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The force between two charged wires

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For two circular cross-section "infinitely long" wires charged to opposite electric potential, it is a standard problem to calculate the potential field, the attractive force per unit wire length, etc. If, however, the potentials are not exactly opposite, the problem is not even well defined. As shown here, the problem becomes well defined when the physical environment of the wires is considered. An effective electrical ground is imposed on the problem either by the presence of nearby conductors or due to the finite length of the wires.

I. INTRODUCTION AND OVERVIEW

A standard configuration in electrostatics involves two long parallel conductors charged to opposite electrical potentials $+V$ and $-V$. If the length of the conductors is much larger than their separation (labeled d in Fig. 1), the standard approach is to take the conductors to be infinitely long and thereby to reduce the problem to one in two-dimensional electrostatics. If the conductor cross sections are circular, the fields can be found in closed form[1] with the use of complex variable techniques, image line charges, or bipolar coordinates. In SI units, λ, the charge per unit

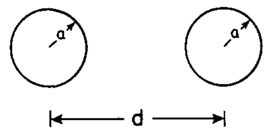

Fig. 1. Two infinitely long parallel conducting cylinders with circular cross sections. If the cylinders have opposite potentials imposed on them, the induced charge per unit length and the force of attraction per unit length can be found in terms of elementary functions.

length, and F, the force of attraction per unit length, are

$$\lambda = 4\pi\epsilon_0 V/\cosh^{-1}[(d^2/2a^2) - 1]$$
$$\approx 2\pi\epsilon_0 V/\ln(d/a),$$
$$F = (8\pi\epsilon_0/d\sqrt{1 - 4a^2/d^2})$$
$$\times\{V/\cosh^{-1}[(d^2/2a^2) - 1]\}^2$$
$$\approx (2\pi\epsilon_0/d)[V/\ln(a/d)]^2. \quad (1)$$

The approximate expressions given above are the limiting forms that apply when the wires are far apart compared to their radii, that is, when $d \gg a$.

It is easy to lose sight of the crucial role played in this configuration by the choice that the potentials on the two conductors have opposite polarity.[2] Consider, for example, the case in which the same potential $+V$ is applied to both wires. Two different viewpoints lead to very different predictions about the consequence of the applied potential. A "practical" scientist, especially one familiar with electroscopes, would argue that charging two closely spaced wires to the same potential would cause the wires to repel each other.[3] A "mathematical" scientist would argue, quite differently, that the problem is described by Laplace's equation, $\nabla^2\Phi = 0$, with the boundary conditions that the electrostatic potential Φ is $+V$ on the boundaries. Therefore, the unique solution is that $\Phi = +V$ everywhere. But if that were correct, there would be no electric fields (since $\nabla\Phi$ would vanish), no electrical charges on the wires (from Gauss' law), and no force between the wires.

Clearly, some information is missing; the physical problem is incompletely specified. It might seem that what is missing is the charge per unit length on the wires, but to specify that would be to sidestep the real issue: Experimentally, we impose "voltage" on the wires, not charge. Nature figures out what the charge must be. The charge per length should then be a result contained in the solution of a correctly posed mathematical problem, not an input into such a problem.

The difficulty has to do with boundary conditions,[4] in particular with the spatial location of ground, i.e., of zero potential. For the case of opposite potentials on the wires, it is clear that ground is located at the median plane between the wires. If, on the other hand, both cylinders are at $+V$, there is no natural location for ground. In particular, ground cannot in any simple way be at spatial infinity since the mathematics of two-dimensional electrostatics demands that the potential difference between spatial infinity and the wires is infinite, except in the single case that the wires have opposite potentials or, more generally (that is, if the wires have different cross sections), except in the case that the sum of the linear charge densities on the wires is zero.[2]

Where, then, is ground? Let us suppose that the wires are $L = 1$ m long and are separated by $d = 1$ mm. We must distinguish two different kinds of experimental environments for these wires. In the first environment [Fig. 2(a)], there are large conductive elements (tables, laboratory apparatus, or whatever) at a distance R_∞, say 1 cm, from the wires. These external conducting elements may be far away in comparison with the separation between the wires, but they are close compared to the wire length. More generally this case is characterized by $L \gg R_\infty$. When this condition applies, it is justified to continue to use a two-dimensional ("infinitely long") viewpoint for the wires,[5] but the large external element must be taken into consideration. This element introduces a large equipotential surface into the neighborhood of the wires, a surface that we can define as "ground." (That is, the voltage on the wires should be taken with reference to the external conductive element. In determining the charge on the wires or the force between the wires, the voltage relative to some nominal circuit ground is irrelevant. It is the difference between the potential of the nearby conductive element and the potential of the wires that has physical consequences.) What is interesting and usually unappreciated about this sort of configuration is that the charges induced on the wires, and the forces between the wires, depend crucially on the location of the external conductive element. Without the location and shape of that element, the problem is incompletely specified.

