

P1 Antena lineal $\bar{J} = I_0 \delta(x') \delta(y') \sin\left(\frac{2\pi z'}{d}\right) e^{i\omega t} \hat{z}$ $\oplus \left(\frac{d}{2} - z\right) \oplus \textcircled{1}$

② Por la ecuación de continuidad $\partial_t p(x', t) + \nabla' \cdot \bar{J}(x', t) = 0$ sabemos que para

i) $p(x', t) = p(x') e^{i\omega t} \Rightarrow i\omega p(x') e^{i\omega t} = -e^{i\omega t} I_0 \delta(x') \delta(y') \frac{\partial \sin\left(\frac{2\pi z'}{d}\right)}{\partial z'}$

$$\Rightarrow p(x') = \left[I_0 \frac{2\pi}{d} \delta(x') \delta(y') \cos\left(\frac{2\pi z'}{d}\right) \right] = -I_0 \delta(x') \delta(y') \frac{2\pi}{d} \cos\left(\frac{2\pi z'}{d}\right)$$

$$= \frac{I_0}{2} \frac{2\pi}{d} \delta(x') \delta(y') e^{\frac{i\pi}{2}} \left[e^{i\frac{2\pi z'}{d}} + e^{-i\frac{2\pi z'}{d}} \right]$$

$$= \frac{I_0}{2} \frac{\pi}{d} \delta(x') \delta(y') \left[e^{i\left(\frac{2\pi z'}{d} + \frac{\pi}{2}\right)} + e^{-i\left(\frac{2\pi z'}{d} - \frac{\pi}{2}\right)} \right] \oplus \left(\frac{d}{2} - z\right) \oplus \textcircled{2}$$

ii) El potencial retardado $\bar{A}(r', t)$ está dado por (MKS)

$$\bar{A} = \frac{\mu_0}{4\pi} \int \frac{\bar{J}(r', t - \frac{r'}{c})}{r'} d^3 r' \quad \text{dónde } k = \frac{2\pi}{d} = \frac{2\pi}{\lambda} = \frac{\omega}{c} \Rightarrow \boxed{d = \lambda}$$

$$= \frac{\mu_0}{4\pi} I_0 e^{i\omega t} \hat{z} \int_{-\lambda/2}^{\lambda/2} \frac{\sin(kz')}{r'} e^{ikr'} dz'$$

$$\text{con } r' = \sqrt{r^2 - 2rz' \cos\theta + z'^2}$$

Para los términos de radianes que vienen como $\frac{1}{r'}$, podemos ignorar todos los términos más grandes que $\frac{1}{r} \Rightarrow \frac{1}{r'} \approx \frac{1}{r}$

$$r' \approx r - z' \cos\theta \leftarrow \text{término de interferencia}$$

Luego, $\bar{A} = \frac{\mu_0}{4\pi} I_0 \frac{e^{i(\omega t - kr)}}{r} \int_{-\lambda/2}^{\lambda/2} \sin(kz') e^{ikz' \cos\theta} dz'$

(2)

$$x = kz, \frac{dx}{k} = dz$$

$$x\left(\frac{d}{2}\right) = \pm k\frac{d}{2}$$

$$\bar{A} = \hat{z} \frac{\mu_0 k^2}{4\pi} I_0 \frac{e^{i(wt-kr)}}{r} \int_{-\frac{kd}{2}}^{+\frac{kd}{2}} \sin(x) e^{\frac{i k x \cos\theta}{2}} dx$$

$$= \hat{z} \frac{\mu_0 k^2}{4\pi} I_0 e^{i(wt-kr)} \int_0^{\frac{kd}{2}} \sin(x) \sin(x \cos\theta) dx$$

cos(x cos\theta) + i sin(x cos\theta)

uso:

$$\int_{-\pi}^{\pi} \sin z \neq e^{-iaz} dz = \frac{2i \sin(\pi \alpha)}{\alpha^2 - 1}$$

