AMERICAN JOURNAL of PHYSICS

B and H, the intensity vectors of magnetism: A new approach to resolving a century-old controversy

John J. Roche

Citation: American Journal of Physics **68**, 438 (2000); doi: 10.1119/1.19459
View online: http://dx.doi.org/10.1119/1.19459
View Table of Contents: http://scitation.aip.org/content/aapt/journal/ajp/68/5?ver=pdfcov
Published by the American Association of Physics Teachers

B and H, the intensity vectors of magnetism: A new approach to resolving a century-old controversy

John J. Roche[a)]
Linacre College, Oxford OX1 3JA, United Kingdom

(Received 7 June 1999; accepted 15 September 1999)

The B and H controversy, which has persisted for more than a century, is at bottom a debate over the structure of the macroscopic magnetic field, both in a vacuum and in a magnetized body. It is also a controversy over units and notation. It is paralleled by the problem of D and E in dielectrics. Its origins are traced to a dual magnetic field concept of William Thomson, to an altogether different dual field concept of Faraday, and to Maxwell's attempt to bind the concepts of Thomson and Faraday together. The author argues that severe ambiguities were inadvertently introduced to this subject during its foundational period and subsequently, and that many of these still remain embedded in the present-day interpretation of the subject. The article attempts to clear up a long history of misunderstanding by dealing with each difficulty in the same sequence in which it was introduced to electromagnetism. © *2000 American Association of Physics Teachers.*

I. INTRODUCTION

Such is the subtlety of electromagnetism that it is hardly surprising that despite three centuries of intense research certain concepts remain unclear. The problem of interpretation of B and H is, perhaps, the most complex of all and has attracted a considerable literature. The caption to a *Physics World* article relating to this subject in 1994 described it as a ''magnetic Tower of Babel.''[1] It is a frustrating problem because, although the physics involved is quite well understood, an agreed interpretation has never been found. It offers a marvelous challenge, nevertheless, constantly disclosing new levels of difficulty. A resolution of this problem is important for pedagogy, for the coherence and clarity of macroscopic electromagnetic theory, and for those who have to wrestle with a wide variety of competing systems of units and notation. I believe that in this, as in so many other continuing problems of interpretation in classical electromagnetism, a new approach is called for. That which I have adopted involves establishing when and how each ambiguity first arose. Unraveling the problem in this manner makes it much easier to resolve. I have attempted to be faithful to the notation used by each author.

II. MAGNETIC INTENSITY THEORY FROM POISSON TO KELVIN: DIFFICULT BEGINNINGS

The concept of a ''magnetic intensity'' may have first appeared in a publication of 1769 by the Swiss investigator Jacques Mallet-Favre (1740–1780).[2] It was at first measured comparatively by relating frequencies of vibration of a given magnet at different stations or times.[3] The magnetic intensity was thought of as the magnetic force (or torque) exerted on each individual element of the body rather than as the total force on the body. Until the middle of the 19th century the magnetic intensity was not understood as a property of any magnetic ''field'' but as a force somehow applied directly by the distant source to the local test body.

Simeon Denis Poisson (1781–1840), in a series of seminal publications in 1826 and 1827, laid the foundations for the mathematical study of magnetized bodies. Basing his mathematical and conceptual techniques on Laplace's study of gravity, he showed how the ''intensity of magnetic action'' (today, the ''magnetic field strength'') of a magnetized body could be calculated at an external point from a knowledge of the ''intensity of magnetism'' (the magnetization) of each element of the source body.[4] Although he began his analysis by assuming magnetic molecules, Poisson was responsible for introducing the strategy—still followed today—for effecting a coherent transition from a discrete molecular medium to an idealized continuum.[5] He also introduced the scalar function to magnetism, originally invented by Laplace for gravity, which George Green of Nottingham (1793–1841) was to call the ''potential.''[6] His study of the magnetization of ellipsoidal bodies is of particular importance for the present study. He showed that a homogeneous ellipsoid (including, of course, a sphere) placed in a region of uniform magnetic intensity—such as that provided by the earth—becomes uniformly magnetized. He showed how the magnetic intensity of such an ellipsoid may be precisely calculated at an external point. Poisson's investigations also led to the recognition that the magnetic intensity in a small ellipsoidal hollow in a magnetized body is uniform in magnitude and direction.[7]

One of Poisson's greatest triumphs was his rigorous mathematical explanation why, even though every elementary part of one bar magnet exerts a force and a torque on every part of a neighboring magnet, the force seems to be an interaction between the poles only. To explain this he did not require Coulomb's implausible hypothesis that the end faces of polar magnetic molecules are contiguous and mutually canceling.[8] By a suitable transformation of his volume integral over the dipole medium, Poisson arrived at an integral which could be interpreted to mean that the force between two magnets was partly due to an interaction between a layer of ''free'' magnetic monopole fluid on the ends of each magnet and partly due to an interaction between ''free'' monopole fluids distributed throughout the body of each magnet. In a homogeneous body the latter vanished. The density of the surface distribution was equal to the normal component of the intensity of magnetization and the density of the volume distribution was proportional to the negative divergence of the magnetization.[9] William Thomson in 1849, following Gauss,[10] took pains to emphasize that Poisson's boundary distribution of ''free'' magnetism is ''imaginary magnetic matter.'' Forces between imaginary magnetic matter on the poles of the magnets, though ''convenient,'' are ''very artificial'' and are ''not the same as the real mutual action between the different parts of the magnets themselves.''[11] Ottaviani Mossotti (1791–1863), who applied Poisson's analysis and transformation to dielectrics in 1850, stated that he was not suggesting that the ''electrical stratum...existed in fact on the surface of the dielectric body.''[12] The physical reality or otherwise of properties which emerge as a result of mathematical transformations remains a debated issue today.

Poisson had no difficulty in defining the magnetic intensity at a point external to a magnetized body, but had much more difficulty specifying the magnetic intensity at a mathematical point M inside a magnetized body—and here we arrive at the beginning of another aspect of the modern problem of B and H. In his most refined attack on the problem, Poisson distinguished between the magnetic intensity produced at M (a) by the magnetic molecule in which it was located, (b) by the magnetic matter in an infinitesimal but macroscopic element of the body around M, and (c) by the rest of the magnetized body. He found that the contribution from a shell of (b), however small its volume, was just as important as the contribution from a shell of similar shape centered on M, however large it was and however remote from M. He also found that it was dependent on the shape of the shell but not on its ''absolute dimension.'' All of this arises because the magnetic intensity of dipoles obeys a complex inverse cube law. Being unable to specify either (a) or (b) in the general case Poisson abandoned the attempt to specify the magnetic intensity at points inside magnetized bodies, asserting that only external points matter.[13]

Green in 1828 also struggled with the same problem and made an important advance. Instead of attempting to determine the magnetic intensity acting on a mathematical point, he concentrated instead on calculating the magnetic intensity experienced by an infinitesimal (but macroscopic) element of the magnetized body.[14] Green argued that, in general, the magnetic intensity in the interior of such an element *due to the rest of the medium* varies in magnitude and direction from point to point of the element, but that a spherical element experiences a uniform applied intensity. (It is not clear that he was aware that all ellipsoidal elements experience uniform intensity.) Green then showed how the magnetic intensity applied by the medium to this spherical element could be calculated in principle.[15] By defining the magnetic intensity within a magnetized body in terms of the intensity experienced by a spherical test element of the medium itself, Green eliminated those of Poisson's difficulties which arose from mixing the macroscopic and microscopic in a single theory, and also from attempting to specify the internal magnetic intensity at a mathematical point. Although Green's spherical element definition was not widely accepted, his approach had a considerable influence on William Thomson.

In 1832 Karl Friedrich Gauss (1777–1855) defined unit magnetic pole absolutely as that which exerts a unit force on an equal pole at unit distance. Gauss's force was also measured in absolute units.[16] Gauss employed magnetic fluids more as a calculating device than as a physical hypothesis, referring to them as ''fictive,'' and his metrology actually measured magnetic moments as well as magnetic intensities.[17] Before Gauss, in the work of mathematical analysts such as Poisson and Green, for example, magnetic units were not clearly defined and the algebra was quite abstract.[18]

William Thomson (1824–1907)—later Lord Kelvin—during the period 1845 to 1872 struggled to define the magnetic intensity inside magnetized bodies.[19] He was committed to a fully macroscopic approach to magnetism. In 1850 he defined the magnetic intensity acting ''upon any small portion of an inductively magnetized substance'' as equivalent to ''the actual resultant force which would exist within the hollow space that would be left if the portion considered were removed and the magnetism of the remainder constrained to remain unaltered.''[20] The magnetic intensity in the cavity was further specified in terms of the force on a ''very small bar magnet...placed in a definite position in this space.''[21] The macroscopic magnetic intensity inside a magnetized body primarily meant, therefore, for Thomson as it had for Green, the intensity experienced by an element of the medium. It is important to recognize that Thomson's famous cavity definitions were notional devices introduced to give a sharper definition to the concept of the magnetic intensity experienced by macroscopic elements of the medium itself. He brings this out even more clearly in an article on magnetic permeability published in 1872.[22]

This did not solve Thomson's problem, however, because he recognized that the form of the portion chosen, however small, would influence the intensity that it experienced and consequently he believed that the magnetic intensity defined in this manner ''has no determinate value.''[23] He also writes that ''The resultant force at a point situated in space occupied by magnetized matter is an expression the signification of which is somewhat arbitrary.''[24] Despite these reservations, his studies eventually led him to two definitions of the magnetic intensity inside magnetized bodies that will now be examined carefully.

