FÍSICA TEÓRICA 1 – 1er. Cuatrimestre 2016

Dos problemas

El monstruo relativista o lo que mata es el relumbrón

Example 3.7 A sphere subtends an angle 2θ when seen from a point P in its rest frame. An observer O passes through P with velocity $\boldsymbol{v}$. Show that to O, the sphere appears to subtend an angle 2α such that

$$c\,\cot\alpha = \gamma(v)(c\cot\theta + \boldsymbol{e}\cdot\boldsymbol{v}\,\mathrm{cosec}\,\theta),$$

where $\boldsymbol{e}$ is a unit 3-vector in the frame of the sphere in the direction from P to the centre of the sphere.

A disconcerting consequence of the formula in this example is that $\alpha \to \pi$ as $\boldsymbol{v} \to -c\boldsymbol{e}$. If O accelerates instantaneously directly ***away*** from the sphere, and increases his speed towards the velocity of light, then the area of the sky filled by the sphere grows. When his speed is $c\cos\theta$, $\alpha = \pi/2$, and the sphere occupies half the sky. At higher speeds, it occupies more than half the sky, and in the limit, only a small hole in the observer's forward direction is left uncovered.

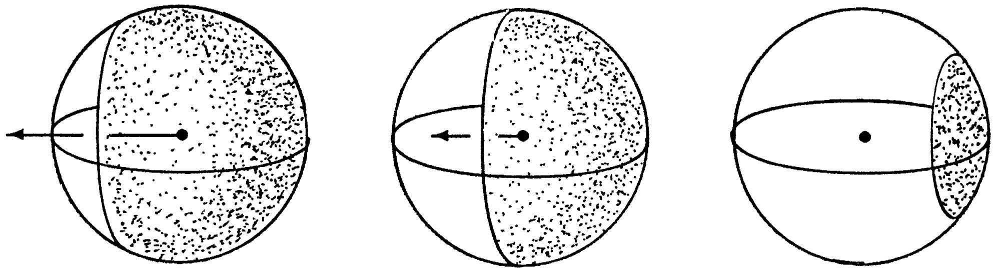


Fig. 3.6. The outline of the sphere as O accelerates away from it to the left. The spheres in the figure represent the sky, with the observer at the centre. The shaded portions are the areas covered by the images of the solid sphere

If an astronaut sees a space monster approaching, with jaws open ready to swallow his spaceship, and if he attempts to accelerate sharply away from the danger, then, when his speed exceeds $c\cos\theta$, where 2θ is the angle originally subtended by the jaws, he appears to be inside the monster's mouth. As he accelerates further, the jaws close around him. He is eventually killed by the glint on the monster's teeth, which is blue-shifted to high-frequency gamma radiation (exercise).

(Woodhouse, *Special Relativity*, 1992.)

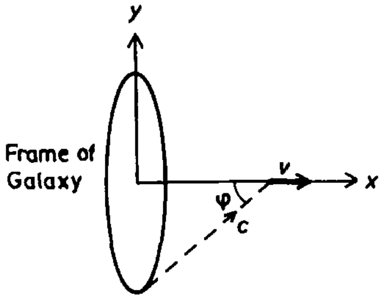
La apuesta de Feynman

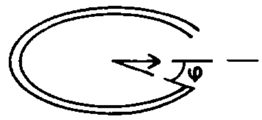
In a simplified version of the ending of one of Fred Hoyle's novels, the hero, traveling at high Lorentz factor at right angles to the plane of our galaxy (Fig. 3.12), said he appeared to be inside and heading toward the mouth of a "goldfish bowl" with a blue rim and a red body (Fig. 3.13). Feynman betted 25 cents that the light from the galaxy would not look that way. We want to see who was right. Take the relative speed to be $\beta = 0.99$ and the angle φ in the frame of the galaxy to be 45° (Fig. 3.12).

(a) Derive (or recall) an expression for the relativistic aberration and use it to calculate (Fig. 3.13) the direction from which light from the edge of the galaxy appears to come when viewed in the spacecraft.

(b) Derive (or recall) the relativistic Doppler effect and use it to calculate the frequency ratio ν'/ν for light from the edge.

(c) Calculate φ' and ν'/ν at enough angles φ to decide who won the bet.





(Yung-Kuo Lim, *Problems and solutions on Mechanics*, 1994.)