$$\frac{kd}{2} = \frac{2\pi}{\alpha} \cdot \frac{\alpha}{2}$$

$$= \hat{z} \frac{\mu_0}{4\pi k} I_0 \frac{e^{i(wt-kr)}}{r} \frac{2i \sin(\pi \cos\theta)}{\cos^2\theta - 1}$$

$$\alpha = -\cos\theta$$

$$\vec{A} = \hat{z} \frac{\mu_0}{4\pi k} 2I_0 e^{i(wt-kr + \frac{\pi}{2})} \frac{\sin(\frac{kd}{2} \cos\theta)}{(\cos^2\theta - 1)}$$

$$\vec{A} = \hat{z} \frac{\mu_0 2I_0}{4\pi k r} e^{i(wt-kr + \frac{\pi}{2})} \frac{-\sin^2\theta}{\frac{\sin(\frac{kd}{2} \cos\theta)}{\sin\theta}}$$

Como $\hat{R} = k \hat{r}$ y $\hat{z} = (\cos\theta, -\sin\theta, 0)$ en coordenadas esféricas ($\hat{r}, \hat{\theta}, \hat{\phi}$)

$$\Rightarrow \vec{B} = \vec{\nabla} \times \vec{A} = -i \hat{R} \times \vec{A} = -\frac{\mu_0}{2\pi} \frac{I_0}{r} \frac{\sin(\frac{kd}{2} \cos\theta)}{\sin\theta} \hat{\phi}$$

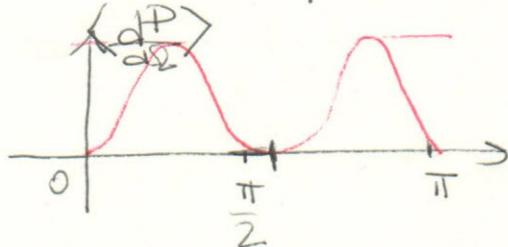
Por la ec. de Maxwell $\vec{\nabla} \times \vec{H} = \vec{D}$ ó $\vec{\nabla} \times \vec{B} = -i \hat{R} \times \vec{E} = i \frac{\mu_0 c}{k} \vec{E}$

$$\Rightarrow \vec{E} = -c \frac{\hat{R}}{k} \times \vec{B} = c \vec{B} \times \hat{r}$$

$$\langle \vec{S} \rangle = \frac{1}{2\mu_0} \operatorname{Re} [\vec{E}^* \times \vec{B}] = \frac{c}{2\mu_0} |\vec{B}| \hat{r} = \frac{\mu_0}{8\pi^2} \frac{c I_0^2}{r^2} \hat{r} \left[\frac{\sin(\frac{kd}{2} \cos\theta)}{\sin\theta} \right]$$

La potencia por unidad de ángulo sólido

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{\langle \bar{s} \rangle \cdot \bar{F}}{r^2} = \frac{I_0}{8\pi r^2} \sqrt{\frac{\mu_0}{\epsilon_0}} \left[\frac{\sin(\pi n \cos \theta)}{\sin \theta} \right]^2$$



(b) Si trabajamos en la aproximación multipolar

$$\Rightarrow e^{-ikr} \approx 1 - ikr + \dots$$

$$e^{-ikz' \cos \theta} \approx 1 + i \frac{kz' \cos \theta}{1!} + \left(i \frac{kz' \cos \theta}{1!} \right)^2 + \dots$$

