Fenómenos dependientes del tiempo

$$\boldsymbol{\nabla} \cdot \boldsymbol{D} = 4\pi\rho_L$$
$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}$$
$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$$
$$\boldsymbol{\nabla} \times \boldsymbol{H} = \frac{4\pi}{c}\boldsymbol{J_L} + \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t}$$

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$$\boldsymbol{D} = \epsilon \boldsymbol{E}$$
$$\boldsymbol{J_L} = \sigma \boldsymbol{E}$$
$$\boldsymbol{B} = \mu \boldsymbol{H}$$

Balance de energía

$$\begin{cases} \boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \\ \boldsymbol{\nabla} \times \boldsymbol{H} = \frac{4\pi}{c}\boldsymbol{J_L} + \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t} \end{cases}$$

Balance de energía

$$\begin{cases} \boldsymbol{H} \cdot \boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\mu}{c} \boldsymbol{H} \cdot \frac{\partial \boldsymbol{H}}{\partial t} \\ \boldsymbol{E} \cdot \boldsymbol{\nabla} \times \boldsymbol{H} = \frac{4\pi}{c} \boldsymbol{E} \cdot \boldsymbol{J_L} + \frac{\epsilon}{c} \boldsymbol{E} \cdot \frac{\partial \boldsymbol{E}}{\partial t} \end{cases}$$

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$$\boldsymbol{\nabla} \cdot (\boldsymbol{E} \times \boldsymbol{H}) = \boldsymbol{H} \cdot \boldsymbol{\nabla} \times \boldsymbol{E} - \boldsymbol{E} \cdot \boldsymbol{\nabla} \times \boldsymbol{H}$$

Balance de energía

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$$\downarrow \quad \boldsymbol{\nabla}\cdot(\boldsymbol{E}\times\boldsymbol{H}) = \boldsymbol{H}\cdot\boldsymbol{\nabla}\times\boldsymbol{E} - \boldsymbol{E}\cdot\boldsymbol{\nabla}\times\boldsymbol{H}$$

$$\begin{aligned} \boldsymbol{\nabla}\cdot(\boldsymbol{E}\times\boldsymbol{H}) &= -\frac{\mu}{c}\boldsymbol{H}\cdot\frac{\partial \boldsymbol{H}}{\partial t} - \frac{4\pi}{c}\boldsymbol{E}\cdot\boldsymbol{J_L} - \frac{\epsilon}{c}\boldsymbol{E}\cdot\frac{\partial \boldsymbol{E}}{\partial t} \\ &= -\frac{1}{2c}\frac{\partial}{\partial t}\left[\boldsymbol{H}\cdot\boldsymbol{B} + \boldsymbol{E}\cdot\boldsymbol{D}\right] - \frac{4\pi}{c}\boldsymbol{E}\cdot\boldsymbol{J_L} \end{aligned}$$

Balance de energía

$$\boxed{\boldsymbol{\nabla} \cdot \boldsymbol{S} + \boldsymbol{J_L} \cdot \boldsymbol{E} + \frac{\partial u_{\mathrm{EM}}}{\partial t} = 0}$$

$$\boldsymbol{S} = \frac{c}{4\pi} \boldsymbol{E} \times \boldsymbol{H} = 0$$

$$u_{\mathrm{EM}} = \frac{1}{8\pi} \left(\boldsymbol{D} \cdot \boldsymbol{E} + \boldsymbol{B} \cdot \boldsymbol{H} \right)$$

Balance de energía

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$A\in\mathbb{C}$

$B\in\mathbb{C}$

$$F\left(t\right)=\mathfrak{Re}\left\{Ae^{-i\omega t}\right\}\mathfrak{Re}\left\{Be^{-i\omega t}\right\}$$

Balance de energía

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$$\left\langle F\left(t\right)\right\rangle = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \mathrm{d}t F\left(t\right)$$

Balance de energía

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&= \frac{\omega}{8\pi} \int_0^{2\pi/\omega} \mathrm{d}t \left[ABe^{-2i\omega t} + AB^* + A^*B + A^*B^*e^{2i\omega t}\right]
\end{aligned}$$

