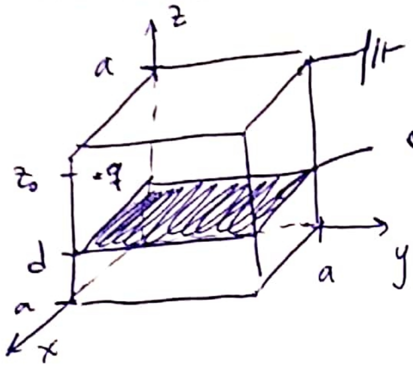
P3)

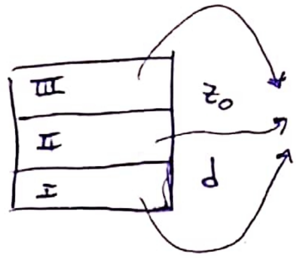


$\cdot$ (q en (x_0, y_0, z_0))

σ : cuadrado uniforme en $z = d$

$\boxed{\text{opción / método } 1}$: Dividir en 3

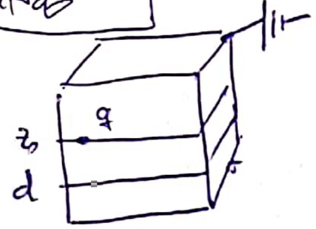
(ver siguiente pág.)



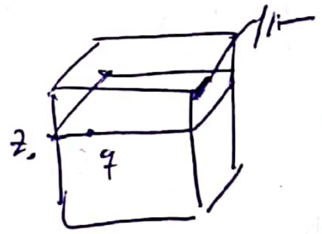
$\nabla \phi^2 = 0$ en I, II, III

usando continuidad y salto en ς superficie : $z = d$ y $z = z_0$

$\boxed{\text{opción / método } 2}$: superposición de problemas mas sencillos

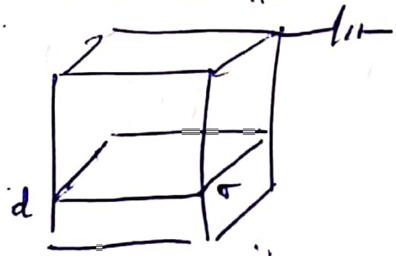


$=$



(ya lo hicimos en zoom)

$+$



uniforme

(muy parecido: ahora $\sigma(x,y) = \sigma$)

$$\phi = \phi_q + \phi_\sigma$$

$z_{\gtrless} = \{z, z_0\}$

$$\phi_q = \sum \overbrace{s(k_n x)\, s(k_m y)}^{\text{base en } x \text{ e } y}\, s(k_n x_0)\, s(k_m y_0) \cdot 4\pi q \cdot \left(\frac{2}{a}\right)^2 \cdot \frac{sh[\gamma_{nm}(a - z_>)]\, sh[\gamma_{nm} z_<]}{\gamma_{nm} \cdot sh[\gamma_{nm} a]}$$

$$\phi_\sigma = \sum \sin(k_n x) \sin(k_m y) \cdot A_{nm} \cdot \underbrace{sh[\gamma_{nm}(a - z_>)]\, sh[\gamma_{nm} z_<]}_{\text{continuo en } z=d \text{ garantizada}}$$

$\downarrow$

falta salto:

$$- \partial_z \phi_\sigma \Big|_{z=d^+} + \partial_z \phi_\sigma (z = d^-) = 4\pi\sigma$$

$z_{\gtrless} = \{z, d\}$

$$\left(\frac{a}{2}\right)^2 \gamma_{nm}\, sh(\gamma_{nm} a) A_{nm} = \sigma\, 4\pi \int_0^a dx\, s(k_n x) \int_0^a dy\, s(k_m y) = \sigma \left(\frac{-c k_n x}{k_n}\right)\Big|_0^a \left(\frac{1 - c(k_m a)}{k_m}\right) ; \; k_n a = n\pi$$

$$\boxed{A_{nm} = 4\pi\sigma \left[1 - (-1)^n\right]\left[1 + (-1)^{m+1}\right] \cdot (k_n \cdot k_m)^{-1} \left(\frac{2}{a}\right)^2 \cdot \left[\gamma_{nm} \cdot sh(\gamma_{nm} a)\right]^{-1}}$$