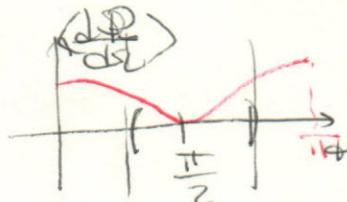
Despreciando términos de orden $\left(\frac{z'}{\lambda}\right)^2$ (sólo efecto dipolar)

$$\bar{A} \approx \frac{\mu_0 I_0}{4\pi r} e^{i(wt - kr)} \approx \left[\int_{-\frac{kd}{2}}^{\frac{kd}{2}} \sin(kz') dz' \right] + \int_{-\frac{kd}{2}}^{\frac{kd}{2}} dz' \sin(kz') \frac{ikz' \cos \theta}{2!}$$

$$= \frac{i \mu_0}{4\pi} I_0 e^{i(wt - kr)} \frac{\lambda \cos \theta}{r} \approx$$

$$B = -i \bar{r} \times \bar{A} = \mu_0 \frac{I_0}{r} e^{i(wt - kr)} \cos \theta \hat{r}$$

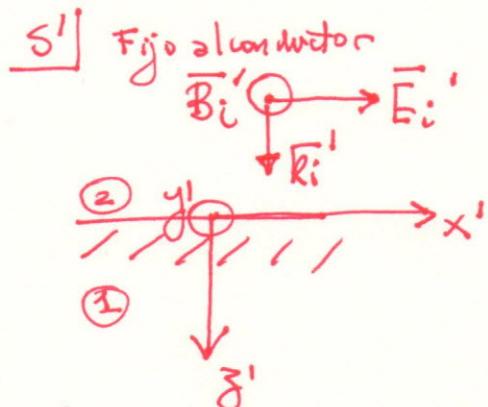
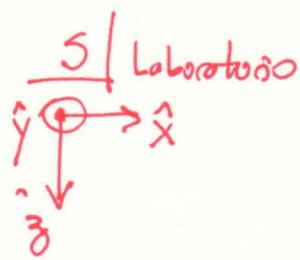
$$\Rightarrow \left\langle \frac{dP}{d\Omega} \right\rangle = \frac{\mu_0}{8} C I_0^2 \cos^2 \theta$$



Sólo en un entorno reducido de $\frac{\pi}{2} = \theta$ la < $\frac{dP}{d\Omega}$ > se parecen!!

P2] conductor perfecto

①



②

$$\text{Según } S' \quad \bar{E}_{in}' = \hat{x}' E_i' e^{i(k'z' - \omega t')} \quad \text{intensidad normal} \quad k'i' = k' \hat{z}'$$

$$\bar{E}_{ref}' = \hat{x}' E_r' e^{i(-k'z' - \omega t')}$$

$$\bar{E}'_{trans} = 0 \quad \text{dado que es un conductor perfecto.}$$

Línea normal al conductor $\hat{n}_{12}' = -\hat{z}'$. La intersección $\hat{z}' = 0$

$$\text{Las condiciones de contorno son } ① \quad (\bar{E}_1' - \bar{E}_2') \times \hat{n}_{12}' = 0$$

$$② \quad (\bar{H}_1' - \bar{H}_2') \times \hat{n}_{12}' = \bar{g}_{12}'$$

$$\text{Los campos totales son: } \bar{E}_{2,\text{tot}}' = \bar{E}_{in}' + \bar{E}_{ref}' \\ \bar{E}_{1,\text{tot}}' = 0$$

$$\text{Según } ①: [-\hat{x}' (E_i' e^{i(k'z' - \omega t')} + E_r' e^{i(-k'z' - \omega t')})] \times (-\hat{z}') = 0 \\ \Rightarrow \boxed{E_r' = -E_i'}$$

$$\bar{B}_i' = \sqrt{\mu_0 \epsilon_0} \hat{R}_i' \times \bar{E}_i', \quad k'i' = k' \hat{z} \therefore \hat{R}_i' = \hat{z}'$$

$$= \sqrt{\mu_0 \epsilon_0} \hat{z}' \times \hat{x}' E_i' e^{i(k'z' - \omega t')}$$

$$\bar{H}_i' = \frac{\bar{B}_i'}{\mu_0}$$

$$= \sqrt{\mu_0 \epsilon_0} \hat{y}' E_i' e^{i(k'z' - \omega t')}$$

$$\bar{B}_r' = \sqrt{\mu_0 \epsilon_0} \hat{y}' E_i' e^{i(k'z' - \omega t')} \quad \bar{B}_t' = 0 \quad \bar{H}_r' = \frac{\bar{B}_r'}{\mu_0}$$