The second type of electrostatic environment [Fig. 2(b)] is that in which the wires are isolated; there are no relevant external conductive elements. More realistically this means that $L \ll R_\infty$, as would be the case for our 1-m wires if they were many meters from any other conductive element. In this case, we must recognize the fact that the wires exist in a three-dimensional world. At points much closer to the wires than 1 m, and not too near the ends of the

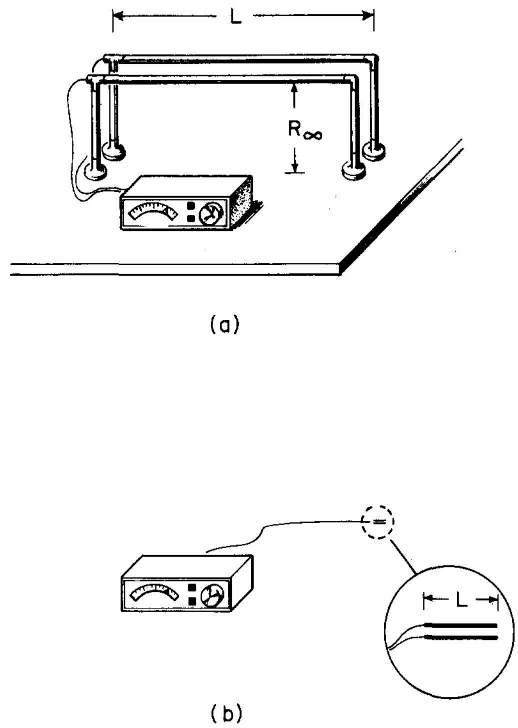

Fig. 2. The two electrostatic environments of two "infinite" parallel wires. In (a) a large metal laboratory table is at a distance R_∞ that is small compared to the wire length L. In (b) the wires are isolated; there are no other electrical elements within distances from the wires many times L.

wires, the field structure is approximately two-dimensional, but from many meters away the wires look like a "point" source, not a "line" source. For an isolated point source, of course, the appropriate ground is at spatial infinity. Thus three-dimensional reality can be inserted into the two-dimensional mathematics by imposing an effective ground surface at a distance from the wires of several times L.

The rest of this article will elaborate, with details and model problems, on the central ideas above. Section II deals with the influence of an external conductive element and presents a model problem for which a quantitative description can be given. Section III deals in an approximate quantitative manner with the case of isolated parallel wires.

II. THE INFLUENCE OF GROUND

We consider here the way in which a nearby external conductive element affects the electrical charge induced on wires and the forces on them. An electrostatic configuration with a realistically irregular nearby conducting element does not typically lead to a simple solution, and the need for a numerical solution might obscure the insights that are our goal here. We will therefore use a model problem that has enough flexibility to show the effect of changing the distance to ground and other interesting effects, but which allows a reasonably simple solution. To describe this model problem, in its simplest form, we start with the cylinders in Fig. 1 set to the same potential V. We then choose one of the equipotentials of that solution to be defined as ground, and we characterize the distance to ground as shown in Fig. 3. Once this equipotential is specified as zero potential, the relationship between the potential on the wires and the charge per unit length of the wires is fixed, and the force on each wire can be found.