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\end{aligned}$$

Aproximación cuasi-estacionaria

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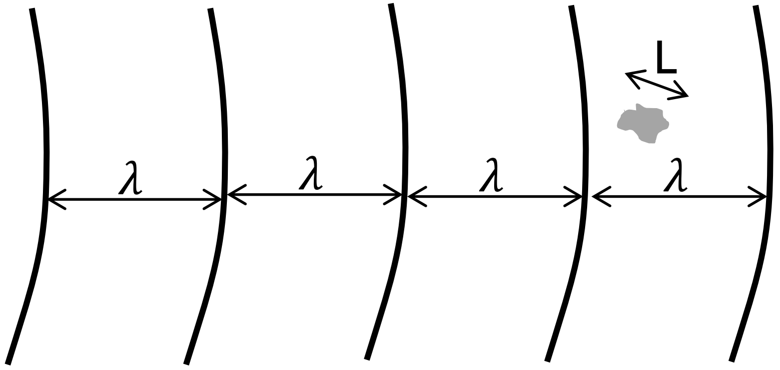

$$1 \gg \frac{L}{\lambda} = \frac{L}{\frac{c}{\nu}} = \frac{L\nu}{c} = \frac{L\omega}{2\pi c} = \frac{L}{c\tau}$$

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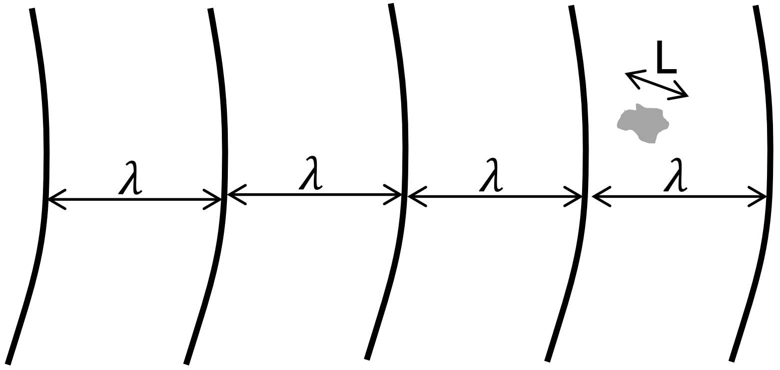

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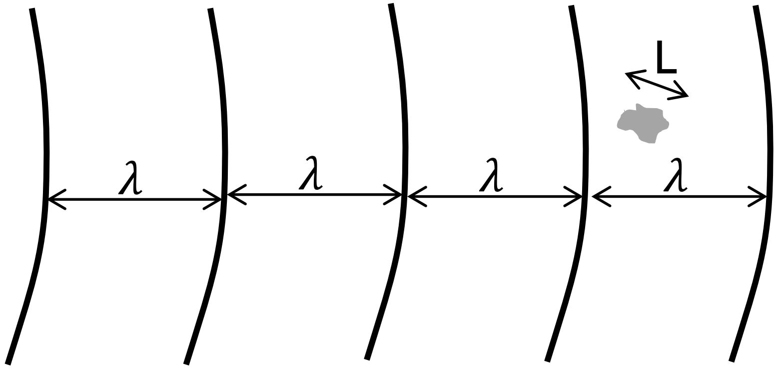
Orden 1 en $\frac{\omega}{c}$



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Tips cuasi-generales:

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① Identificar cual es la parte *prescripta* que esta variando en el tiempo
(a veces son las fuentes, a veces son los campos, etc)

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1. Identificar cual es la parte *prescripta* que esta variando en el tiempo (a veces son las fuentes, a veces son los campos, etc)