Según (B) $\vec{H}_2^1 = 0$ para ser conductor y además $\hat{n}_{12} = -\hat{z} \Rightarrow$

$$\vec{g}_e^1 = -\vec{H}_2^1 \times \hat{n}_{12} \Big|_{\vec{z}^1=0}$$

$$\cdot \quad \vec{g}_e^1 = \frac{1}{\mu_0} (\sqrt{\mu_0 \epsilon_0}) E_i^1 \left[e^{i(k^1 z^1 - \omega^1 t^1)} + e^{i(k^1 z^1 + \omega^1 t^1)} \right] \hat{z}^1 \times \hat{z}^1$$

$$\boxed{\vec{g}_e^1 = \sqrt{\frac{\epsilon_0}{\mu_0}} E_i^1 e^{-i\omega^1 t^1} \hat{x}^1} \rightarrow J^\mu|_{S^1} = (0, g_e^1 \delta(z), 0)$$

(b) Paseaje de $S' \rightarrow S$ varas de transformación inversa

$$A^\mu|_{S^1} = L^{\mu\nu}(-\beta) A^\nu|_{S^1}$$

Por el caso de un boost en \hat{x}' : • $L^{\mu\nu}(-\beta) = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}_{2 \times 2}$
dónde seguimos el ordenamiento $\{t, x, y, z\}$.

Los vectores $k^{\mu i}$ según $\underline{S'}^1$ son: $k^{\mu i} = k^1 (1, 0, 0, 1)$, $k^1 = \frac{\omega^1}{c}$

$$k^{\mu i} = k^1 (1, 0, 0, 1)$$

La cuadrivectora $\underline{S'}^1$ es: $J^{\mu i}|_{S^1} = (0, g_e^1 \delta(z), 0, 0)$

$$\text{Por ejemplo, } k^{\mu i}|_{S^1} = L^{\mu\nu}(-\beta) k^{\nu i}|_{S^1} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= k^1 \begin{pmatrix} \gamma \\ -\beta\gamma \\ 0 \\ 1 \end{pmatrix}$$

$$J^{\mu i}|_{S^1} = (-\beta\gamma g_e^1 \delta(z), \gamma g_e^1 \delta(z), 0, 0)$$

Para los campos usamos que $F^{\alpha\beta}|_{S^1} = (L^{-1})^\mu_\alpha F^{\mu\nu}|_{S^1}, (L^{-1})^\nu_\beta$,

$$\text{y se obtiene } E_x = E_x^1 \quad E_z = \gamma (E_z^1 - \beta B_y^1) \\ E_y = \gamma (E_y^1 + \beta B_z^1) \quad E_z = \gamma (E_z^1 - \beta B_y^1)$$

$$B_x = B_x^1, \quad B_y = \gamma(B_y^1 - \beta E_z^1), \quad B_z = \gamma(B_z^1 - \beta E_y^1) \quad (3)$$

Para los ondas incidente tenemos $\vec{E}_i^1 = E_i^1 \hat{x}^1$
 $\vec{B}_i^1 = B_i^1 \hat{y}^1$

$$\Rightarrow E_{x^{inc}} = E_{x^{inc}}^1, \quad E_{y^{inc}} = \gamma(0 + \beta \cdot 0) = 0$$

$$E_{z^{inc}} = \gamma(0 - \beta B_{y^1}^1) \quad \underline{E_{z^{inc}} = \gamma(0 - \beta B_{y^1}^1)}$$

$$\Rightarrow \underline{E_{inc} = E_{x^{inc}}^1 \hat{x} + E_{z^{inc}}^1 \hat{z} = E_{x^{inc}}^1 \hat{x} + (-\gamma \beta) B_{y^{inc}}^1 \hat{z}}$$

$$B_{x^{inc}} = 0, \quad B_{y^{inc}} = \gamma(B_{y^1}^{inc} - \beta \cdot 0) = \gamma B_{y^1}^{inc}$$

$$\underline{\underline{B_{z^{inc}} = \gamma(0 - \beta E_{y^{inc}}^1) = 0}} \quad B_{z^{inc}} = \gamma B_{y^1}^{inc} \hat{y}$$

Lo mismo para los campos reflejados.