In the limiting case $a \ll d \ll R_\infty$, it is not difficult to find an approximate solution. We can, in this limit, treat the

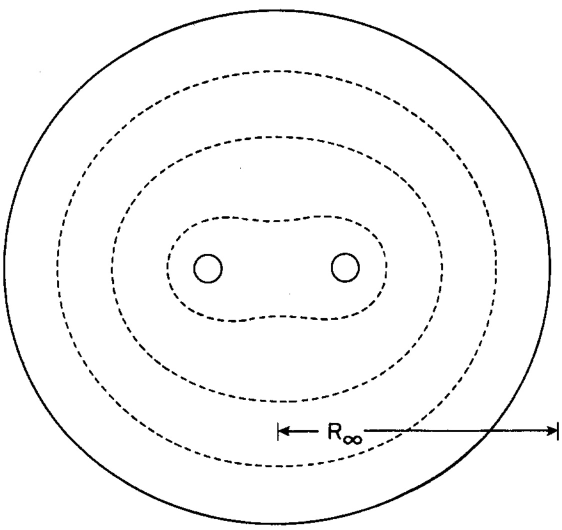

Fig. 3. A model problem for studying the influence of the location of ground. A closed-form solution exists for the potential field of two symmetrically charged circular cross-section conductors. Ground is chosen to be one of the equipotentials and is characterized by the horizontal distance R_∞ from the center of the figure to the equipotential. In the figure, $d/a = 10$, and for the solid curve, $R_\infty/d = 2$. The dashed curves represent other equipotentials, any of which could also be chosen as the zero-potential surface.

fields as if they were due to a uniform surface charge distribution, on each cylinder, with charge per unit length λ. Equivalently, we can view the fields as those due to infinitesimal line charges λ at the cylinder axis. (In reality, the charge distributions will not be uniform, but the nonuniformity can be ignored for $d \gg a$ and $d \ll R_\infty$.) The contribution to the potential from each "line charge" is then $\Phi = (\lambda/2\pi\epsilon_0)\ln(R/r)$, where r is the distance from the line charge, and R is an arbitrary constant. The potential V on the surface of one of the cylinders has contributions from both line charges:

$$V \approx \frac{\lambda}{2\pi\epsilon_0}\ln\frac{R}{a} + \frac{\lambda}{2\pi\epsilon_0}\ln\frac{R}{d}.$$

The potential on the grounding sphere at (large) distance R_∞ is

$$0 \approx \frac{\lambda}{2\pi\epsilon_0}\ln\frac{R}{R_\infty} + \frac{\lambda}{2\pi\epsilon_0}\ln\frac{R}{R_\infty}.$$

From these, we find that $R \approx R_\infty$, and that the induced linear charge density is

$$\lambda \approx \{2\pi\epsilon_0 V/[2\ln(R_\infty/d) + \ln(d/a)]\}, \tag{2a}$$

and that the repulsive force per unit length is[6]

$$F \approx \frac{\lambda^2}{2\pi\epsilon_0 d} \approx \frac{2\pi\epsilon_0}{d}\left(\frac{V}{2\ln(R_\infty/d) + \ln(d/a)}\right)^2. \tag{2b}$$

Though the conditions $a \ll d \ll R_\infty$ allow a usefully simple approximation, they turn out to be unnecessary constraints; the electrostatics problem can be solved in closed form for arbitrary values of a, d, and R_∞. The details of the solution are given in the Appendix. Some numerical results for λ and F are presented in Fig. 4 with dashed lines representing the limiting approximations of Eqs. (2). These results show that the approximations are quite accurate except for the case $d/a = 2.1$, in which case the wires are very nearly touching each other, or the case that R_∞/d is near the limiting value ($R_\infty/d = \frac{1}{2} + a/d$) at which the ground surface is touching the wires. Equations (2), as well as the curves in Fig. 4, show that the closer the grounding surface is, the stronger the force between the wires. This agrees with the simple intuitive picture that a close grounding surface implies large potential gradients and therefore large electrical fields, with the consequence of large charges and forces.

We are now in a position to compare the (repulsive) force between the two wires of Fig. 1 symmetrically charged to $+V$ and $-V$. The mathematics in the Appendix covers the general case, but is rather complicated. The essence of the comparison can be seen in the limiting case of widely separated wires ($d \gg a$) with ground a large distance away ($R_\infty \gg d$). From Eqs. (1) and (2), we see that the forces are approximately equal if $(R_\infty/d)^2 \ll d/a$. But for $(R_\infty/d)^2 \gg d/a$, the force of repulsion in the symmetric case is much less than the force of attraction in the antisymmetric case. There is a competition of two geometric effects: the weakening of the force due to a wide separation and the weakening due to a large distance to ground. If the former one dominates, then the forces in the symmetric and antisymmetric cases have the same magnitude.