We have seen that, by notionally replacing the real dipole medium by an appropriate layer of imaginary monopole magnetic matter on the poles (and in the interior for heterogeneous bodies), Poisson obtained the correct calculated result for the magnetic intensity at external points.[25] Thomson now looked at the effect of this substitution on the internal field of the magnet. He found that there was no ambiguity about the magnetic intensity inside the surrogate body: It was simply that due to the boundary monopole layers and the internal monopole distribution.[26] The magnetic intensity experienced by an element of the surrogate medium was now independent of the shape of that element since the medium was no longer magnetized. The latter intensity was also equal to the negative gradient of Poisson's scalar potential function and was well-defined mathematically.[27] Thomson's theoretical investigations also led him to the discovery that the intensity inside this fictional medium was the same in magnitude and direction as that which would be measured in the real medium in a ''split''—which he soon called a ''crevasse''—along the lines of magnetization.[28] In 1871 he termed this the ''polar'' definition of the magnetic intensity inside the magnetized body.[29]

Thomson's second magnetic intensity definition arose from a new integral transformation that he discovered in 1849. He developed Ampère's discovery that a closed current loop is equivalent to a magnetic shell bounded by the loop.[30] While mathematically analyzing magnetized bodies, which could be partitioned into magnetic shells perpendicular to the magnetization (''lamellar'' magnets), Thomson found that the magnetic intensity at external points was the same as that produced by imaginary surface currents numerically equal to the component of the magnetization parallel to the surface (together with an interior current intensity equal to the curl of the magnetization, for heterogeneous bodies).[31] When the real medium was systematically replaced by this second surrogate medium a unique but different value of the magnetic intensity results at each internal point—again because the medium is no longer magnetized. Thomson termed this new definition the ''electromagnetic definition.''[32] It was well-defined mathematically in terms of the imaginary currents. He also found that his ''electromagnetic'' intensity was the same as that existing in the real medium ''in an infinitely small crevasse *perpendicular* to the lines of magnetization.''[33] It was also, of course, the intensity experienced by the lamellar element of the medium that actually filled the crevasse. The ''polar definition'' he found useful for magnets partitioned into fine tubes or ''solenoids'' (from the Greek for tube) parallel to the magnetization, the ''electromagnetic definition'' for lamellar partitioning, and for electromagnets.[34]

Thomson made many further contributions to the theory of magnetization. He advanced considerably Poisson's theory of the magnetization of ellipsoidal cavities and bodies.[35] In 1871 Thomson worked out in detail the analytical properties of his two magnetic intensities. In his units he found that the ''electromagnetic'' intensity was larger than the ''polar'' intensity, by the added amount $4\pi\times$(magnetization).[36] The ''polar'' intensity, ''electromagnetic'' intensity, and magnetization were all in the same direction in isotropic bodies.[37] Thomson found that the divergence of ''polar'' intensity was equal to the negative divergence of the magnetization and that the divergence of the ''electromagnetic'' intensity was always zero, as was the curl of the ''polar'' intensity in the absence of macroscopic currents.[38] The curl of the ''electromagnetic intensity'' was equal to $4\pi\times$(curl of magnetization). It also followed from his investigations that the normal component of the ''electromagnetic'' intensity and the tangential component of the ''polar'' intensity were continuous across the boundary of a magnetized body.[39] Thomson always expresses these relationships using Cartesian components since he does not use vectors. He represents his electromagnetic intensity by X, Y, Z or by F, G, H and the polar intensity by Gothic versions of the same letters. The intensity of magnetization, or magnetic moment per unit volume, is represented by α, β, γ.[40]

For Thomson the two magnetic intensities were qualitatively the same, they were measured by similar procedures in identical units and dimensions, and they reduced to a single intensity in a vacuum.[41] Thomson's study of what is now termed B and H established a tradition of interpretation that is still highly respected. Although he found each of his definitions useful, he never withdrew his earlier statement that the intensity definitions inside a magnetized body were ''somewhat arbitrary.''[42] Some authors this century have agreed with Thomson that the definition of the field intensities inside a material medium is a matter of convention.[43] This issue will be examined carefully below.

III. FARADAY'S MAGNETIC ''INTENSITY'' AND ''QUANTITY''

We now turn to Michael Faraday (1791–1867) for a quite independent and entirely different conception of dual magnetic intensities. The discovery and investigation of electromagnetic induction by Faraday had convinced him by 1851 that a wire moving ''transversely across the lines of force'' measured a property of the field that was ''very different'' both quantitatively and qualitatively from that measured by a vibrating needle in the same field.[44] He came to regard the moving wire as the proper measure of the magnetic field and the magnetic needle an imperfect measure.[45] This view was supported by his discovery that magnetic lines of force, as measured by a transversely moving wire, formed continuous closed tubes circulating through and around the body of a magnet.[46] In his most persuasive experiment Faraday noted that a magnetic needle vibrates more rapidly in water (which is diamagnetic) than in air, when it is placed between the poles of an electromagnet. However, the lines of force are then more spread out in water than in air, or—equivalently—the charge that would circulate through a wire moving transversely across the field is then less than in air. Exactly the reverse occurred when the fluid became paramagnetic.[47] Faraday found many other reasons for supposing that a magnetic needle and a moving wire measured different physical properties of the field.[48]

Thomson could have suggested to Faraday in the 1850s that the vibrating needle creates a longitudinal cavity which experiences a stronger magnetic intensity in a diamagnetic medium than that experienced in the transverse cavity created by the moving wire. However, there is no evidence of any communication between Thomson and Faraday on this matter. Also, it would take more than 50 years before Hendrik Lorentz (1853–1928) proved that the induction of a current in a wire cutting across a field is caused by exactly the same property of the magnetic field—the Lorentz force—as that which causes the torque on a vibrating magnetic needle.[49] It was perfectly reasonable for Faraday in his day, therefore, to suppose that needle and moving wire measured different properties of the field. Faraday seems to suggest that the needle measures what he terms the ''tension'' or ''intensity'' while the transverse wire measures what he terms the ''power'' or ''quantity'' of the field.[50] The 19th century had given the medieval distinction between ''intensity'' and ''quantity'' new meanings.[51] ''Intensity'' and ''quantity'' were now used to distinguish between electric tension and electric charge, between pressure gradient and fluid flow,[52] and also between voltage (or tension) and electric current.[53] It seems quite possible that Faraday had the latter analogy in mind, given his use of the term ''tension'' for the magnetic intensity and his statement that the relationship between magnetic intensity and quantity is controlled by the ''conducting power'' of the magnetic medium.[54] This suggests that Faraday may have thought of the magnetic ''intensity'' as somehow the cause of the magnetic ''quantity'' but, as always, he is hesitant about making assertions of this sort.[55]

IV. MAXWELL MERGES THE THEORIES OF THOMSON AND FARADAY

James Clerk Maxwell (1831–1879), during 1855–56, readily accepted Faraday's theory of two magnetic field properties.[56] He modified and quantified Faraday's magnetic ''quantity'' or ''power'' and it became a new property defined at every point of the field that he termed the ''magnetic induction.'' Maxwell, in accordance with his hydrodynamic model, cautiously hypothesized the magnetic field as a flowing process of some sort in a resisting medium.[57] The ''magnetic intensity'' was a kind of pressure gradient,[58] the medium offered a resistance (even the vacuum contains ether),[59] and the ''magnetic induction'' was the flow per unit area that results.[60] The ''magnetic induction'' he represented by the Cartesian symbols a, b, c and later by the vector symbol $\mathbf{B}$.[61] The magnetic intensity he represented by the Cartesian symbols α, β, γ, and later by the vector symbol $\mathbf{H}$.[62] It is difficult to find a clear justification and explanation of the distinction between B and H in Maxwell's writings, beyond Faraday's experiments and considerations and Maxwell's own analogies with electric currents, hydrodynamics, or mechanics.[63] Here we have the origin of that present-day tradition of interpretation that regards H as the cause of B and interprets B as a flux density. It is also the origin of the magnetomotive force analogy and the theory of the magnetic circuit.[64]

In his great *Treatise on Electricity and Magnetism*, first published in 1873, Maxwell is uncertain about the distinction between B and H. Many parts of his text do sharply emphasize a qualitative distinction. He writes that ''... [both] magnetic force and magnetic induction...are supposed to be observed in a space from which the magnetic matter has been removed.''[65] He also writes that ''magnetic force...produces magnetic induction'' and ''the magnetic induction'' is a directed quantity of the nature of a flux and it satisfies the same conditions of continuity as electric currents and other fluxes do... .[66] Also, Maxwell's measuring definitions of magnetic induction and magnetic intensity are quite different from each other and are strongly influenced by Faraday. B is measured by the electromotive force induced per unit length in a wire that cuts the lines of force perpendicularly.[67] According to Maxwell, ''when the magnetic field is explored by a moving wire...it is the magnetic induction and not the magnetic [intensity] which is measured.''[68] He also makes the magnetic induction responsible for the force acting on a current-bearing conductor placed in the magnetic field.[69] However, in Maxwell's day both of these procedures were recognized measures of the older magnetic intensity H, as was the force on a magnetic pole.[70] Maxwell seems to have found it necessary to redefine H so that it would no longer be responsible for any effect other than the production of B. This is a gradual development in Maxwell's thought which appears to be complete by the end of his *Treatise*. It is the origin of the modern ''source'' definition of H in terms of the current configuration that produces it, rather than in terms of any magnetic action.[71] Since Maxwell makes $\nabla\times\mathbf{H}=4\pi\mathbf{C}$ (the current intensity), in all systems of units, H now received units and dimensions which generally differed from those of B, although not in the case of the electromagnetic system of measurement.[72]