Problema ③ (a)



$$\bar{B}_{\text{ext}} = B_e e^{-i\omega t} \hat{z}$$

Planteamos las ecuaciones de Maxwell dentro del conductor, considerando que

$$\bar{J} = \sigma \bar{E}_T$$

$$\begin{cases} \bar{\nabla} \cdot \bar{E}_T = 0 \\ \bar{\nabla} \times \bar{E}_T = -\frac{1}{c} \frac{\partial \bar{B}_T}{\partial t} \end{cases} \quad \begin{cases} \bar{\nabla} \cdot \bar{B}_T = 0 \\ \bar{\nabla} \times \bar{B}_T = \frac{4\pi}{c} \sigma \bar{E}_T + \frac{1}{c} \frac{\partial \bar{E}_T}{\partial t} \end{cases}$$

donde $\bar{E}_T(\vec{r}, t)$ y $\bar{B}_T(\vec{r}, t)$ son el campo eléctrico y magnético TOTAL

Dado que la excitación externa es armónica (proporcional a $e^{-i\omega t}$) planteamos como siempre el siguiente ansatz para los campos:

$$\bar{E}_T(\vec{r}, t) = \bar{E}(\vec{r}) e^{-i\omega t}$$

$$\bar{B}_T(\vec{r}, t) = \bar{B}(\vec{r}) e^{-i\omega t}$$

A su vez para la partes espaciales de los campos ($\bar{E}(\vec{r})$ y $\bar{B}(\vec{r})$) expandimos

$$\bar{E}(\vec{r}) = \sum_{m=0}^{\infty} \bar{E}_m(\vec{r})$$

$$\bar{B}(\vec{r}) = \sum_{m=0}^{\infty} \bar{B}_m(\vec{r}).$$

donde $\bar{E}_m(\vec{r})$ es de orden $(\frac{\omega}{c})^m$ y $\bar{B}_m(\vec{r})$ también.

Al reemplazar el ansatz en las ecuaciones de Maxwell nos queda el siguiente sistema:

$$\bar{\nabla} \cdot \bar{E}_m(\vec{r}) = 0 \quad \forall m \in \mathbb{N}_0$$

$$\begin{cases} \bar{\nabla} \times \bar{E}_0(\vec{r}) = 0 \\ \bar{\nabla} \times \bar{E}_m(\vec{r}) = \frac{i\omega}{c} \bar{B}_{m-1}(\vec{r}) \quad \forall m \in \mathbb{N} \end{cases}$$

$$\bar{\nabla} \cdot \bar{B}_m(\vec{r}) = 0 \quad \forall m \in \mathbb{N}_0$$

$$\begin{cases} \bar{\nabla} \times \bar{B}_0(\vec{r}) = \frac{4\pi}{c} \sigma \bar{E}_0(\vec{r}) \\ \bar{\nabla} \times \bar{B}_m(\vec{r}) = \frac{4\pi}{c} \sigma \bar{E}_m(\vec{r}) - \frac{i\omega}{c} \bar{E}_{m-1}(\vec{r}) \quad \forall m \in \mathbb{N}. \end{cases}$$

Ahora pasamos a resolver orden a orden hasta llegar al orden más bajo no trivial (ie: sin contar el orden cero, el orden más bajo).