If the potentials V_1 and V_2 on the two wires in Fig. 1 are neither equal nor opposite, it will be convenient to describe their potentials by

$$V_{\text{av}} \equiv \tfrac{1}{2}(V_1 + V_2), \quad V_{\text{diff}} \equiv \tfrac{1}{2}(V_1 - V_2),$$

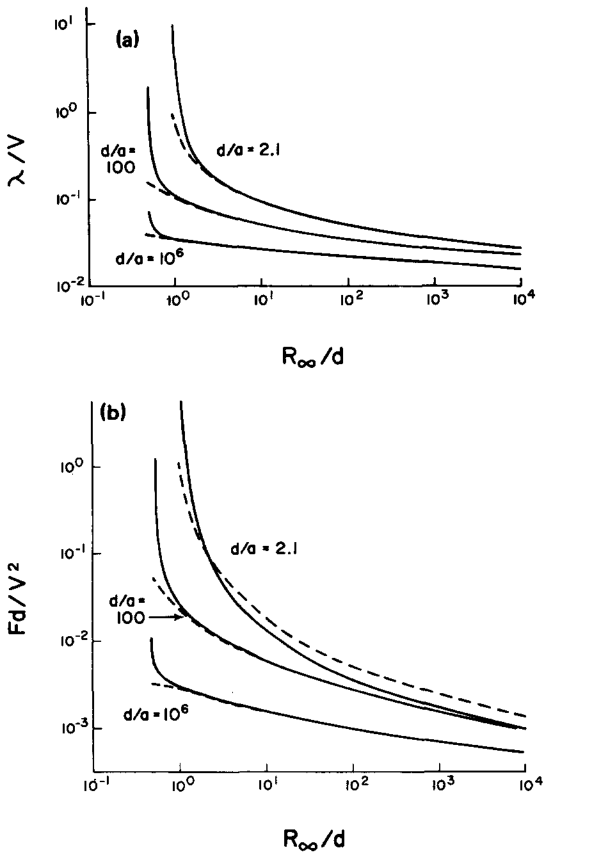

Fig. 4. (a) Induced charge per unit length λ, and (b) force per unit length F, on two wires, both at potential $+V$. Values are given as a function of R_∞/d, the distance to the grounding surface in units of the wire separation. Results are presented for three values of d/a, the separation of the wires in units of the wire radius. Dashed curves show the values given by the limiting form appropriate to large R_∞/d and d/a.

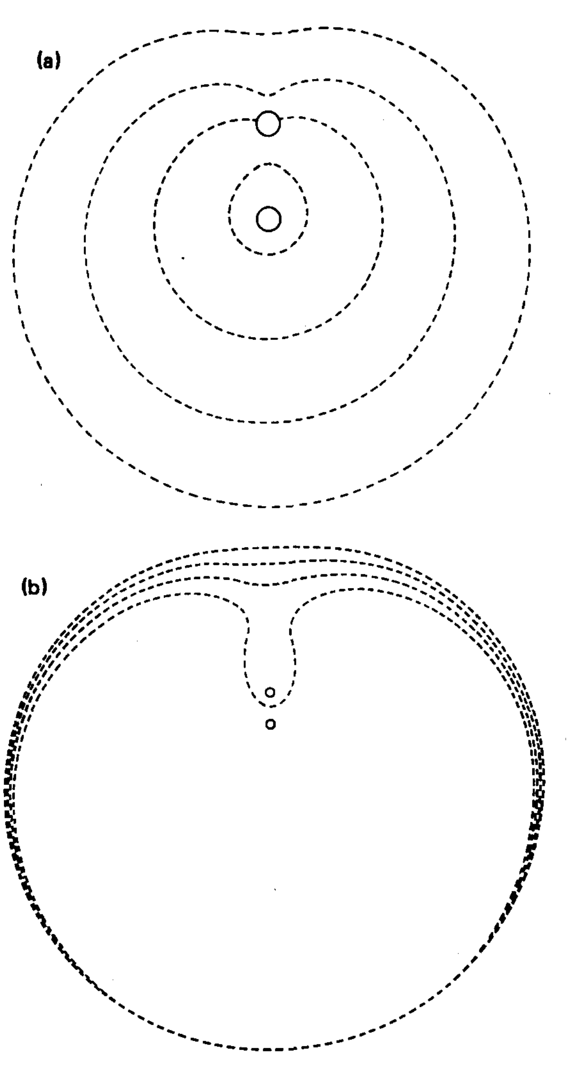

Fig. 5. Equipotentials around two circular cross-section wires with $d/a = 8$, and with $V_{\rm diff} = V_{\rm av}$ in (a) and $V_{\rm diff} = 2V_{\rm av}$ in (b).

