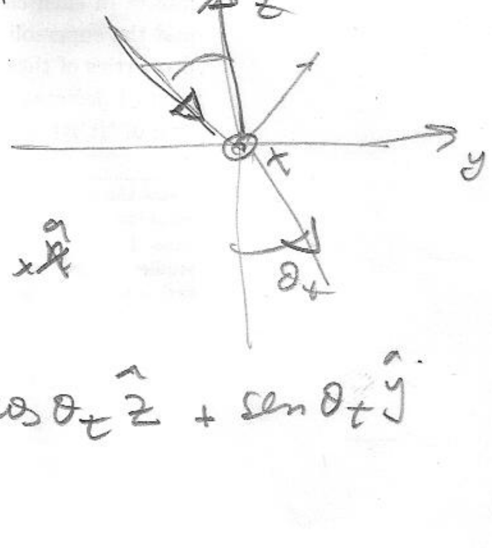
(No. maneja x teorema armónico.

$$\langle F_z \rangle = \int_{S_1} \langle T_{zz} \rangle \Big|_{z=0} dS_1 - \int_{S_2} \langle T_{zz} \rangle \Big|_{z=0} dS_2$$

$$T_{zz} = \frac{1}{4\pi} \left[\epsilon E_z^2 + \frac{1}{\mu} B_z^2 - \frac{1}{2} \left(\epsilon E^2 + \frac{1}{\mu} B^2 \right) \right]$$

<u>BEB</u>

Escribamos todo usando el TE



$$\bar{B} = n \hat{k} \times \bar{E} \quad \left(\frac{c\bar{k}}{\omega} \times \bar{E} \right)$$

$$\bar{E}_i = E_i\, e^{i(\bar{k}_i \cdot \bar{r} - \omega t)} \hat{x}$$

$$\bar{B}_i = \sqrt{\mu_1 \epsilon_1}\, E_i\, e^{i(\bar{k}_i \cdot \bar{r} - \omega t)}\, \hat{k}_i \times \hat{x}$$

$$\hat{K}_i = -\cos\theta_i \hat{z} + \operatorname{sen}\theta_i \hat{y} \quad ; \quad \hat{K}_{Tt} = -\cos\theta_t \hat{z} + \operatorname{sen}\theta_t \hat{y}$$

$$K_n = \cos\theta_i \hat{z} + \operatorname{sen}\theta_i \hat{y}$$

$$\hat{k}_i \times \hat{x} = (-\cos\theta_i\, \hat{z} + \operatorname{sen}\theta_i\, \hat{y}) \times \hat{x} = (-\cos\theta_i\, \hat{y} - \hat{z}\, \operatorname{sen}\theta_i)$$

$$= -(\cos\theta_i\, \hat{y} + \hat{z}\, \operatorname{sen}\theta_i)$$

$$\bar{E}_1 = \left[E_i\, e^{i(k_i \cdot \bar{r} - \omega t)} + E_R\, e^{i(\bar{k}_R \cdot \bar{r} - \omega t)}\right]\hat{x}$$

$$\bar{E}_R = E_R\, e^{i(\bar{k}_R \cdot \bar{r} - \omega t)}\,\hat{x}$$

$$\bar{B}_R = \sqrt{\mu_1 \epsilon_1}\, E_R e^{i(k_R \cdot \bar{r} - \omega t)} (\cos\theta_i\, \hat{z} + \operatorname{sen}\theta_i\, \hat{y}) \times \hat{x}$$

$$\bar{B}_R = \sqrt{\mu_1 \epsilon_1}\, E_R\, e^{i(\bar{k}_R \cdot \bar{r} - \omega t)} \left[\cos\theta_i\, \hat{y} + \operatorname{sen}\theta_i\, (-\hat{z})\right]$$

$$\bar{B}_i = \sqrt{\mu_1 \epsilon_1}\, E_i\, e^{i(\bar{k}_i \cdot \bar{r} - \omega t)} \left[\cos\theta_i\, \hat{y} + \hat{z}\, \operatorname{sen}\theta_i\right]$$

$$\bar{B} = \sqrt{\mu_1 \epsilon_1} \left\{ \cos\theta_i\, \hat{y} \left[E_R\, e^{i(\bar{k}_R \cdot \bar{r} - \omega t)} + E_i\, e^{i(\bar{k}_i \cdot \bar{r} - \omega t)}\right] + \operatorname{sen}\theta_i\, \hat{z} \left(-E_R e^{ik_1} + E_i e^{ik_i}\right) \right.$$