2. Asumir que todas las cantidades varían como $\sim e^{-i\omega t}$ (o sea, como la parte prescripta)

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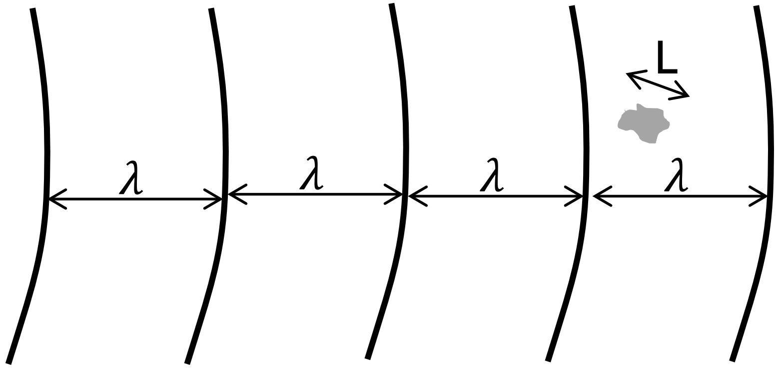
3. Usar simetrías!!!

4. Elegir uno de los 2 métodos…

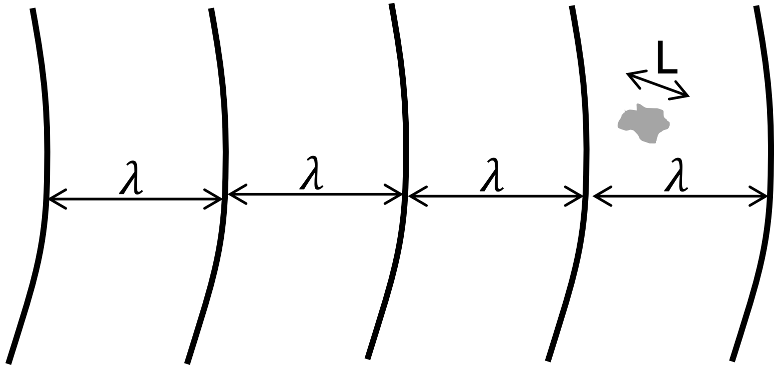
Método “orden a orden”



$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}^{(0)}(\boldsymbol{r},t) + \boldsymbol{E}^{(1)}(\boldsymbol{r},t) + \boldsymbol{E}^{(2)}(\boldsymbol{r},t) + \cdots$$

$$\boldsymbol{B}(\boldsymbol{r},t) = \boldsymbol{B}^{(0)}(\boldsymbol{r},t) + \boldsymbol{B}^{(1)}(\boldsymbol{r},t) + \boldsymbol{B}^{(2)}(\boldsymbol{r},t) + \cdots$$

Método "orden a orden"



$$1 \gg \frac{L}{\lambda} = \frac{L}{c/\nu} = \frac{L\nu}{c} = \frac{L}{c\tau}$$

A orden cero:

$$\nabla \times B^{(0)} = \frac{4\pi}{c} J + \mathcal{O}_0\left(\frac{1}{c}\frac{\partial E}{\partial t}\right)$$

$$\nabla \times E^{(0)} = \mathcal{O}_0\left(-\frac{1}{c}\frac{\partial B}{\partial t}\right)$$

$$\nabla \cdot E^{(0)} = 4\pi\rho$$

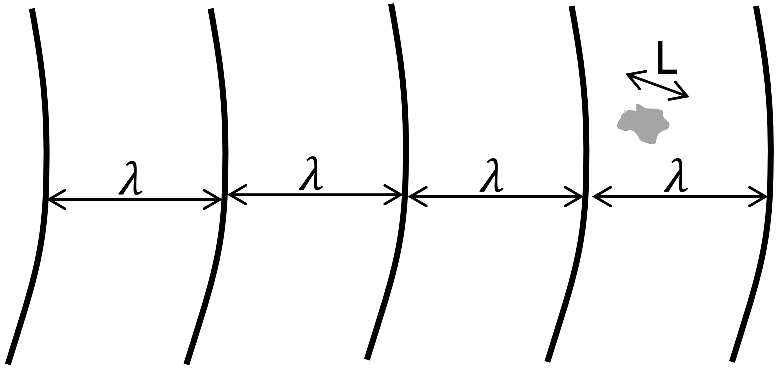
$$\nabla \cdot B^{(0)} = 0$$

Estos no cambian el orden

Estos suben el orden de a **1**

Método "orden a orden"

$$\nabla \times \boldsymbol{B}^{(0)} = \frac{4\pi}{c}\boldsymbol{J}^{(0)}$$
$$\nabla \times \boldsymbol{E}^{(0)} = 0$$
$$\nabla \cdot \boldsymbol{E}^{(0)} = 4\pi\rho^{(0)}$$
$$\nabla \cdot \boldsymbol{B}^{(0)} = 0$$