Método ①

$$\phi = \underbrace{\sum s(\,)s(\,)}_{\text{base}} \begin{cases} A^{I}_{nm}\ \mathrm{sh}(\gamma_{nm} z) & 0 \le z \le d \\ A^{II}_{nm}\cdot \mathrm{sh}(\gamma_{nm} z) + B^{II}_{nm}\cdot \mathrm{sh}[\gamma_{nm}(a-z)] & : d \le z \le z_0 \\ A^{III}\ \mathrm{sh}[\gamma_{nm}(a-z)] & : z_0 \le z \le a \end{cases}$$

Continuidad del potencial:

cont. $z=d$: $\Rightarrow$ $\left[A^{I} = \dfrac{A^{II}\,\mathrm{sh}\gamma d + B^{II}\,\mathrm{sh}[\gamma(a-d)]}{\mathrm{sh}\,\gamma d}\right.$

cont. $z=z_0$: $\Rightarrow$ $\left[A^{III} = \dfrac{A^{II}\,\mathrm{sh}\gamma z_0 + B^{II}\,\mathrm{sh}[\gamma(a-z_0)}{\mathrm{sh}[\gamma\,(a-z_0)]}\right.$

Saltos de la derivada normal en $z=d$ y $z=z_0$:

salto: $z=d$: $\left(-\partial\phi_{II} + \partial_z\phi_{I}\right)\Big|_d = 4\pi\sigma$; $\sigma = \sum s(\,)s(\,)\sigma_{nm}$

$$\gamma\left[\cancel{A^{II}(-\mathrm{ch}\gamma d)} + B\,\mathrm{ch}\,\gamma(a-d) + \left(\frac{A\,\cancel{\mathrm{sh}\gamma d} + B\,\mathrm{sh}[\,]}{\cancel{\mathrm{sh}\gamma d}}\right)\mathrm{ch}\,\gamma d\right] = 4\pi\sigma_{nm}$$

$$\Rightarrow \left[B^{II}_{nm} = 4\pi\sigma_{nm}\cdot \frac{1}{\gamma_{nm}}\,\frac{\mathrm{sh}(\gamma_{nm} d)}{\mathrm{sh}(\gamma_{nm} a)}\right.$$

(falta despejar \sigma_{nm})

salto $z=z_0$: $\left(-\partial\phi_{III} + \partial_z\phi_{II}\right)\Big|_{z_0} = 4\pi\ \ q\cdot\delta(x-x_0)\,\delta(y-y_0)$

$$\frac{\gamma}{\mathrm{sh}[\gamma(a-z_0)]}\left[\left(A\,\mathrm{sh}\gamma z_0 + B\,\cancel{\mathrm{sh}\gamma(a-z_0)}\right)\mathrm{ch}[\gamma(a-z_0)] + \left(A\,\mathrm{ch}\,\gamma z_0 - B\,\cancel{\mathrm{ch}[\gamma(a-z_0)]}\right)\overset{\mathrm{sh}[\gamma(a-z_0)]}{}\right]$$

$$\Rightarrow \left[A^{II}_{nm} = \underbrace{4\pi\,\sin(k_n x_0)\,s(k_m y_0)\left(\frac{2}{a}\right)^2}_{(\sigma_q)_{nm}}\cdot\frac{1}{\gamma_{nm}}\,\frac{\mathrm{sh}[\gamma_{nm}(a-z_0)]}{\mathrm{sh}(\gamma_{nm} a)}\right.$$

- chequear unidades de ϕ : $[\phi] = [\text{carga}]\cdot[\text{longitud}]^{-1}$
- chequear que coincide con la superposición del método ②
- chequear cond. de contorno
- " simetrías (si las hubiera)