ORDEN 0:

$$\begin{cases} \bar{\nabla} \bar{E}_0(F) = 0 \\ \bar{\nabla} \times \bar{B}_0(F) = 0 \end{cases} \rightarrow \text{Esto no implica que } \bar{E}_0 \text{ sea nulo, pero el problema es, en el caso est\'atico magn\'etico y } \Rightarrow \text{ no hay raz\'on para que exista } \bar{E} \text{ a orden cero. } \Rightarrow \boxed{\bar{E}_0(F) = 0}$$

$$\begin{cases} \bar{\nabla} \bar{B}_0(F) = 0 \\ \bar{\nabla} \times \bar{B}_0(F) = 4\pi\sigma \bar{E}_0(F) = 0 \end{cases} \downarrow \quad \rightarrow \text{En el caso est\'atico el campo magn\'etico tiene que ser } \bar{B}_e \hat{z} \text{ ya que cumple las condiciones de contorno en la superficie de la esfera. (continuidad de campo normal y tangencial de } \bar{B} \text{) y en el } \infty \text{ (} \bar{B} = \bar{B}_e\text{). Luego el orden cero del campo tiene que ser: } \boxed{\bar{B}_0(F) = \bar{B}_e \hat{z}}$$

ORDEN 1:

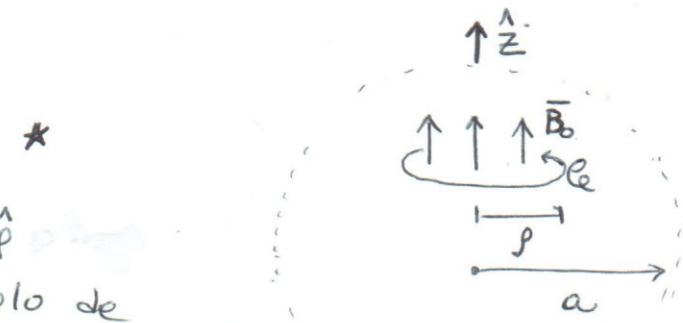
$$\begin{cases} \bar{\nabla} \bar{E}_1(F) = 0 \\ \bar{\nabla} \times \bar{E}_1(F) = i\omega \bar{B}_0(F) = i\frac{\omega}{c} \bar{B}_e \hat{z} \end{cases} *$$

Para resolver supongo $\bar{E}_1(F) = E_1(r)\hat{\phi}$

Sea C la curva dada por un c\'irculo de

radio $\rho = \sqrt{x^2 + y^2} < a$ y $S(C)$ la superficie plana que encierra. Integro * en $S(C)$ orientada seg\'un la normal \hat{z} :

$$\underbrace{\iint_{S(C)} \bar{\nabla} \times \bar{E}(F) \cdot d\bar{s}}_{= \int_0^{2\pi} E(r) \rho d\varphi} = \frac{i\omega}{c} \bar{B}_e \iint_{S(C)} \hat{z} \cdot d\bar{s}$$



$$\iint_{S(C)} d\bar{s} = \pi \rho^2.$$

$$\Rightarrow E(r) = \frac{i\omega}{c} \bar{B}_e \frac{\pi \rho^2}{\pi r^2} = i \bar{B}_e \left(\frac{\omega \rho}{2c} \right)$$

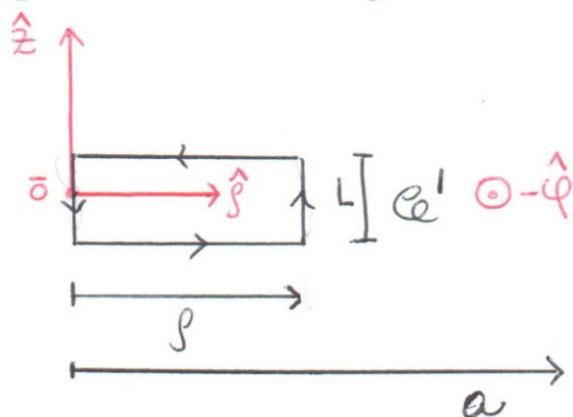
$$\Rightarrow \boxed{\bar{E}_1(F) = i \bar{B}_e \left(\frac{\omega \rho}{2c} \right) \hat{\phi}}$$