There are other parts of the *Treatise*, however, where Maxwell is very uncomfortable about claiming that B and H are qualitatively different. It is clear that he was strongly influenced in this by Thomson's study of magnetized bodies published in 1872.[73] Since the latter appeared so shortly before the publication of the *Treatise*, it is possible that Maxwell did not have time to digest it properly. Maxwell identified H with Thomson's polar intensity and B with Thomson's electromagnetic intensity and applied Thomson's cavity definitions to H and B.[74] This led him to state that ''The magnetic force and the magnetic induction are identical outside the magnet,'' which seems to contradict the above-quoted statements.[75] He also writes that, at a molecular level, ''...the magnetic force and the magnetic induction are everywhere identical,'' but that he will retain the factor μ linking B and H even in that case ''In order...to be able to make use of the electrostatic or of the electromagnetic system at pleasure.''[76]

The *Treatise*, therefore, contains two very different interpretations of H and B, namely Thomson's theory and Maxwell's cause–effect theory. In Thomson's theory, of course, both H and B are qualitatively the same and are jointly caused by the external field and by the rest of the magnetized medium: Thomson's H is not the cause of Thomson's B. Another ambiguity arose because Maxwell gave Thomson's polar intensity H a different measuring definition, units, and dimensions than those of his electromagnetic intensity B. There are further difficulties. If the qualitative difference between B and H in a vacuum is maintained, there should now be four intensity vectors in a magnetized medium since there will be two qualitatively different field strengths in each of Thomson's cavities or test elements. Of course, Maxwell's ambivalence protects him from this implication.

Instead of Thomson's ''crevasses,'' Maxwell's cavities are a narrow cylinder (later to be called a needle[77]) and a thin disc, respectively.[78] Unlike Thomson, however, Maxwell does not state that the fields in the cavities are to be thought of as the fields experienced by the cylindrical and disc elements, respectively, of the medium which actually fill these cavities. This has led to considerable further difficulties in defining the field intensities inside magnetized bodies. Generally, of course, there are no cavities and they can seem an unnatural way of defining the field inside a magnetized medium.[79] Also, the approach which had been developed by Green and Thomson—to specify macroscopic fields inside magnetized bodies in terms of the macroscopic elements of the medium which actually experience these fields—seems to have slipped away, although remnants of it can be found in polarization theory.[80]

Given Maxwell's own ambivalence over the matter and the rather unfinished character of his theory, it is not surprising that the theory that B and H (and D and E) were different in a vacuum became very controversial. In 1890 Heinrich Hertz (1857–1894) wrote that ''For the determination of the electrical as well as the magnetic state [in the ether] the specification of a single directed magnitude is sufficient to determine completely the change of state under consideration.''[81] Many physicists followed Hertz in this. Richard Becker (1887–1955) wrote in 1932 that ''the distinction in principle between D and E [and between B and H] ...in empty space...has been absolutely abandoned in modern physics.''[82] The distinction had not been abandoned, however. In that same year, in the course of an informal meeting of British and Continental physicists in Paris in July, Sir Richard Glazebrook

> referred to the fact that he was one of the last surviving pupils of Maxwell and he felt convinced from recollections of Maxwell's teaching that [Maxwell] was of the opinion that B and H were quantities of a different kind. When a vote was taken nine were in favor of treating B and H as quantities of a different nature, whilst three were in favor of regarding B and H as quantities of the same nature.[83]

Four years later, in 1936, a subcommittee of the International Electrotechnical Commission (IEC) proposed the names ''gauss'' and ''oersted,'' respectively, for the cgs electromagnetic units of B and H, respectively, even though B and H in a vacuum have the same numerical values and dimensions in that system.[84] The enduring belief that B and H in a vacuum, irrespective of the symbols and units used, are different in kind was again illustrated when a vote on the Paris motion was taken in London at a British Institute of Physics meeting on Faraday in 1991, with a similar outcome. There is no suggestion that this is how physics decides on the interpretation of its concepts, but it does provide a snapshot of the progress of a debate at a particular moment for a particular body of physicists.

As is so often the case with Maxwell, theories that seem quite ambiguous nevertheless turn out to be extraordinarily fruitful. His merging of Faraday and Thomson here led to Maxwell's macroscopic equations inside a magnetized and dielectric medium.[85] Maxwell's theory that H is defined "with reference to a line" and B "with reference to an area" turns out to have particular relevance for a magnetic medium.[86] Also, Maxwell's choice of different units for B and H, even if they are taken to represent the same physical quantity in a vacuum, can be partly justified on the grounds of notational convenience. This can be seen in Box 1.

BOX 1. NOTATION IN MAXWELL'S EQUATIONS

The standard form of Maxwell's vorticity equations in SI, with H measured in A/m and B in T, is as follows:

$$\nabla\times\mathbf{H}=\mathbf{j}+\partial\mathbf{D}/\partial t, \tag{1}$$

$$\nabla\times\mathbf{E}=-\partial\mathbf{B}/\partial t. \tag{2}$$

If both **H** and **B** are measured, say, in tesla and both **E** and **D** in N/coulomb, the equations become

$$\nabla\times\mathbf{H}=\mu_0\mathbf{j}+1/c^2\partial\mathbf{D}/\partial t, \tag{3}$$

$$\nabla\times\mathbf{E}=-\partial\mathbf{B}/\partial t. \tag{4}$$

H and **B** (and **D** and **E**), of course, remain quantitatively different in the new units (except in a vacuum). Clearly, the magnetic vorticity equation now loses its notational simplicity. This suggests that, in certain applications, the use of inconsistent units here may be convenient.

V. INTERPRETING *B* AND *H*, *D* AND *E* AFTER MAXWELL

Many physicists throughout the twentieth century attempted to resolve the interpretative difficulties with B and H. Some have adhered closely to Thomson's interpretation, others to Maxwell's, and yet others have introduced new interpretations. The most important new interpretation of the problem was that introduced by Lorentz and his followers. While accepting in principle Maxwell's theory of dual intensity vectors, Lorentz, like Hertz before him, in practice employed only one vector in free space.[87] From 1902 Lorentz further argued that there is also only one physically significant field vector in a magnetized medium. He postulated that B is the volume average of the microfields and the only authentic field intensity in the medium. H became a mathematical artifact defined by $\mathbf{H}=\mathbf{B}-4\pi\mathbf{M}$, where M is the intensity of magnetization.[88] In a publication of 1909, however, he muddied the waters a little by postulating that H was the average of the microfields.[89] Lorentz also replaces the real medium by a surrogate medium that contains a macroscopic current density $\mathbf{rot\,M}$ (curl $\mathbf{M}$)[90] and a macroscopic surface current of density $\mathbf{n}\times\mathbf{M}$. Richard Becker explains this in more detail in 1932: atomic currents "certainly neutralise each other in the interior [of homogeneous bodies], but there will be left over on the curved surface [of the magnet] a finite, superficially distributed current encircling the cylinder." This current is "actually" present and, together with external currents, is responsible for B. He concludes, "*Not the magnetic force H but the induction B is the primary magnitude* [Becker's italics]. The vector $\mathbf{H}=\mathbf{B}-4\pi\mathbf{I}\ldots$ must be regarded as purely artificial." [91]

Quite sophisticated averaging strategies were used by Lorentz and others who have followed his approach to prove that curl $\mathbf{M}$ is a real current density; nevertheless each strategy at some point introduces a mathematical transformation from which curl $\mathbf{M}$ emerges and is then treated as real.[92] As we have seen with the Poisson transformation, which appears to result in magnetic monopoles at the end-faces of magnets, the results of such transformations must be interpreted with caution. Indeed, the great majority of recent writers insists that these currents are fictional. Also, I have never come across a defense of the claim that electron spin moments of neighboring atoms cancel, or even that orbital spin moments cancel. The basic understanding appears to be that the replacement medium is macroscopically equivalent to the real medium in its magnetic effects.[93] If this is correct, then, as Thomson discovered, the only field in the surrogate medium—which is unmagnetized and a kind of superconductor—is, indeed, B, and H is an artifact and not a physical field intensity. This claim will be carefully examined below.

In Maxwell's *Treatise* of 1873 there is little symmetry between the treatment of dielectrics and magnetized bodies. Symmetrical treatment appears to have been begun by Lorentz in his doctoral thesis of 1875 in explicit analogy with Mossotti's theory of dielectrics and William Thomson's theory of magnetization. Maxwell's E became the field intensity in a needle cavity and the only physical field intensity in a dielectric.[94] Lorentz also postulates that E is the volume average of the microfields in the dielectric.[95] He attempts to justify all of this in various publications.

Lorentz applied Poisson's transformation to a dielectric, thereby replacing the real medium by an imaginary monopole charge distribution of density P_n on the boundary, together with a volume distribution of charge equal to $-\mathrm{div}\,\mathbf{P}$.[96] What the Poisson transformation seems to do here is to replace the real atomic and molecular dipoles by a continuum of infinitesimal volume elements with charges on the end-faces of these elements, faces that are in contact with those of neighboring elements. This, of course, causes sequential canceling or partial canceling of charges, leaving a charge distribution P_n on the dielectric boundary and an internal volume distribution of charges $-\mathrm{div}\,\mathbf{P}$ if the dielectric is inhomogeneous. The resultant medium is not polarized and it has a unique field intensity E caused partly by the applied field, partly by the surface distribution P_n, and partly by the volume density $-\mathrm{div}\,\mathbf{P}$.