Beamos a $z = 0^+$

$$E_z = 0$$

$$B_z = B_R^z\Big|_{z=0} + \tilde{B}_i^z\Big|_{z=0} = \sqrt{\mu_1 \epsilon_1} \left\{ -E_R \operatorname{sen}\theta_i\, e^{ik_1 \operatorname{sen}\theta_i y} - E_i \operatorname{sen}\theta_i\, e^{ik_1 \operatorname{sen}\theta_i y} \right\} = -\sqrt{\mu_1 \epsilon_1}\, \operatorname{sen}\theta_i\, e^{ik_1 \operatorname{sen}\theta_i y} (E_R + E_i)$$

$$\langle B_z^2 \rangle = \mu_1 \epsilon_1 \operatorname{sen}^2\theta_i\, (E_R + E_i)(E_R^* + E_i^*)/2$$

$$= \frac{\mu_1 \epsilon_1}{2} \operatorname{sen}^2\theta_i \left[|E_R|^2 + |E_i|^2 + 2\operatorname{Re}(E_R E_i^*)\right]$$

$$E \cdot E^*\Big|_{z=0^+} = \left(E_i\, e^{ik \operatorname{sen}\theta_i y} + E_R\, e^{ik \operatorname{sen}\theta_i y}\right)\left(E_i^*\, e^{-ik \operatorname{sen}\theta_i y} + E_R^*\, e^{-ik \operatorname{sen}\theta_i y}\right)$$

$$= |E_i|^2 + |E_R|^2 + 2\operatorname{Re}(E_i E_R^*)$$

$$\langle E^2 \rangle = \frac{1}{2}\left[|E_i|^2 + |E_R|^2 + 2\operatorname{Re}(E_i E_R^*)\right]$$

$$B \cdot B^* = \mu_1 \epsilon_1 \left\{ \cos^2\theta_i \left(E_R e^{ik_1 \operatorname{sen}\theta_i y} + E_i e^{ik_1 \operatorname{sen}\theta_i}\right)\left(E_R^* e^{-ik \operatorname{sen}\theta} + E_i^* e^{-ik \operatorname{sen}\theta_i}\right) + \operatorname{sen}^2\theta_i \left(-E_R e^{ik \operatorname{sen}\theta_i} + E_i e^{ik \operatorname{sen}\theta_i y}\right. \right.$$

$$\times\left(-E_r^{*}\, e^{-ik_1 \operatorname{sen}\theta_i y} + E_i^{*}\, e^{-ik\operatorname{sen}\theta_i y}\right)\Big]$$

$$= \mu_1\epsilon_1\Big\{\cos^2\theta_i\left[|E_r|^2+|E_i|^2+2\,\mathrm{Re}(E_r E_i^{*})\right]+$$

$$\operatorname{sen}^2\theta_i\left(\left[|E_r|^2+|E_i|^2-2\,\mathrm{Re}\left(E_r E_i^{*}\right)\right]\right)\Big\}$$

$$\langle B^2\rangle = \frac{1}{2}\mu_1\epsilon_1\left\{|E_r|^2+|E_i|^2 - 2\,\mathrm{Re}[E_r E_i^{*}]\left(\cos^2\theta_i-\operatorname{sen}^2\theta_i\right)\right\}$$