Calculamos $\bar{B}_1(F)$:

$$\left\{ \begin{array}{l} \bar{\nabla} \bar{B}_1(F) = 0 \\ \bar{\nabla} \times \bar{B}_1(F) = \frac{4\pi\sigma}{c} E_1(F) - i\omega \bar{E}_1(F) = \frac{4\pi}{c} \Gamma iBe \left(\frac{\omega}{2c} \right) \hat{\varphi}. \end{array} \right.$$

Propongo $\bar{B}_1(F) = B_1(\rho) \hat{z}$ con $B_1(0) = 0$ ya que el orden 1 comienza a corregir el orden 0 fuera del eje \hat{z} .

Sea C' la curva dada por un rectángulo de altura L y base ρ contenida en un plano $\varphi = \text{cte}$ como en el dibujo:



Integro \heartsuit en $S(C')$, la superficie^{Plana} de normal $-\hat{\varphi}$ encerrada por C' :

$$\iint_{C'} \bar{\nabla} \times \bar{B}_1(F) d\bar{s} = \underbrace{\frac{4\pi}{c} \Gamma iBe \frac{\omega}{2c}}_{\int \int} \underbrace{\left[\int_0^{\rho} \int_0^{L/2} d\rho dz (-\hat{\varphi}) \right] \rho \hat{\varphi}}_{d\bar{s}} - L \frac{\rho^2}{2}.$$

$$\Rightarrow A \cdot B_1(\rho) = - \frac{4\pi}{c} \Gamma iBe \frac{\omega}{2c} \frac{\rho^2}{2}$$

$$- \frac{2\pi}{c} \Gamma iBe \left(\frac{\omega \rho^2}{2c} \right)$$

$$\Rightarrow B_1(\rho) = - \frac{2\pi\sigma}{c} iBe \frac{\omega \rho^2}{2c}$$

$$\frac{\sigma \omega}{c^2} = \frac{1}{\rho^2}$$

$$\therefore \bar{B}_1(F) = - \frac{2\pi\sigma}{c} iBe \frac{\omega \rho^2}{2c} \hat{z}$$

Conclusion: $\bar{B}_T(Ft) = Be \left(1 - \frac{2\pi\sigma}{c} i \frac{\omega \rho^2}{2c} \right) e^{-i\omega t} \hat{z}$

$$\bar{E}_T(Ft) = iBe \frac{\omega \rho}{2c} e^{-i\omega t} \hat{\varphi}$$

Problema 3 (b).

Potencia : $P(t) = \int \bar{J}(\vec{r}t) \bar{E}(\vec{r}t) d\vec{r}^3 = \int \sigma \bar{E}(\vec{r}, t) \cdot \bar{E}(\vec{r}, t) d\vec{r}^3$

$$\bar{E}(\vec{r}, t) = \operatorname{Re} \left(i B e \frac{\omega \rho}{2c} e^{-i\omega t} \hat{\varphi} \right) = B e \frac{\omega}{2c} \rho \sin \omega t \hat{\varphi}.$$

$$\rightarrow P(t) = \int_0^a dr \int_0^{2\pi} d\varphi \int_0^\pi d\theta r^2 \sin \theta \left(\frac{B e \omega}{2c} \right)^2 \sigma r^2 \sin^2 \theta \sin^2 \omega t.$$

$$\rho = r \sin \theta$$

$$= \underbrace{\left(\int_0^a dr r^4 \right)}_{\frac{a^5}{5}} \underbrace{\left(\int_0^{2\pi} d\varphi \right)}_{2\pi} \underbrace{\left(\int_0^\pi d\theta \sin^3 \theta \right)}_{4/3} \cdot \left(\frac{B e \omega}{2c} \right)^2 \sigma \sin^2 \omega t.$$

$$= \frac{a^5}{5} \cdot 2\pi \cdot \frac{4}{3} \frac{B e \omega^2}{4 c^2} \sigma \sin^2 \omega t.$$

$$P(t) = \frac{2\pi}{15} B e^2 \frac{\omega^2 \sigma}{c^2} a^5 \sin^2 \omega t$$

Balance de energía: $V = \text{esfera de radio } a$.