Lorentz passed beyond the Poisson transformation to a more sophisticated statistical strategy and appears to have progressively grown to believe that $-\mathrm{div}\,\mathbf{P}$ was a real charge distribution. He justifies the latter by a partitioning of the dielectric medium that involves cutting electric dipoles notionally in two and a subsequent transformation to $-\mathrm{div}\,\mathbf{P}$. Today, the "Lorentz cut" would mean that electrons and nucleons are notionally cut in two. In the Lorentz tradition the displacement vector $\mathbf{D}=(\mathbf{E}+4\pi\mathbf{P})$ seems to be generally thought of as a mathematical artifact.[97]

Various authors from Mossotti onwards have indicated or emphasized the fictional character of the surface and volume charge densities P_n and $-\mathrm{div}\,\mathbf{P}$. Mason and Weaver wrote in 1929 that

> ...it is not correct to say that a non-uniformly polarized body has a volume density of charge given at any point by $-\mathrm{div}\,\mathbf{P}$ and a surface density of charge given by P_n; but it is true to say that any polarized body can be viewed as a non-polarized body having a volume density of charge $-\mathrm{div}\,\mathbf{P}$ and a surface density of charge P_n.[98]

Others, however, seem to have treated these charges as real. Abraham and Becker, for example, wrote in 1932 that ''the surface of a polarized body...carries a surface charge...''[99] and Richard Feynman (1918–88) wrote in 1969 ''We emphasize that this is a perfectly real charge density: we call it the 'polarization charge' only to remind ourselves how it got there.''[100] This is quite difficult to understand since it is well known that the displacement of the positive nucleus within its electron cloud under polarization is of the order of 10^{-6} of the diameter of that cloud.[101] Sequential charge cancellation clearly cannot occur. Furthermore, the real medium has a mean charge density everywhere of zero—even in a heterogeneous medium—if the molecules are electrically neutral and if there are no free charges. However, as in the corresponding magnetic case, the great majority of recent authors assert that these charges are fictional. The substantial claim again appears to be that the electric field of the substitute medium is macroscopically equivalent to that of the real medium. This, together with various other issues, will now be closely examined.

VI. RECONSIDERING *B* AND *H*, AND *D* AND *E*

Three major traditions of interpretation of B and H have now been identified, that of William Thomson which gives H and B equal status as field intensities acting on different ''free-body'' elements of the medium, that of Faraday and Maxwell which makes H the cause of B (and, for some authors, independent of the medium[102]), and that of Lorentz which interprets B as the average of the microfields and H an artifact. Is it possible to resolve these interpretative difficulties?

Were Faraday and Maxwell correct in assuming that two intensity vectors are required to specify the magnetic field in a vacuum? It has been well known since Hertz that only one magnetic field vector is required in practice to specify the vacuum field, and that a second vector seems redundant. This is increasingly chosen to be B. It might be objected, however, that the difference in units and dimensions between B and H (tesla and ampere/meter, respectively, in SI) proves that they are physically different quantities. But does this difference in units necessarily mean that a different physical property is being measured? It is well known, for example, that the units of positive electrical charge in cgs electromagnetic units and cgs electrostatic units are different in magnitude and dimensions but we do not suppose that this means there are two kinds of positive charge.[103] For physical quantities to be distinct, surely they should have different physical properties? Is the physical information about the magnetic field that is provided by H ever different from that provided by B in a vacuum? Suppose the values of H and B are given in magnitude and direction, for a given magnetic field, 3 A/m for example, and 3.77 μT (approximately), respectively. Measurement provides us with information that allows us to reconstruct or recognize a given physical state. Are the physical states that we can reconstruct from H and B in a vacuum ever different? If a current of 3 A turns per m is passed through a solenoid in an appropriate sense, with its axis pointing in the direction of H, a magnetic field will be produced along that axis of the solenoid with the value H. If the various properties of this field are measured, such as direction, ability to exert a torque or a force or to induce motional emf, they will be found to be exactly the same as those of the magnetic field that is reconstructed from the corresponding value of B. This is true in all circumstances. This suggests that the information content about the field provided by H in a vacuum is always exactly the same as that provided by B and that they are simply different measures of exactly the same field property.[104] H measures it by its cause, B by its effect. It is also surely significant that almost 150 years after Faraday no such pair of distinguishable vacuum field intensities has been revealed experimentally, nor is there any theoretical basis for such a distinction.

Suppose, for the sake of argument, we now accept that there is only one magnetic field in a vacuum. In order to resolve the residual ambiguities caused by the two distinct measuring definitions which now exist for this field, it seems appropriate that the B measure should be taken as standard and the H measure as auxiliary—useful in certain circumstances. A similar argument shows that D measures the electric field in a vacuum by its cause (the charge per unit area on capacitor plates required to reproduce the field) and E by its effect (the force exerted by the field on unit charge). Let us further assume for the rest of this article that there is only one electric field in a vacuum and that the E measure is the standard with the D-measure auxiliary. From this point onwards, therefore, both H and B will be measured in T (tesla) and D and E in N/C (newton per coulomb).

Even if we assume that there is only one magnetic field strength in a vacuum, and only one unit and measuring definition for field strength, several problems of interpretation arise in a magnetized medium: What is the appropriate specification of the macroscopic field strength in such a medium?; what is the average strength of the microfields?; is the present notation the most convenient or is it misleading?

Chiefly within the tradition of interpretation derived from Maxwell—in which H is believed to be the cause of B—the relationship $\mathrm{curl}\,\mathbf{H}=\mu_0\mathbf{j}$ in a steady medium appears to have persuaded many authors that H is ''independent, therefore, of the medium.''[105] However, this only proves that $\mathrm{curl}\,\mathbf{H}$ is independent of the medium, not H itself. This was clearly pointed out by N. Capaldi and W. James in 1968.[106] As William Thomson and many other authors have recognized, both H and B, however they are interpreted, are, in general, caused partly by external fields, partly by the magnetization of the medium, and partly by any macroscopic currents within the medium.

The Lorentz tradition of interpretation this century has been characterized by the belief that there is only one physical field strength in a magnetized medium; the other is a mathematical artifact.[107] This is, of course, incompatible with both the Thomson and Maxwell traditions that treat both B and H as true physical properties of the medium. Which, if any, of these interpretations is correct? Thomson recognized that H is the intensity experienced by each filament, if the medium is partitioned into solenoids parallel to the magnetization, and that B is the intensity experienced by each lamella in the corresponding lamellar partitioning. Thomson also introduced definitions of both B and H in terms of macroscopic ''free-body'' elements of the medium.

The investigations of Poisson and Thomson have shown that such elements will experience uniform fields from the rest of the medium only if they are ellipsoidal in shape. Strictly speaking, therefore, the needle and disc elements introduced in the Thomson tradition should be limiting ellipsoids if the internal macroscopic fields are to be well specified.

If we measure the field strength acting on a disc element (or in the corresponding cavity) that is perpendicular to the magnetization to determine *B*, we have not adequately specified the magnetic field in the medium. The same value of *B* is compatible with a wide range of values of the field experienced by a needle element of the medium, or by elements of other shapes. Indeed, each elementary ellipsoid with a different shape experiences a different uniform field intensity. We need the measurements of the field strength experienced by any two such known elements to fully specify, or to reconstruct, the state of the magnetic field in a magnetized medium.[108] Other branches of physics, of course, such as stress theory, analyze the macroscopic behavior of a medium in terms of such ''free-body'' elements of the medium.[109] The uniform macroscopic magnetic field experienced by elements of a magnetized medium at any point is not, therefore, single valued. It has a range of values within the limiting values *B* and *H*. This suggests that the macroscopic field in a magnetized body is duplex in structure and is not a simple vector field, contrary to the Lorentz assumption. It reduces to a simple vector field in cavities, at boundaries, and on foreign bodies in the medium. The definition of the macroscopic field remains conventional in that any two distinct ellipsoidal elements would suffice to specify it. However, *B* and *H* appear to be the principal field intensities in the medium and the most appropriate, therefore, to choose as measures of the macroscopic medium field. Since they are the extreme field intensities in the medium, there is some slight analogy here with c_p and c_v, the principal specific heat capacities of thermodynamics.[110]

The interpretation of *H* as an artifact has meant that, in Lorentz electromagnetism, its considerable physical importance has often been overlooked. For example, solutions to the wave equation naturally contain *H* rather than *B*[111] because *H*—like *E*—is defined along a wave front. For a similar reason, *H* is the vector that appears in Poynting's energy and momentum flux theory. Again, *H* appears with equal status with *B* in the field energy expression and, of course, in Maxwell's equations.[112] The component of *H* along its length is the field intensity experienced along its length by a needle element or filament of any orientation in the medium. Similarly, the component of *B* perpendicular to its area is the axial component of the magnetic intensity experienced by a disc element or lamella of any orientation in the medium. In this interpretation both *H* and *B* are necessary for a complete description of the field in the medium; they are qualitatively identical and appear to be equally significant.

The reasons for the difference between *H* and *B* become clearer if a notional sphere is drawn around both the needle element and the disc element, respectively, assuming the length of the former is equal to the diameter of the latter and that they have a common center. The contribution of the medium external to the sphere will be common to both *H* and *B* and any difference will be due to the neighboring spherical medium. The element experiencing *H* receives the depolarizing field of the sphere while that experiencing *B* receives the stronger end-on field of the sphere. This explains why, in a ferromagnetic or paramagnetic medium, *B* is the maximum and *H* the minimum field in the medium. Ultimately, of course, *B* and *H* differ because of the difference between the axial and equatorial fields of a magnetic dipole.