so that symmetric and antisymmetric effects will be easily identifiable. If neither $V_{\rm av}$ nor $V_{\rm diff}$ vanishes, several factors complicate the analysis. Foremost is the fact that the equipotentials are now of complex shape. Figure 5 shows examples of this for $V_{\rm diff} = V_{\rm av}$ and $2V_{\rm av}$, both with $d = 8a$. The equipotentials are asymmetric and of complex shape, but in both figures, and in every case except that of $V_{\rm av} = 0$, at sufficient distances the field of the two wires looks like the field of a single wire of potential $V_{\rm av}$. It follows that at sufficiently large values of R_∞/d, equipotentials will always (except in the single case $V_{\rm av} = 0$) be approximately circular and that we can (as we did above for the case $V_{\rm diff} = 0$) use the sufficiently distant equipotentials to represent the location of ground.

The force between the wires, for arbitrary values of R_∞/d, d/a, and $V_{\rm av}/V_{\rm diff}$, is described in the Appendix. As above, the nature of the answer is most easily seen in the limiting case. For widely separated ($d \gg a$) wires with a distant ground ($R_\infty \gg d$), the force of repulsion is

$$F = \frac{2\pi\epsilon_0}{d}\left[\left(\frac{V_{\rm av}}{2\ln(R_\infty/d) + \ln(d/a)}\right)^2 - \left(\frac{V_{\rm diff}}{\ln(d/a)}\right)^2\right]. \tag{3}$$

Note that Eq. (3) agrees with Eq. (1) in the limit $V_{\rm av} = 0$, and with Eq. (2) in the limit $V_{\rm diff} = 0$.

Some interesting patterns can be seen in Eq. (3). In particular, for very thin wires, specifically when $d/a \gg (R_\infty/d)^2$, the attractive and repulsive forces are on an equal footing and approximately cancel if $|V_{\rm av}| \approx |V_{\rm diff}|$. If, on the other hand, $d/a \ll (R_\infty/d)^2$, the attractive part of the force tends to be stronger than the repulsive part, and the overall force can be attractive, even if $V_{\rm av} \gg V_{\rm diff}$.

III. ISOLATED WIRES

If two wires are isolated in space with no nearby grounding electrode, we can consider that there is an effective grounding surface at a value of R_∞ on the order of L, the length of the wires. To see why this is so, we first note that at distances r such that $r \gg d$, but $r \ll L$, the electric potential has the form

$$\Phi = -(1/2\pi\epsilon_0)\lambda_{\rm tot}\ln r + \text{const}. \tag{4}$$

Here, $\lambda_{\rm tot}$ is the sum of the charge per unit length of both wires. If there were a grounding surface at some large $r = R_\infty$, the constant could be expressed in terms of R_∞ to give

$$\Phi = (1/2\pi\epsilon_0)\lambda_{\rm tot}\ln(R_\infty/r). \tag{5}$$

At distances $r \gg L$, the wires can be considered to be a point source with total charge $\lambda_{\rm tot}L$ and therefore with a potential

$$\Phi = \lambda_{\rm tot} L / 4\pi\epsilon_0 r. \tag{6}$$

The actual potential in the median plane of the wires (the plane that bisects the wires and is orthogonal to the wires) can be reasonably well approximated by Eq. (5) for $r \ll L$. That is, Eq. (5) should be correct to order of magnitude until r is considerably larger than L. Similarly, Eq. (6) should be approximately true for $r \gg L$. At $r = L$, both equations must be approximately true and this can only be the case if $\ln(R_\infty / L) \approx 1$, i.e., for

$$R_\infty \approx L. \tag{7}$$

The reasoning here is based on the matching of a two-dimensional "line charge" source to its large-distance, three-dimensional "point source" form, and this reasoning can be checked with a relatively simple closed-form solution. We consider the case of a single isolated thin wire of length L. We model this wire as a conducting prolate spheroid of extreme eccentricity (see Appendix). The model has the disadvantage that the cross-sectional radius a is not constant, but varies along the wire approximately as $a \sim \sqrt{(L/2)^2 - z^2}$, where z, the coordinate along the wire, varies from $-L/2$ to $L/2$. This disadvantage is outweighed by two advantages. First, though the "wire" is not constant in cross section, it *is* constant in charge per unit length λ. This would seem to be just as natural a requirement for the three-dimensional extension of a wire that we have previously considered "infinitely long." And we cannot have it both ways. The wire can be constant in cross section *or* in λ, but not in both. The second advantage of this choice of wire, of course, is that the external field can be expressed in closed form.