$$\underbrace{\int_V \bar{J} \bar{E} dV}_{\textcircled{1}} + \underbrace{\frac{d}{dt} \int_V \frac{1}{8\pi} (\bar{E}^2 + \bar{B}^2) dV}_{\textcircled{2}} = - \oint_S \vec{n} \cdot \left(\frac{c}{4\pi} \bar{E} \times \bar{B} \right) d\vec{S} \quad \textcircled{3}$$

(1) es $P(t)$ y lo calculamos.

Calculamos (2): $\bar{B}(r, t) = B e \left(\omega \omega t - \frac{2\pi \sigma}{c} \frac{\omega \rho^2}{2c} \sin \omega t \right) \hat{z}$

$$E^2 + B^2 = \left(\frac{Be\omega}{2c}\right)^2 \int r^2 \sin^2 \omega t + Be^2 \omega^2 \sin^2 \omega t - Be^2 \frac{4\pi\sigma}{c} \frac{\omega}{2c} \int r^2 \sin^2 \omega t + Be^2 \left(\frac{2\pi\sigma}{c} \frac{\omega}{2c}\right)^2 \int r^4 \sin^2 \omega t$$

$$\Rightarrow \frac{1}{8\pi} \int_V E^2 + B^2 dV = \frac{1}{8\pi} \left[\left(\frac{Be\omega}{2c} \right)^2 \sin^2 \omega t \left(\int_V r^2 dV \right) + Be^2 \omega^2 \sin^2 \omega t \left(\int_V r^4 dV \right) \right. \\ \left. - Be^2 \frac{4\pi\sigma}{c} \frac{\omega}{2c} \int_V r^2 \sin^2 \omega t \left(\int_V r^2 dV \right) + Be^2 \left(\frac{2\pi\sigma}{c} \frac{\omega}{2c} \right)^2 \sin^2 \omega t \int_V r^4 dV \right]$$

$$\int_V r^2 dV = 2\pi \int_0^\pi d\theta \int_0^a dr r^2 \sin \theta \quad r^2 \sin^2 \theta = 2\pi \frac{a^5}{5} \frac{4}{3} = \frac{8\pi}{15} a^5.$$

$$r = \rho \sin \theta$$

y ya integrar
en θ

$$\int_V dV = \frac{4}{3} \pi a^3$$

$$\int_V r^4 dV = 2\pi \int_0^\pi d\theta \int_0^a dr r^4 \sin^2 \theta \quad r^4 \sin^4 \theta = 2\pi \frac{a^7}{7} \cdot \frac{16}{15}.$$

$$\Rightarrow \frac{1}{8\pi} \int_V E^2 + B^2 dV = \frac{1}{8\pi} \left[\left(\frac{Be\omega}{2c} \right)^2 \frac{8\pi}{15} a^5 + Be^2 \left(\frac{2\pi\sigma}{c} \frac{\omega}{2c} \right)^2 \cdot \frac{2\pi a^7}{7} \frac{16}{15} \right] \sin^2 \omega t.$$

$$+ \frac{1}{8\pi} Be^2 \frac{4}{3} \pi a^3 \omega^2 \sin^2 \omega t - Be^2 \frac{4\pi\sigma}{c} \frac{\omega}{2c} \frac{8\pi}{15} a^5 \omega \sin^2 \omega t.$$

Derivando respecto a t obtenemos ②.

Análogamente se hace integral ③ y se comprueba la relación. ❤️ =
Primer orden.