$$\langle T_{zz}\rangle_1 = \frac{1}{4\pi}\Big\{\frac{\epsilon_1}{2}\operatorname{sen}^2\theta_i\left[|E_r|^2+|E_i|^2+2\,\mathrm{Re}(E_r E_i^{*})\right]+$$

$$-\frac{1}{2}\epsilon_1\frac{1}{2}\left[|E_r|^2+|E_i|^2+2\,\mathrm{Re}\left(E_i E_r^{*}\right)\right]-$$

$$-\frac{1}{2}\frac{1}{2}\epsilon_1\left[|E_r|^2+|E_i|^2-2\,\mathrm{Re}(E_r E_i^{*})\left(\cos^2\theta_i-\operatorname{sen}^2\theta_i\right)\right]\Big\}$$

$$=\frac{1}{2}\frac{\epsilon_1}{4\pi}\Big\{\operatorname{sen}^2\theta_i\left(|E_r|^2+|E_i|^2\right)-\left(|E_r|^2+|E_i|^2\right)+$$

$$+\mathrm{Re}(E_r E_i^{*})\underbrace{\left[\operatorname{sen}^2\theta_i-\frac{1}{2}+\frac{1}{2}\left(\cos^2\theta_i-\operatorname{sen}^2\theta_i\right)\right]}_{=0}\Big\}$$

$$=\frac{1}{2}\frac{\epsilon_1}{4\pi}\left(|E_r|^2+|E_i|^2\right)\cos^2\theta_i$$

Vamos sobre el medio 2.

$$\bar{E}_2 = E_t\, e^{i(\bar{k}_t\cdot\bar{r}-\omega t)}\,\hat{x}$$

$$\bar{B}_2 = \sqrt{\mu_2\epsilon_2}\,E_t\, e^{i(\bar{k}_t\cdot\bar{r}-\omega t)}\left(\hat{k}_t\times\hat{x}\right)$$

$$\hat{k}_t = -\cos\theta_t\,\hat{z}+\operatorname{sen}\theta_t\,\hat{y}$$

$$\hat{k}_t \times \hat{x} = (-\cos\theta_t \hat{z} + \operatorname{sen}\theta_t \hat{y}) \times \hat{x}$$

$$= -\cos\theta_t \hat{y} - \operatorname{sen}\theta_t \hat{z}$$

$$E_z\big|_{z=0^-} = 0$$

$$E \cdot E^*\big|_{z=0} = |E_t|^2$$

$$B \cdot B^*\big|_{z=0} = \mu_2 \epsilon_2 |E_t|^2 (\cos^2\theta_t + \operatorname{sen}^2\theta_t)$$

$$= \mu_2 \epsilon_2 |E_t|^2$$

$$B_z^2 = B_z B_z^* = \mu_2 \epsilon_2 |E_t|^2 \operatorname{sen}^2\theta_t$$

$$\langle T_{zz}\rangle_2 = \frac{1}{4\pi}\left\{ \frac{\epsilon_2}{2}|E_t|^2 \operatorname{sen}^2\theta_t - \frac{1}{2}\left(\frac{\epsilon_2 |E_t|^2}{2} + \frac{\epsilon_2}{2}|E_t|^2\right)\right\}$$

$$= \frac{1}{4\pi}\left\{ \frac{\epsilon_2}{2}|E_t|^2 \operatorname{sen}^2\theta_t - \frac{1}{2}\epsilon_2 |E_t|^2\right\}$$

$$= -\frac{\epsilon_2}{8\pi}|E_t|^2 \cos^2\theta_t$$

$$p_{rad} = -\frac{F_z}{dS} = \frac{-\langle T_{zz}\rangle_{z=0^+}}{dS} + \frac{\langle T_{zz}\rangle_{z=0^-}}{dS}$$

$$\boxed{p_{rad} = \frac{\epsilon_1}{8\pi}(|E_r|^2 + |E_i|^2)\cos^2\theta_i - \frac{\epsilon_2}{8\pi}|E_t|^2 \cos^2\theta_t}$$