In the median ($z = 0$) plane of the wire, the external field is

$$\Phi = (\lambda / 4\pi\epsilon_0) \times \ln\{[\sqrt{(2r/L)^2 + 1} + 1][\sqrt{(2r/L)^2 + 1} - 1]\}. \tag{8}$$

The $r \gg L$ limit gives $\Phi = \lambda L / 4\pi\epsilon_0 r$, which agrees, as it must, with Eq. (6). In the opposite limit, $r \ll L$, Eq. (8) becomes $\Phi = (\lambda / 2\pi\epsilon_0)\ln(L/r)$, and we can infer [by comparison with Eq. (5)] that the effective grounding surface for the wire is at $R_\infty = L$. That is, if we are interested in the fields near the wire (at $r \ll L$) where we consider the wire to be two-dimensional, we must locate a cylindrical grounding surface at $r \equiv R_\infty = L$. This is a specific example of the more general conclusion in Eq. (7).

APPENDIX

The detailed calculations justifying the results reported in the text are best done with bipolar coordinates[7] u and v, which are related to Cartesians by

$$x = c \sinh u / (\cosh u - \cos v),$$

$$y = c \sin v / (\cosh u - \cos v). \tag{A1}$$

The coordinate lines for both u and v are circles, as shown in Fig. 6. The cross sections of the wires are represented by the curves $u = +u_0$ and $-u_0$, where the bipolar quantities u_0 and c are related to the quantities a and d of Fig. 1 by

$$u_0 = \cosh^{-1}(d/2a), \quad c = \sqrt{(d/2)^2 - a^2}. \tag{A2}$$

The potential outside the cylinders at $u = \pm u_0$ must satisfy Laplace's equation $\nabla^2\Phi = 0$, which in bipolar coordinates has the form

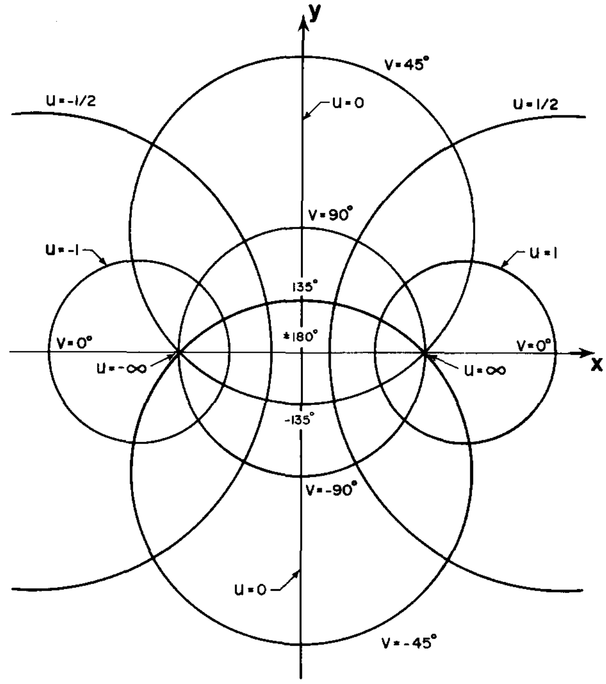

Fig. 6. Bipolar coordinates.

$$\frac{\partial^2\Phi}{\partial u^2} + \frac{\partial^2\Phi}{\partial v^2} = 0. \tag{A3}$$