For a coil wound evenly around a toroidal ring, or around a long narrow specimen, *H* is effectively the applied field intensity H_a, and is entirely caused by macroscopic currents. *B* is then made up of H_a together with a contribution from the medium. A plot of the magnetization *I* against H_a will then most effectively display the magnetization characteristic of the material.

VII. ARE THERE ''BOUND'' CHARGES AND CURRENTS?

Is the magnetized medium equivalent in all significant respects to an unmagnetized medium with appropriate permanent macroscopic currents on its boundary and in its interior? The external field produced by such a medium is indeed equivalent to that of the real medium, but what of the internal fields? In the substitute medium the magnetic field experienced by a disc element of the medium is the same as that experienced by a needle element, or indeed by any element. The field in all cavities is the same and *H* becomes a mathematical artifact. The surrogate field, therefore, does not have the duplex structure of the field in the real medium. Again, even when a uniform magnetizing field is applied to a homogeneous and isotropic medium, stresses—which do not exist in the real medium—will appear in the substitute medium caused by forces on the fictional currents.[113] The Thomson substitution does not seem to create a valid equivalent, therefore, of the macroscopic field structure in the real medium, although it is very useful in certain circumstances.

If a similar analysis is applied to dielectrics, *E* is found to be the field intensity experienced by a filament or needle element of the medium parallel to the polarization, and also the field in the corresponding cavity. *D* is the intensity experienced by a corresponding disc element or lamella and the field in a disc cavity. When a Poisson transformation is carried out on the dielectric, *E* becomes the only field intensity experienced by any element of the medium or in any cavity, and *D* an artifact. Fictional stresses would also appear in the substitute medium.[114] The Poisson transformation, therefore, fundamentally alters the electrical properties of the medium. A comparison of the dielectric with the corresponding magnetic case shows that, in certain respects, *H* and *B* are analogous to *E* and *D* and in other respects to *D* and *E*. This arises because of the different properties of electric and magnetic fields and of electric and magnetic dipoles.

How should the relations $\operatorname{div}\mathbf{H}=-\mu_0\operatorname{div}\mathbf{M}$, $\operatorname{div}\mathbf{E}=-1/\epsilon_0\operatorname{div}\mathbf{P}$, $\operatorname{curl}\mathbf{B}=\mu_0\operatorname{curl}\mathbf{M}$, and $\operatorname{curl}\mathbf{D}=1/\epsilon_0\operatorname{curl}\mathbf{P}$ be interpreted if there are no ''bound'' magnetic poles, charges, or currents, as the above critique suggests? Surely exactly as $\operatorname{div}\mathbf{H}=-\mu_0\operatorname{div}\mathbf{M}$ is interpreted. There are no magnetic monopoles in magnets. This means that it is a false analogy to interpret $-\operatorname{div}\mathbf{M}$ as a density of magnetic poles. This suggests that $\operatorname{div}\mathbf{H}=-\mu_0\operatorname{div}\mathbf{M}$ is an abstract relationship between functions of *H* and *M*. I believe much the same holds well for the other above-mentioned relationships. Take the special case of $\operatorname{div}\mathbf{E}=-1/\epsilon_0\operatorname{div}\mathbf{P}$. In a random dielectric only, $-\mathbf{P}$, in the expression $\int\mathbf{E}\cdot\mathbf{dS}=\int\mathbf{D}\cdot\mathbf{dS}-1/\epsilon_0\int\mathbf{P}\cdot\mathbf{dS}$, when applied to an internal portion of the medium, can indeed be interpreted as a negative surface charge density, but only if electrons and protons are notionally divided in two by **dS**. To deduce from this that $-\mathrm{div}\,\mathbf{P}$ is a charge density seems a further step away from physical reality because it means replacing the purely notional surface density by a notional volume density of charges.[115] Particularly in introductory teaching, when focusing on what is going on physically, might it not be best to leave the magnetized and also the polarized medium alone—just as they are—explain their action directly, and not transform them into something which, though mathematically convenient, is controversial in interpretation and, perhaps, unphysical?

VIII. FIELD AVERAGES

A self-consistent and fully interpreted macroscopic theory is quite possible without a consideration of microscopic averages but it is illuminating to relate the macroscopic to the microscopic. Also, although it is not part of macroscopic theory, the passage of subatomic particles through a magnetized or polarized medium clearly requires a consideration of field averages. A rigorous study needs a quantum mechanical treatment but it has been customary to adopt a preliminary approach that treats atoms as though they contain classical charge and current distributions and argues that quantum theory leads to essentially the same results.[116] There now appear to be two approaches to averaging the microfields in magnetized and polarized bodies, one deriving directly from Lorentz, which today is the most popular in advanced textbooks, and the other apparently based mainly on William Thomson's theory of magnetization. In general, it seems to me that the theory of field averages is rather recondite. I will confine my discussion to isotropic media.

I find certain developments of Lorentz averaging theory quite perplexing. It seems to postulate that B is equal to the volume average of the magnetic microfields and attempts to prove this postulate by a mathematical averaging process.[117] However, since B within the medium does not appear to be provided with a macroscopic definition in some of these approaches, it is difficult to see how the result can be established. How can one know that the volume average of the microfields is equal to the established macroscopic quantity B, if B is not defined macroscopically? This puzzlement may, of course, represent my failure to understand some subtle point of the argument.

It is also common in this approach to assume that Maxwell's magnetic vorticity equation in the form

$$\nabla\times\mathbf{f}_m=\mu_0\mathbf{j}+1/c^2\partial\mathbf{f}_e/\partial t \tag{5}$$

(where $\mathbf{f}_m$ and $\mathbf{f}_e$ are the microscopic magnetic and electric field intensities, respectively) applies at an atomic level and can be averaged.[118] However, W. G. V. Rosser argues very persuasively that

$$\nabla\times\mathbf{f}_m=1/c^2\partial\mathbf{f}_e/\partial t \tag{6}$$

is the correct form at that level and that the **j** term appears only in the macroscopic version of the equation.[119] Other authors use this equation in the form

$$\nabla\times\mathbf{f}_m=\mu_0\Sigma_n\mathbf{q}_n\mathbf{v}_n\,\delta(\mathbf{x}_n-\mathbf{x})+1/c^2\partial\mathbf{f}_e/\partial t, \tag{7}$$

where $\delta(\mathbf{x}_n-\mathbf{x})$ is Dirac's delta function.[120] I feel that the presence of the delta function here transforms the expression into an analytical device rather than a physical law, with the logical possibility that it may have been introduced simply to return the desired macroscopic outcome. It is also important to point out here that Maxwell's macroscopic equations, both for a vacuum and for material bodies, remain to this day the foundation of classical electromagnetism and have far greater authority than any set of equations from which they may be derived.

Again, the Lorentz approach always transforms the real atomic charge and current distributions into expressions containing imaginary charge and current densities $-\mathrm{div}\,\mathbf{P}$ and $\mathrm{curl}\,\mathbf{M}$, respectively.[121] As we have seen, the physical interpretation of such transformations is controversial. Finally, Lorentz theory here assumes that neither H nor D represents field averages, but there is an alternative tradition which maintains that they do.

The second approach to field averages appears to be rather more satisfactory in that it provides measuring specifications, at least in principle, for both microfield and macrofield, it securely links the macroscopic to the microscopic, it distinguishes between line, surface, and volume averages, and it introduces few controversial assumptions. It is less well-developed mathematically, however, in the literature. I have been unable to discover who first introduced it, but it is present in a rudimentary form in the 1912 edition of Sydney Starling's *Electricity and Magnetism*[122] and it may be much older.

Using a needle cavity and the relation $\mathrm{curl}\,\mathbf{H}=0$ (in the absence of real currents in the medium), it can be shown classically that the integral of the microscopic magnetic field strength $\int\mathbf{f}_m\cdot\mathbf{dl}$ along an interstitial line is equal to the integral $\int\mathbf{H}\cdot\mathbf{dl}$ along that line.[123] It follows that H is equal to the interstitial line average, h, along the direction of magnetization. $\mathrm{Curl}\,\mathbf{H}$ is independent of the medium, therefore, precisely because it does not link any microscopic current loops.[124] Straight interstitial paths will presumably occur in ordered media, and an equivalent path can be contrived even in a random medium. h is defined for all media, whether ordered or disordered, but it is a restricted average in that it does not pass through atoms. Presumably, the component of h along its path will be the average of the microfields along its path experienced by a subatomic particle or ion migrating interstitially through the medium parallel to the magnetization. This will not cause a deflection but it will have some bearing on magnetic spin orientation.

It is easy to demonstrate classically, using a disc cavity, $\mathrm{div}\,\mathbf{B}=0$, and an imaginary infinitesimal box located partly in the medium and partly in the cavity, that

$$\int\mathbf{f}_m\cdot\mathbf{dS}=\int\mathbf{B}\cdot\mathbf{dS}, \tag{8}$$

where $\mathbf{f}_m$ is the microscopic field intensity at any point of the chosen surface. It follows from this that the component of B perpendicular to any surface is equal to the mean component b of the microfields, perpendicular to that surface, whether it intersects molecules or not. It also follows from this alone that B is equal to the volume average. It does not follow from this argument, however, that B is equal to the line average of the microfields, and in general it is not. b and h, as defined above, are general averages in that they apply to all media, whether ordered or disordered. h is clearly more restricted than b but it may on occasion have more physical significance, microscopically, since it is difficult to see how a surface average might act microscopically. It also seems that both h and b are necessary to specify a medium with a given macroscopic magnetic structure.