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Método "orden a orden"

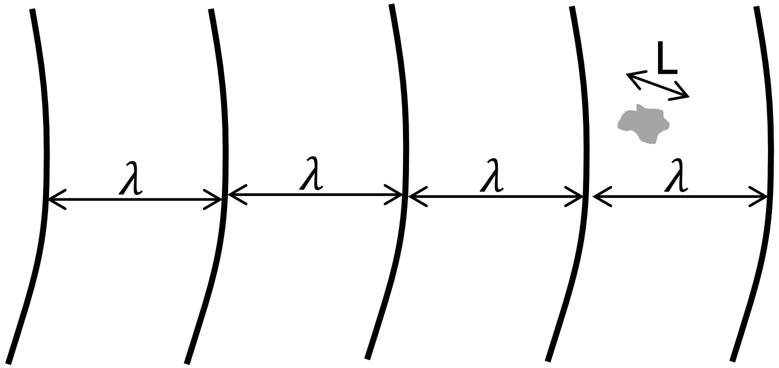
$$\nabla \times \boldsymbol{B}^{(0)} = \frac{4\pi}{c}\boldsymbol{J}^{(0)}$$

$$\nabla \times \boldsymbol{E}^{(0)} = 0$$

$$\nabla \cdot \boldsymbol{E}^{(0)} = 4\pi\rho^{(0)}$$

$$\nabla \cdot \boldsymbol{B}^{(0)} = 0$$

$$\boldsymbol{J}^{(0)} = \sigma \boldsymbol{E}^{(0)}$$



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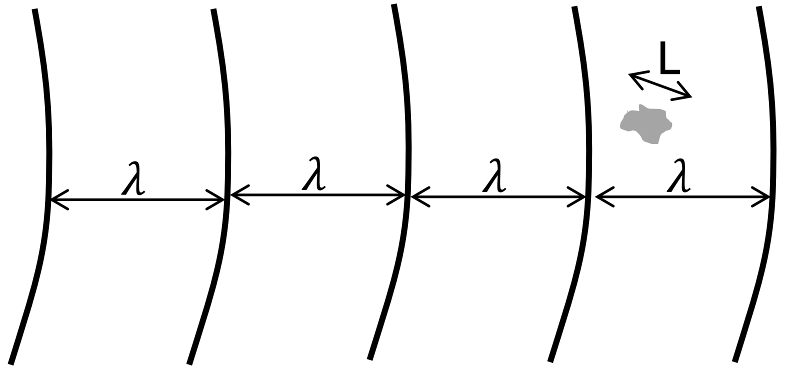
Método "orden a orden"

$$\nabla \times \boldsymbol{B}^{(0)} = \frac{4\pi}{c}\boldsymbol{J}^{(0)}$$

$$\boxed{\begin{aligned}\nabla \times \boldsymbol{E}^{(0)} &= 0\\ \nabla \cdot \boldsymbol{E}^{(0)} &= 4\pi\rho^{(0)}\end{aligned}}$$