When the cylinders have opposite potentials $\Phi = \pm V_{\rm diff}$, the appropriate solution to Eq. (A3) is simply

$$\Phi = K_1 u. \tag{A4}$$

Since $\Phi = V_{\rm diff}$ on the $u = u_0$ circle, the constant K_1 is fixed, and the potential becomes

$$\Phi = V_{\rm diff}(u/u_0). \tag{A5}$$

The charge per unit length on the cylinders can be found by computing the charge density $\sigma = -\epsilon_0 \mathbf{n}\cdot\nabla\Phi$, where $\mathbf{n}$ is the unit outward normal. The result of integrating σ around the circumference of the $u = u_0$ circular conductor is a charge per unit length $\lambda_{\rm diff}$ as given (with V in place of $V_{\rm diff}$) by Eq. (1). The value of $\lambda_{\rm diff}$ can be inferred more immediately by noticing that the potential in Eq. (A4) can be viewed as that arising from image line charges $\pm\lambda_{\rm diff}$ at $x = \pm c$, $y = 0$. The electrostatic force between the wires can be found by taking the derivative with respect to the separation d, of the electrostatic energy per unit length $V\lambda_{\rm diff} = (\lambda^2_{\rm diff}/4\pi\epsilon_0)\cosh^{-1}[(d^2/2a^2) - 1]$. Alternatively, and more simply, the force is that between the two image line charges, $(\lambda_{\rm diff})^2/4\pi\epsilon_0 c$, with c given by Eq. (A2).

For two cylinders at $u = \pm u_0$, both charged to the same potential $V_{\rm av}$, a solution to Eq. (A3) must be found that is even in u, which gives $\Phi = V_{\rm av}$ at $u = \pm u_0$, and for which the equipotentials at large distances approach circles about the origin. This solution[8] turns out to be

$$\Phi = V_{\rm av} + K_2\left(|u| - u_0 - 2\sum_1^\infty \frac{e^{-n|u|}}{n}\cos nv + 2\sum_1^\infty \frac{e^{-nu_0}}{n}\frac{\cosh nu}{\cosh nu_0}\cos nv\right). \tag{A6}$$

By computing $-\mathbf{n}\cdot\nabla\Phi$, we find that $K_2 = \lambda_{\rm av}/2\pi\epsilon_0$.

The general potential case, that of the cylinders at two potentials V_1 and V_2, is solved by the superposition of Eqs. (A5) and (A6):

$$\Phi = V_{\rm diff}\frac{u}{u_0} + V_{\rm av} + \left(\frac{\lambda_{\rm av}}{2\pi\epsilon_0}\right)\times\left(|u| - u_0 - 2\sum_1^\infty \frac{e^{-n|u|}}{n}\cos nv + 2\sum_1^\infty \frac{e^{-nu_0}}{n}\frac{\cosh nu}{\cosh nu_0}\cos nv\right). \tag{A7}$$

In this general solution, however, the constraint $\lambda_{\rm av}$ has not been resolved. To do this we must specify the location of ground. In the prescription of the text, this means choosing a point on the x axis of Fig. 6 to be at $x = R_\infty$, the point that defines the equipotential at $\Phi = 0$. The $x = R_\infty, y = 0$ point has bipolar coordinate values

$$v = 0, \quad u = u_\infty \equiv 2\tanh^{-1}(c/R_\infty), \tag{A8}$$

where c is given in terms of a and d by Eq. (A2). By setting Φ to zero at these values of u and v, we find the relationship that determines $\lambda_{\rm av}$ in terms of the potentials:

$$0 = V_{\rm diff}\frac{u_\infty}{u_0} + V_{\rm av} + \left(\frac{\lambda_{\rm av}}{2\pi\epsilon_0}\right)\left(u_\infty - u_0 - 2\sum_1^\infty \frac{e^{-nu_\infty}}{n} + 2\sum_1^\infty \frac{e^{-nu_0}}{n}\frac{\cosh nu_\infty}{\cosh nu_0}\right). \tag{A9}$$

In the case of distant ground ($u_\infty \ll 1$) and thin wires ($u_0 \gg 1$), this reduces to

$$\lambda_{\rm av} \approx 2\pi\epsilon_0\{[V_{\rm diff}(u_\infty/u_0) + V_{\rm av}]/(u_0 - 2\ln u_\infty)\} \approx 2\pi\epsilon_0\{[V_{\rm diff}(u_\infty/u_0) + V_{\rm av}]/[\ln(d/a) + 2\ln(R_\infty/d)]\}. \tag{A10}$$

If $V_{\rm diff}$ (which makes little difference in any case) vanishes, we get the first relation in Eq. (2).