An average which seems more useful than either the above-defined h or b is the line average of the microfields perpendicular to the magnetization, since this will determine the magnetic deflection of a charged subatomic particle moving rapidly through the medium. I have not found a rigorous argument to prove that this is equal to b, and there are good reasons for supposing that it will not be b, in general, for an ordered medium. However, if the medium is random, there seem good informal reasons to suppose it will be equal to b over a finite path, since the series of microscopic paths described effectively covers all field possibilities. This appears to have been confirmed experimentally.[125]

What is the line average of the magnetic field parallel to the magnetization when the path does not avoid atoms? In an ordered medium such an average of the microfields can have a wide range of values. Only in a random medium does an arbitrary straight path thread the right proportion of atomic current loops for the line average to be equal to B.[126] Line integrals are highly abstract concepts, but a quantum analysis suggests that b or B will be the intensity experienced by a subatomic particle along such a path only in a random medium at high speeds, when atomic current linkages or encounters are in the correct proportion.[127]

The corresponding analysis for uncharged but polarized dielectrics shows that E is equal to e, the general line average along the line of polarization, and is therefore also equal to a volume average of the electric microfields. This suggests that a charged subatomic particle moving along the line of polarization will experience e. In other directions the component of the field along the path of the particle will be equal to the component of E along that path.

D can be shown to be equal to the average d of the microfields across a smooth—or equivalent—interstitial surface.[128] This explains most simply, perhaps, why div $\mathbf{d}$ (and div $\mathbf{D}$) is zero in the absence of real charges: The closed surface used to calculate the flux of the electric microfields across such a surface will contain no unbalanced charges, since it respects the integrity of atoms, which means that the total flux—and div $\mathbf{d}$—will be zero.

What is the macroscopic significance of these averages? It is clear that B and H are numerically and directionally equal to different sorts of microscopic field averages, b and h, respectively. However, this does not seem to be an appropriate way of defining B and H. It seems to me that a coherent macroscopic theory should be provided with macroscopic specifications for its field intensities and, historically, such specifications were indeed provided by Maxwell. If we accept the above-discussed Thomson specification of the macroscopic field, as Maxwell did, then the principal field intensities B and H experienced by free-body elements of the medium are uniform on a microscopic scale and are not averages.

IX. NOMENCLATURE AND NOTATION

What of nomenclature and notation? Tampering with these is a daunting prospect given the vast literature that employs the present usage. Change—even for the remainder of this article—may be a necessity, however, to clear up stubborn ambiguities: A good terminology and notation greatly assists in the clarity of interpretation. It appears to me that there are at least three possibilities if the above analysis is valid. One is to retain existing notation, including B and H with their different units, but to read a different interpretation into it. The disadvantage of this is that the notation and units strongly suggest that there are two distinct field intensities in a vacuum and that B and H are qualitatively different—and not simply quantitatively different—in a magnetic medium. Another possibility is to retain B and H but to give them the same units. However, this would not allow a smooth notational transition to a vacuum where there would be only one vector, B or H?

The most radical approach would be to eliminate the symbol H, employ only one unit of field strength, and use the symbol B only, with suffixes to distinguish the principal field strengths in a magnetized body. In the latter approach an appropriate notation might be $B_l[B]$ and $B_s[H]$ for the field strengths experienced by appropriate lamellar and solenoidal elements of the medium, respectively, thereby explicitly acknowledging William Thomson's contribution in the notation.

I believe with Purcell[129] that B—like B_l and B_s—should be termed the magnetic field strength or intensity and not the flux density. B, of course, can be reinterpreted as a flux density if the magnetic flux is measured by multiplying B by the transverse section of a flux tube. However, B is primarily measured and defined at a point (in terms of the force on a moving charge), or along a line (in terms of the torque on a magnetic needle, in terms of the force on a current element, or in terms of motional electromotive voltage), and it is only as a result of the latter that it can be measured in terms of an area. B, therefore (together with the flux of the electric and gravitational fields), does not seem primarily to belong to the category of a true flux or property normal to an area such as heat flux or fluid flow or pressure and I do not believe that its basic description should be as a flux density. Similarly, I also believe that E, E_l, and E_s should be termed electric field strengths or intensities.

Some of the nomenclature and notation in this subject seems to be obsolete or misleading. It is now generally admitted, for example, that μ_0 is the magnetic interaction constant (analogous to G of gravitation theory), and that the term ''permeability of free space'' is a redundant term deriving from 19th century ether theory. Also, the ''relative permeability'' μ_r is a material property and not an interaction property and is, therefore, physically very different from μ_0. Furthermore, μ_r is an absolute measure and not a relative measure since its value allows us to reproduce or recognize a material with the same property without the necessity of referring it to another material. It might seem appropriate, therefore, to drop the ''nought'' in μ_0 and also to choose a different symbol and name for μ_r. For the latter, κ_m and ''magnetization constant'' of the medium, which are already in use, seem suitable.[130] Very similar considerations apply to dielectrics, except that, through an historical accident, the electric interaction constant ϵ_0 seems to be upside down: An increase in the force of electric interaction everywhere, for example, would reduce ϵ_0. Although Coulomb forces between moving charges are many orders of magnitude greater than magnetic forces, the present convention might suggest to the unwary that they are much weaker. I will, therefore, replace $1/\epsilon_0$ everywhere by ϵ. This means that the electrical, magnetic, and gravitational constants all now appear in the numerator of their respective force equations.

Some basic electromagnetic equations using this suggested notation follow in Box 2. Changes in notation, of course, are very expensive, they make earlier texts almost unreadable, and they are matters for much debate and very careful consideration by international bodies such as the Symbols, Units and Nomenclature Commission (SUN) of the International Union of Pure and Applied Physics (IUPAP).

BOX 2. SUGGESTIONS FOR CHANGES IN NOMENCLATURE AND NOTATION

I will use vector notation only when it is necessary.

Magnetization and polarization relations

For a simple[131] isotropic medium,

$$M = \chi_m B_s / \mu, \tag{9}$$

$$P = \chi_e E_s / \epsilon, \tag{10}$$

$$B_l = \kappa_m B_s = B_s + \mu M, \tag{11}$$

$$E_l = \kappa_e E_s = E_s + \epsilon P. \tag{12}$$

B_l and B_s are the lamellar and solenoidal magnetic field intensities in the medium, respectively, and E_l and E_s the corresponding electric field intensities. Lamellar means sheet-like and solenoidal pipe-like and refers to the appropriately oriented elements of the medium experiencing these fields. In a vacuum, $E_l = E_s = E$ and $B_l = B_s = B$. B is measured by the force on a unit current element placed perpendicular to the magnetic field, E by the force on unit charge. κ_m, the magnetization constant, controls the relationship between the principal magnetic intensities in the medium. κ_e has an analogous role for dielectrics. $\mu = 4\pi \times 10^{-7}$ [$\mathrm{MLT^{-2}I^{-2}}$; kg, m, s, A] is the magnetic interaction constant and $\epsilon = 4\pi c^2 \times 10^{-7}$ [$\mathrm{ML^3T^{-4}I^{-2}}$; kg, m, s, A] the electric interaction constant, since $\epsilon/\mu = c^2$.

M is the intensity of magnetization, P that of polarization. M (for a permanent magnet) is measured by the torque per unit magnetic field intensity per unit volume experienced by the magnet with its magnetic axis perpendicular to a magnetic field. P can be measured in terms of the reduction in voltage caused by a slab of dielectric material placed between capacitor plates. χ_m and χ_e, the isotropic magnetic and electric susceptibilities, respectively, are specified by applying a uniform field to a long thin cylinder, since self-depolarization is then least. The relationships $M = \chi_m B_s / \mu$ and $P = \chi_e E_s / \epsilon$ clearly also apply to a needle element deep within the medium itself, if it is isotropic and linear and correctly oriented. All of this can be easily generalized for anisotropic media.

Maxwell's vorticity equations

$$\nabla \times \mathbf{B}_s = \mu \mathbf{j} + 1/c^2 \partial \mathbf{E}_l / \partial t, \tag{13}$$

$$\nabla \times \mathbf{E}_s = -\frac{\partial \mathbf{B}_l}{\partial t}. \tag{14}$$

These may be interpreted as equations linking mathematical functions of position and time, as correlations between field properties experienced by a disc element of the medium (which cross the disc or encircle its periphery), or as relationships involving different field averages.

X. CONCLUSIONS

I do not believe there is such a thing as a final or perfect explanation of a concept in physics: As our understanding grows, explanations will surely evolve. However, at a particular moment in time I do believe that a good explanation is possible, one that draws optimally on all of the evidence and analysis then available. Three competing explanations for the same set of magnetic phenomena do not now constitute a good explanation, if they ever did. Indeed, until I systematically studied the origins of this problem, these three interpretations were merged in a very confused manner in my mind. When the various traditions of interpretation of B and H are compared, it does seem to me that the Thomson tradition, with the refinements suggested above, is the most satisfactory. If the hypothesis—deriving from Faraday and Maxwell—that there are two magnetic fields in a vacuum is reverently laid to rest; if we ask for a measuring specification, at least in principle, for the macroscopic magnetic field in a material medium (as Maxwell does), then I believe that the pair of Thomson free-body definitions is almost unavoidable, as is the recognition that the macroscopic field at a point in a medium is many-valued with B and H as its principal values. The Thomson theory as I have presented it may, of course, contain hidden errors or inconsistencies that I have failed to notice and it must be examined carefully before it is accepted. Nevertheless, I do hope that it has become clear that much useful interpretative work needs to be done in electromagnetism and that the close study of early sources, together with the forensic examination of concepts, can be helpful in this endeavour. I believe that clearing up interpretative difficulties in physics—and there are very many—makes it easier for students to understand and enjoy physics, and it may also remove obstacles to further research.