$$\nabla \cdot \boldsymbol{B}^{(0)} = 0$$

$$\boldsymbol{J}^{(0)} = \sigma \boldsymbol{E}^{(0)}$$



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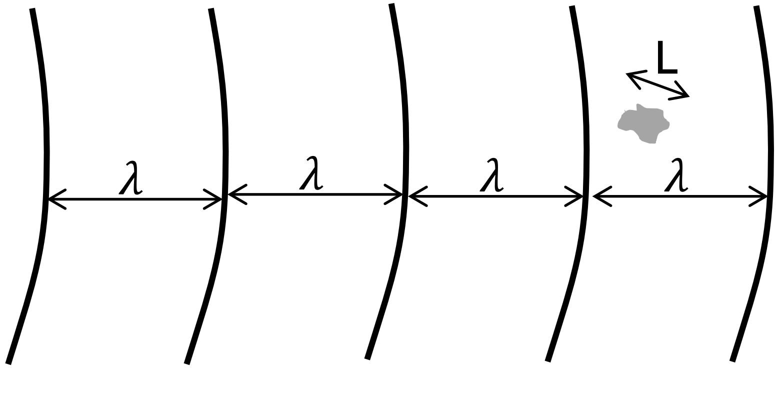
$$\nabla \times \boldsymbol{B}^{(1)} = \frac{4\pi}{c}\boldsymbol{J}^{(1)} - \frac{i\omega}{c}\boldsymbol{E}^{(0)}$$

$$\nabla \times \boldsymbol{E}^{(1)} = -\frac{i\omega}{c}\boldsymbol{B}^{(0)}$$

$$\nabla \cdot \boldsymbol{E}^{(1)} = 4\pi\rho^{(1)}$$

$$\nabla \cdot \boldsymbol{B}^{(1)} = 0$$

Método "orden a orden"



$$1 \gg \frac{L}{\lambda} = \frac{L}{c/\nu} = \frac{L\nu}{c} = \frac{L}{c\tau}$$

$$\nabla \times \boldsymbol{B}^{(0)} = \frac{4\pi}{c}\boldsymbol{J}^{(0)}$$

$$\boxed{\begin{aligned} \nabla \times \boldsymbol{E}^{(0)} &= 0 \\ \nabla \cdot \boldsymbol{E}^{(0)} &= 4\pi\rho^{(0)} \end{aligned}}$$

$$\nabla \cdot \boldsymbol{B}^{(0)} = 0$$

$$\boldsymbol{J}^{(0)} = \sigma \boldsymbol{E}^{(0)}$$

$$\nabla \times \boldsymbol{B}^{(1)} = \frac{4\pi}{c}\boldsymbol{J}^{(1)} - \frac{i\omega}{c}\boldsymbol{E}^{(0)}$$

$$\boxed{\begin{aligned} \nabla \times \boldsymbol{E}^{(1)} &= -\frac{i\omega}{c}\boldsymbol{B}^{(0)} \\ \nabla \cdot \boldsymbol{E}^{(1)} &= 4\pi\rho^{(1)} \end{aligned}}$$

$$\nabla \cdot \boldsymbol{B}^{(1)} = 0$$

$$\boldsymbol{J}^{(1)} = \sigma \boldsymbol{E}^{(1)}$$

Método "orden a orden"

$$\nabla \times \boldsymbol{B}^{(0)} = \frac{4\pi}{c}\boldsymbol{J}^{(0)}$$
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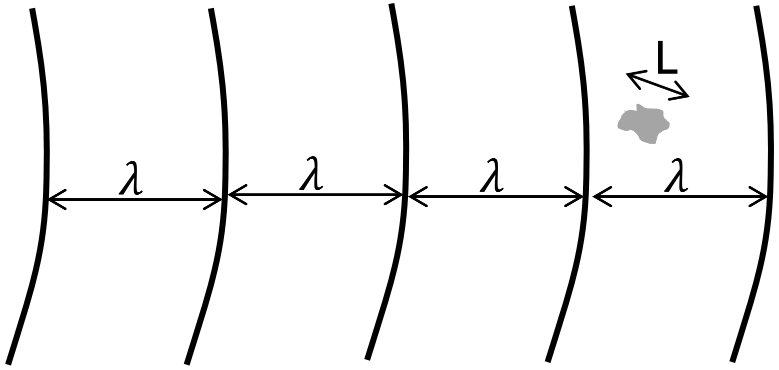

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$$\nabla \times \boldsymbol{B}^{(n)} = \frac{4\pi}{c}\boldsymbol{J}^{(n)} - \frac{i\omega}{c}\boldsymbol{E}^{(n-1)}$$
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Método "orden a orden"