To find the force between the wires, the electrostatic pressure $\epsilon_0 E^2/2$ is computed for the wire on the right. The force per unit area on the wire is then projected in the x direction and integrated over the wire's surface. The force of repulsion is found to be

$$F = \frac{1}{32\pi^2\epsilon_0 c}\int_{-\pi}^{\pi} dv(\cosh u_0\cos v - 1)\times\left(2\lambda_{\rm av} + \frac{4\pi\epsilon_0 V_{\rm diff}}{u_0} + 4\lambda_{\rm av}\sum_1^\infty e^{-nu_0}\cos nv(1 + \tanh nu_0)\right)^2. \tag{A11}$$

In the limit of distant ground ($u_\infty \ll 1$) and thin wires ($u_0 \gg 1$), the force becomes

$$F = \frac{2\pi\epsilon_0}{d}\left[\left(\frac{\lambda_{\rm av}}{2\pi\epsilon_0}\right)^2 - \left(\frac{V_{\rm diff}}{u_0}\right)^2\right]. \tag{A12}$$

Equation (3) follows from Eq. (A12) if $\lambda_{\rm av}$ is replaced with its value from Eq. (A10) and if the negligible term $V_{\rm diff}(u_\infty/u_0)$ in that equation is ignored.

In Sec. III the solution is used for the field outside a charged conducting prolate spheroid. A family of confocal spheroids, with focii at $z = \pm L/2$, can be parametrized with μ and defined in terms of Cartesian coordinates by

$$\frac{x^2 + y^2}{\mu^2 - 1} + \frac{z^2}{\mu^2} = \left(\frac{L}{2}\right)^2. \tag{A13}$$

The degenerate case $\mu = 1$ corresponds to the segment of the z axis from $z = -L/2$ to $+L/2$. The field outside a prolate spheroid is given by

$$\Phi = (Q/4\pi\epsilon_0 L)\ln[(u + 1)/(\mu - 1)], \tag{A14}$$

where Q is the total charge on the spheroid. For the extreme $\mu - 1 \ll 1$ case, Q/L can be viewed as the charge per unit length of the wirelike limiting spheroid. In the median ($z = 0$) plane, $\mu = \sqrt{(2r/L)^2 + 1}$ and the potential has the form in Eq. (8).

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[1] This problem is presented or solved, with varying levels of detail, in many texts, e.g., J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962), 2nd ed., Prob. 2.4; L. Page and N. I. Adams, *Principles of Electricity* (Van Nostrand, Princeton, 1969), 4th ed., Sec. 32; E. M. Pugh and E. W. Pugh, *Electricity and Magnetism* (Addison-Wesley, Reading, MA, 1960), Sec. 4-11; W. R. Smythe, *Static and Dynamic Electricity* (McGraw-Hill, New York, 1939), Sec. 4.13.

[2] More generally, the mathematics in the solutions listed in Ref. 1 allows the wires to be at different potentials if they also have different radii. What is crucial is that the radii and potentials be related such that the charge per unit length on the two wires is equal and opposite.

[3] The practical scientist is correct. If the + lead of a dc voltage supply is attached to an electroscope (the moral equivalent of two long wires) and if the − or "ground" lead is ignored, the electroscope *will* deflect. With 1000 V even a fairly crude electroscope shows a strong deflection.

[4] The importance of boundary conditions, and how easily their importance is overlooked, is evident also in the question of the electric field outside a current-carrying wire. See T. N. Sarachman, Am. J. Phys. **37**, 748 (1969); R. Stoeckly, Am. J. Phys. **38**, 934 (1970); D. Marcuse, Am. J. Phys. **38**, 935 (1970); W. T. Scott, Am. J. Phys. **38**, 936 (1970).

[5] Actually, for the "infinitely long" viewpoint to apply, an additional constraint must be imposed: R_∞ must not vary significantly along the wire or along that length of the wire being considered.

[6] There is an additional contribution to the force on each cylinder due to the charge distribution on the grounding surface, but this is negligible in the limit $R_\infty \gg d$.

[7] H. Margenau and G. M. Murphy, *The Mathematics of Physics and Chemistry* (Van Nostrand, Princeton, 1956), 2nd ed., Sec. 5.14. See also Smythe, Ref. 1.

[8] M. P. Sarma and W. Janischewskyj, "Electrostatic field of parallel cylindrical conductors," IEEE Trans. Power Appar. Syst. **88**, 1069 (1969).