When the Thomson approach is accompanied by a better modern understanding of concepts such as the electrical and magnetic interaction constants it gives rise to a raft of units, definitions, and notation for quantities associated with polarization and magnetization which appears to be straightforward, elegant, and coherent. I have not presented these fully here nor have I given a detailed treatment of dielectrics, because this article is already sufficiently long. There are many other related issues that have not even been touched upon. For example, can Coulomb's law and the corresponding laws for current elements (and magnets) be generalized in the presence of a dielectric or magnetic medium, respectively? I feel, nevertheless, that the above investigation clears the ground a little for a fresh approach to these questions. None of the problems that I have discussed is particularly difficult to deal with, and many of the solutions already exist in the literature. However, there appear to have been far too many competing interpretations and conflicting conventions for a consensus to emerge.

[a]Electronic mail: john.roche@linacre.ox.ac.uk

[1]J. Crangle and M. Gibbs, ''Units and unity in magnetism: A call for consistency,'' Phys. World **3** (11), 31–32 (November, 1994).

[2]J. Mallet-Favre, ''Observationes variae,'' Novi Comment. Acad. Sci. Imp. Petrop. **14**, 34 (1769).

[3]F. H. A. von Humboldt and J. B. Biot, ''Sur les variations du magnétism terrestre a differentes latitudes,'' J. Phys. Chim. Hist. Nat. Arts **59**, 429–450 (1804); F. H. A. von Humboldt, *Cosmos*, translated by E. C. Otté, 5 Vols. (Bohn, London, 1849–1858), Vol. 1, p. 178; Vol. 5, pp. 58–68; J. Cawood, ''Terrestrial magnetism and the development of international collaboration in the early nineteenth century,'' Ann. Sci. **34**, 551–587 (1977).

[4]S. D. Poisson, ''Mémoire sur la théorie du magnétisme,'' Mem. Acad. R. Sci. Inst. France **5**, 247–338 (1821–1822).

[5]See Ref. 4, pp. 271, 272, J. G. Leathem, *Volume and Surface Integrals Used in Physics* (Cambridge U. P., Cambridge, 1913), pp. 2–6.

[6]See Ref. 4, p. 277; P.-S. Laplace, *Mécanique Céleste*, English translation and commentary by N. Bowditch, 4 Vols. (Hillard, Gray, Little and

Wilkins, Boston, 1829–1834), Vol. 2, p. 280; G. Green, *An Essay on the Application of Mathematical Analysis to the Theory of Electricity and Magnetism* (privately published, Nottingham, 1828), p. 9.
[7]S. D. Poisson, in Ref. 4, pp. 291, 308, 318, 338; ''Seconde mémoire sur la théorie du magnétisme,'' Mem. Acad. R. Sci. Inst. France **5**, 488–533 (1821–1822); W. Thomson, *Reprint of Papers on Electrostatics and Magnetism* (Macmillan, London, 1872), pp. 470–471.
[8]C. A. Coulomb, ''Septième mémoire sur l'électricité et le magnetisme: Du magnetisme,'' Mem. Acad. R. Sci. Année **1789**, 455–513 (1789).
[9]See Ref. 4, pp. 296–303; C. Coulson, *Electricity* (Oliver and Boyd, Edinburgh, 1961), p. 138.
[10]C. F. Gauss, *Werke*, 12 Vols. (Königlichen Gesselschaft der Wissenschaften, Leipzig, Göttingen, Berlin, 1863–1929), Vol. 5, p. 87.
[11]See W. Thomson, in Ref. 7, pp. 351–355, 361.
[12]O. Mossotti, ''Sull' influenza che l'azione di un mezzo dieletrico la sulla distribuzione dell'elettrica alla superfice di piu'corpi ellectrici disseminati in esso,'' Memori di Mathematica e di Fisica della Societa' Italiana della Scienza **24** (2) (Modena), 49–74 (1850); **24** (2), 67 (1850).
[13]See S. D. Poisson, in Ref. 4, pp. 4, 272, 276, 292, 298, 301; ''Mémoire sur la théorie du magnétisme en mouvement,'' Mem. Acad. R. Sci. Inst. France **6**, 441–570 (1823); **6**, 453–461 (1823).
[14]See G. Green, in Ref. 6, pp. 50–53.
[15]See G. Green, in Ref. 6, p. 53.
[16]C. F. Gauss, ''Mesure absolue de l'intensité du magnétisme terrestre,'' Ann. Chim. Phys. **57**, 5–69 (1834).
[17]See Ref. 16.
[18]J. Roche, *The Mathematics of Measurement: A Critical History* (Athlone, Springer, London, 1998), pp. 163–166.
[19](a) W. Thomson, ''On the mechanical theory of electrolysis,'' Philos. Mag. **2**, 429–442 (1851); **2**, 243 (1851), **2**, 316–362 (1851); (b) in Ref. 7, pp. 363–364, 379–380, 390, 393–394, 398–401, 424–425; Cambridge Mss Add. 7656/K38.
[20]See W. Thomson, in Ref. 7, p. 362 (footnote).
[21]See W. Thomson, in Ref. 7, p. 361.
[22]See W. Thomson, in Ref. 7, p. 482.
[23]See W. Thomson, in Ref. 19(a), pp. 19, 281.
[24]See W. Thomson, in Ref. 7, p. 361.
[25]See Ref. 4, p. 302.
[26]See W. Thomson, in Ref. 7, pp. 361–368.
[27]See W. Thomson, in Ref. 7, pp. 363–364.
[28]See W. Thomson, in Ref. 7, pp. 362, 389.
[29]See W. Thomson, in Ref. 7, pp. 362 (footnote), 396.
[30]A. M. Ampère, ''Mémoire sur la théorie mathématique des phenomènes électrodynamiques,'' Mem. Acad. R. Sci. Inst. Fr. **6**, 228–232 (1823), and plate 1 Fig. 16 facing p. 388; see W. Thomson, in Ref. 7, p. 424.
[31]See W. Thomson, in Ref. 7, pp. 394–395, 424–425.
[32]See W. Thomson, in Ref. 7, p. 398.
[33]See W. Thomson, in Ref. 7, p. 390.
[34]See W. Thomson, in Ref. 7, pp. 397–398.
[35]See W. Thomson, in Ref. 7, pp. 470–471.
[36]See W. Thomson, in Ref. 7, pp. 389, 397.
[37]See W. Thomson, in Ref. 7, p. 395.
[38]See W. Thomson, in Ref. 7, pp. 395–396, 545–546.
[39]See W. Thomson, in Ref. 7, pp. 395–396, 484–485.
[40]See W. Thomson, in Ref. 7, pp. 394–397, 544.
[41]See W. Thomson, in Ref. 7, p. 397.
[42]See W. Thomson, in Ref. 7, p. 557.
[43]W. Fuller Brown, *Dielectrics in Encyclopedia of Physics*, edited by S. Flugge (Springer, Berlin, 1956), Vol. 17, p. 7; A. Kaufman, ''Definition of the macroscopic electric field,'' Am. J. Phys. **29**, 626–630 (1961).
[44]M. Faraday, *Experimental Researches in Electricity*, 3 Vols. (Taylor and Francis, London, 1849, 1844, 1855), Vol. 3, pp. 328–333.
[45]See Ref. 44, pp. 229–230, 332.
[46]See Ref. 44, pp. 346–347, 349.
[47]See Ref. 44, pp. 204, 228–230.
[48]See Ref. 44, pp. 227, 331–332.
[49]H. A. Lorentz, *The Theory of Electrons* (Teubner, Leipzig, 1909), pp. 14–15.
[50]See Ref. 44, pp. 226, 229, 232, 328, 331–333, 435; D. Gooding, ''Metaphysics versus measurement: The conversion and conservation of force in Faraday's physics,'' Ann. Sci. **37**, 1–29 (1980).
[51]See Ref. 18, pp. 40–41.
[52]See Ref. 44, pp. 516, 519; W. Hackmann, ''Eighteenth century electrostatic measuring devices,'' Annali dell'Istituto e Museo di Storia della Scienza di Firenze **3**, 3–58 (1978); J. L. Heilbron, *Electricity in the 17th and 18th Centuries* (University of California Press, Berkeley, CA, 1979), pp. 452, 457.
[53]W. Wollaston, ''Experiments on the chemical production and agency of electricity,'' Philos. Trans. R. Soc. London **91**, 427–434 (1801); Ref. 44, pp. 1, 79, 82, 300–301, 519; see W. Hackmann, in Ref. 51, p. 8.
[54]See Ref. 44, pp. 229–230, 435.
[55]See Ref. 44, p. 435.
[56](a) J. C. Maxwell, *The Scientific Papers of James Clerk Maxwell*, edited by W. D. Niven, 2 Vols. (Cambridge U. P., Cambridge, 1890), Vol. 1, pp. 158–159, 182, 192; (b) J. C. Maxwell, *The Scientific Letters and Papers*, edited by P. M. Harman, 2 Vols. (Cambridge U. P., Cambridge, 1990–1991), Vol. 1, pp. 371–372.
[57]See Ref. 56(a), pp. 178–179, 182–183, 192, 452–453.
[58]See Ref. 56(a), pp. 164, 171, 174–175, 178–180, 182–183, 192, 197.
[59](a) See Ref. 56(a), pp. 178–180, 182–183, 192; (b) J. C. Maxwell, *A Treatise on Electricity and Magnetism*, 2 Vols. (Clarendon, Oxford, 1873), Vol. 2, pp. 50–51.
[60]See Ref. 56(a), p. 92.
[61]See Ref. 56(a), p. 192; Ref. 59(b), pp. 231–242.
[62]See Ref. 56(a), p. 459; Ref. 59(b), pp. 231–232.
[63]See Ref. 56(a), pp. 453–460.
[64]S. G. Starling, *Electricity and Magnetism* (Longmans, London, 1921), pp. 291–293.
[65]See Ref. 59(b), p. 419; see also p. 51.
[66]See Ref. 59(b), Vol. 1, p. 11; Vol. 2, p. 51.
[67]See Ref. 56(a), pp. 185–187, 194, 204; Ref. 56(b), pp. 368–369, 372–373, 381; Ref. 59(b) pp. 217–218.
[68]See Ref. 59(b), p. 27.
[69]See Ref. 59(b), p. 137.
[70]W. Weber, *Werke*, 6 Vols. (Springer, Berlin, 1892–1894), Vol. 3, p. 16; W. Thomson, ''On the mechanical theory of electrolysis,'' Philos. Mag. **2**, 429–442 (1851); in Ref. 59(b), pp. 21–22.
[71]See Ref. 59(b), pp. 143–144; Ref. 56(b), p. 373; O. Heaviside, *Electromagnetic Theory* (Spon, London, 1951), p. 67; A. W. Russell, *The MKS Approach to Electricity and Magnetism* (University of London Press, London, 1966), pp. 71, 88.
[72]See Ref. 59(b), pp. 231–242.
[73]See Ref. 59(b), pp. 23, 30.
[74]See Ref. 59(b), pp. 21–24, 31–34.
[75]See Ref. 59(b), p. 27.
[76]See Ref. 59(b), p. 234.
[77]J. C. Slater and N. H. Frank, *Electromagnetism* (McGraw–Hill, New York, 1947), pp. 50–51.
[78]See Ref. 59(b), pp. 22–23.
[79]C. Coulson, *Electricity* (Oliver and Boyd, Edinburgh, 1961), p. 67.
[80]P. Debye, *Polar Molecules* (Chemical Catalog Company, New York, 1929), pp. 9–11.
[81]H. Hertz, *Electric Waves* (Macmillan, London, 1893), p. 198.
[82]M. Abraham and R. Becker, *Classical Theory of Electricity and Magnetism*, translated by J. Dougall (Blackie, London, 1932), p. viii.
[83]E. Griffiths, ''Electrical and magnetic units,'' Nature (London) **130**, 987–989 (1932); R. G. Chambers, ''Units-*B, H, D* and *E*,'' Am. J. Phys. **67** (6), 468–469 (1999).
[84]C. Egidi, *Giovanni Giorgi and his Contribution to Electrical Metrology* (Politecnico, Turin, 1990), pp. 53–56.
[85]See Ref. 59(b), pp. 237–238.
[86]See Ref. 59(b), Vol. 1, pp. 11–14; Vol. 2, pp. 143, 216, 230–231, 240.
[87]H. A. Lorentz, *Collected Papers*, 9 Vols. (Nijhoff, The Hague, 1935–1939), Vol. 3, pp. 118–119.
[88]See Ref. 87, pp. 129–130. Lorentz uses Gothic notation.
[89]See Ref. 49, p. 135.
[90]See Ref. 87, p. 128.
[91]See Ref. 82, p. 138.
[92]G. Russakoff, ''Derivation of the macroscopic Maxwell equations,'' Am. J. Phys. **10**, 1188–1195 (1970); G. Walker, *The Electromagnetic Field Vectors* (Typescript, 1994), pp. 3–5.
[93]H. V. Stopes-Roe, ''Essential failure of Giorgi systems of electromagnetism, and a basic error by Sommerfeld,'' Nature (London) **224**, 479–581 (1969).
[94]See Ref. 87, Vol. 1, p. 227; Vol. 2, p. 261.
[95]See Ref. 87, pp. 119, 129.
[96]See Ref. 87, pp. 124–126.
[97]See Ref. 87, p. 129; F. W. Sears, *Electricity and Magnetism* (Addison–

Wesley, Reading, MA, 1958), p. 183; A. Kaufman, ''Definition of the macroscopic electric field,'' Am. J. Phys. **29**, 626–630 (1961).
[98]M. Mason and W. Weaver, *The Electromagnetic Field* (University of Chicago Press, Chicago, 1929), p. 61; E. Durand, *Electrostatique*, 3 Vols. (Masson, Paris, 1960–1964), Vol. 3, pp. 259–260.
[99]See Ref. 82, p. 73.
[100]R. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics*, 3 Vols. (Addison–Wesley, Reading, MA, 1969), Vol. 2, p. 10:5.
[101]W. Schwarz, *Intermediate Electromagnetic Theory* (Wiley, New York, 1964), pp. 79–80.
[102]M. Nelkon, *Advanced Level Physics* (Heinemann, London, 1972), p. 942.
[103]See Ref. 59(b), p. 143.
[104]Association for Science Education, *The 1966 Report on the Teaching of Electricity* (Murray, London, 1966), p. 24.
[105]See Ref. 77, p. 72; W. K. Panofsky and M. Phillips, *Classical Electricity and Magnetism* (Addison–Wesley, Reading, MA, 1956), p. 117; see Ref. 102.
[106]See Ref. 59(b), p. 143; N. Capaldi and W. James, ''Analogies between electrical and magnetic quantities,'' Bull. Inst. Phys. **19**, 86 (1968).
[107]See Ref. 77, p. 69; see Ref. 100, pp. 36:4–36:6; J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), p. 189.
[108]See W. Thomson, in Ref. 7, pp. 470–471; Ref. 59(b), p. 25.
[109]S. C. Hunter, *Mechanics of Continuous Media* (Halsted, Chichester, 1983), pp. 53–56.
[110]M. Zemansky, *Heat and Thermodynamics* (McGraw–Hill, New York, 1957), p. 121.
[111]See J. D. Jackson, in Ref. 107, pp. 283, 287, 297.
[112]See J. D. Jackson, in Ref. 107, pp. 237–239.
[113]See G. Walker, in Ref. 92, p. 2.
[114]See G. Walker, in Ref. 92, p. 2.
[115]See G. Walker, in Ref. 92, pp. 3–5.
[116]C. F. Weizäcker, ''Durchgang schneller Korpuskular strahlen durch ein Ferromagnetikum,'' Ann. Phys. (Leipzig) **17**, 869–896 (1933); G. Russakoff, in Ref. 92, pp. 1189, 1195; J. D. Jackson, in Ref. 107, p. 227.
[117]See J. D. Jackson, in Ref. 107, p. 228.
[118]J. H. Van Vleck, *The Theory of Electric and Magnetic Susceptibilities* (Oxford U.P., Oxford, 1965; 1st ed., 1932), Chap. 1; J. D. Jackson, in Ref. 107, pp. 227–229; A. C. Eringen and C. A. Maugin, *Electrodynamics of Continua* (Springer, New York, 1989), Vol. I, pp. 33–34.
[119]W. G. V. Rosser, *Classical Electromagnetism via Relativity* (Butterworths, London, 1968), pp. 84–102; see also P. A. Wolff, ''Force field inside a magnet,'' Am. J. Phys. **46**, 416–417 (1978).
[120]See A C. Eringen and C. A. Maugin, in Ref. 118, pp. 33–34.
[121]See J. D. Jackson, in Ref. 107, pp. 230–233.
[122]See Ref. 64, pp. 267–268.
[123]See Ref. 64, p. 267; H. F. Biggs, *The Electromagnetic Field* (Clarendon, Oxford, 1934), p. 44; see G. Walker, in Ref. 92, p. 14. See also E. M. Purcell, *Electricity and Magnetism*, Berkeley Physics Course Vol. 2 (McGraw–Hill, New York, 1985), 2nd ed., pp. 369–370, 424–425.
[124]See Ref. 87, pp. 120–121, 128–129.
[125]F. Rassetti, ''Deflections of mesons in magnetized iron,'' Phys. Rev. **66**, 1–5 (1944).
[126]W. F. G. Swann, ''A theoretical discussion of the deviation of high energy charged particles in passing through magnetized bodies,'' Phys. Rev. **49**, 574–582 (1936).
[127]C. F. Weizäcker, in Ref. 116, pp. 873, 896; G. H. Wannier, ''The deflection of charged particles in a ferromagnetic medium,'' Phys. Rev. **72**, 304–312 (1947); A. J. Dahm, ''Simple derivation of the magnetic field inside a long rod of magnetized material,'' Am. J. Phys. **46**, 732 (1978).
[128]See G. Walker, in Ref. 92, p. 7.
[129]See E. M. Purcell, in Ref. 123, p. 434.
[130]See Ref. 77, pp. 43, 69.
[131]For more complex media, see L. Rosenfeld, *Theory of Electrons* (North-Holland, Amsterdam, 1951), pp. 20–27.

MIRACLES

The worst is yet to come. Since sometimes it happens that these abstract theories, independent of any object, nevertheless have some bearing on what happens down below in empirical science—it has to be a *miracle*! Miracle indeed to see a clover-leaf intersection fitting *precisely* with the freeways whose flow it redistributes! It is amusing to see rationalists admire a miracle of that quality while they deride pilgrims, dervishes or creationists. They are so enthralled by this mystery that they are fond of saying, 'The least understandable thing in the world is that the world is understandable.'

Bruno Latour, *Science in Action—How to Follow Scientists and Engineers Through Society* (Harvard University Press, Cambridge, Massachusetts, 1987), p. 242.