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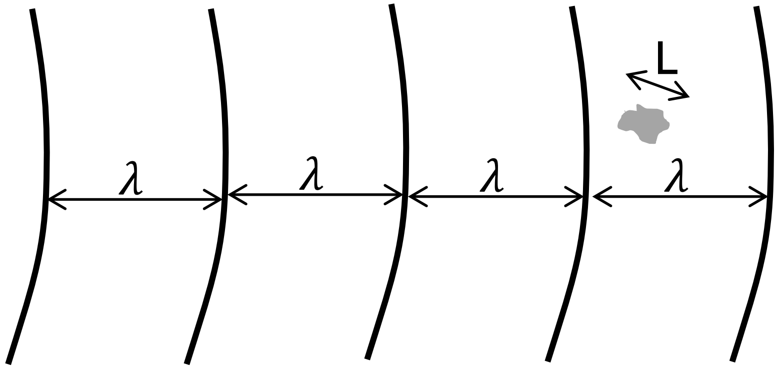


$$1 \gg \frac{L}{\lambda} = \frac{L}{c/\nu} = \frac{L\nu}{c} = \frac{L}{c\tau}$$

$$\nabla \times \boldsymbol{B}^{(n)} = \frac{4\pi}{c}\boldsymbol{J}^{(n)} - \frac{i\omega}{c}\boldsymbol{E}^{(n-1)}$$
$$\boxed{\begin{aligned}\nabla \times \boldsymbol{E}^{(n)} &= -\frac{i\omega}{c}\boldsymbol{B}^{(n-1)} \\ \nabla \cdot \boldsymbol{E}^{(n)} &= 4\pi\rho^{(n)}\end{aligned}}$$
$$\nabla \cdot \boldsymbol{B}^{(n)} = 0$$

$$\boldsymbol{J}^{(n)} = \sigma \boldsymbol{E}^{(n)}$$

Método "tirar la corriente de desplazamiento"

$$\nabla \cdot \hat{\boldsymbol{D}} = 4\pi \hat{\rho_L}$$

$$\nabla \times \hat{\boldsymbol{E}} = -\frac{1}{c}\left(-i\omega\right)\hat{\boldsymbol{B}}$$

$$\nabla \cdot \hat{\boldsymbol{B}} = 0$$

$$\nabla \times \hat{\boldsymbol{H}} = \frac{4\pi}{c}\boxed{\hat{\boldsymbol{J}}_{\boldsymbol{L}}} + \boxed{\frac{1}{c}\left(-i\omega\right)\hat{\boldsymbol{D}}} \simeq \frac{4\pi}{c}\hat{\boldsymbol{J}}_{\boldsymbol{L}}$$

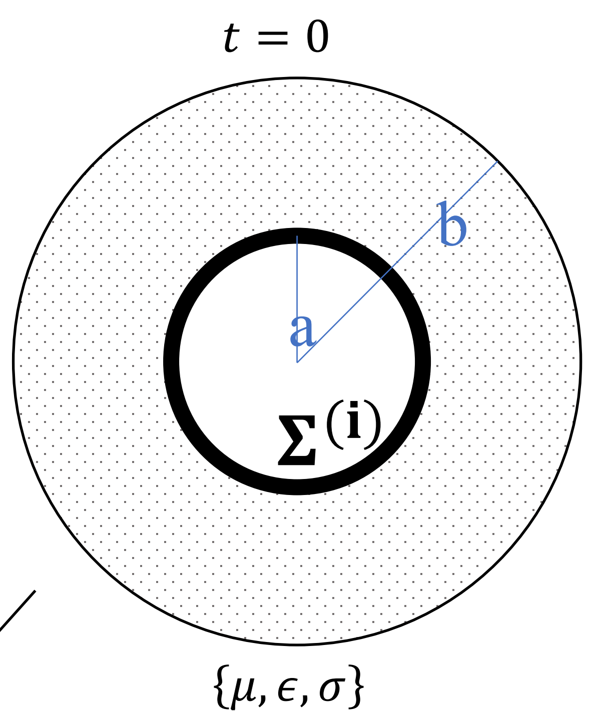
Una cáscara esférica maciza tiene radio interior a y exterior b, y está caracterizada por una conductividad σ, una constante dieléctrica ϵ y una permeabilidad μ. Sobre la cara interior de la cáscara se ha depositado una densidad superficial de carga uniforme Σ. Si a $t = 0$ se permite que el sistema evolucione:

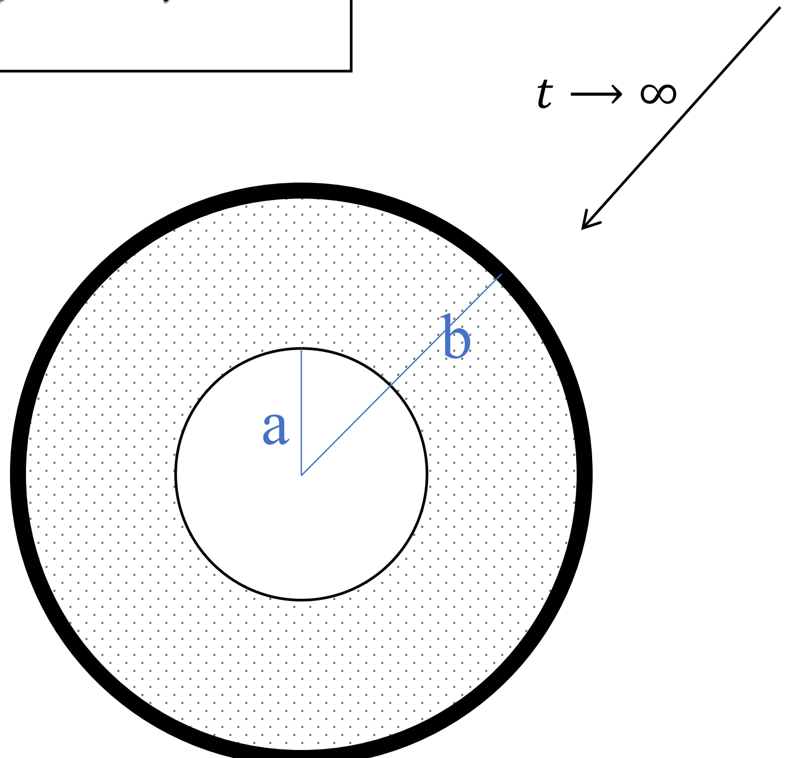
(a) Usando argumentos de simetría, ¿cuánto vale $\mathbf{B}$ en todo el espacio y para todo t? ¿Qué simetría tiene el campo eléctrico?

(b) Teniendo en cuenta lo anterior, encontrar la forma que adoptan las ecuaciones de Maxwell dentro y fuera del conductor.

(c) Encontrar el campo eléctrico, la densidad de corriente y la densidad de carga (superficial y de volumen) en función del tiempo.

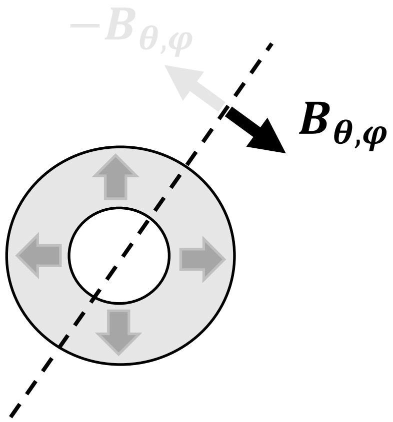
(d) Mostrar que se cumple el teorema de Poynting $\frac{d}{dt}\left(\int_V d^3\mathbf{r}\ u\right)+\int_V d^3\mathbf{r}\ \mathbf{J}\cdot\mathbf{E} = -\oint_S d^2r\ \mathbf{S}\cdot\mathbf{n}$, donde $u = \frac{1}{8\pi}\left(\mathbf{E}\cdot\mathbf{D}+\mathbf{H}\cdot\mathbf{B}\right)$, $\mathbf{S} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{H}$,. En particular, encontrar la evolución de la energía de los campos en función del tiempo y demostrar que la variación de energía entre $t = 0$ y $t = \infty$ es igual a la energía disipada por efecto Joule.





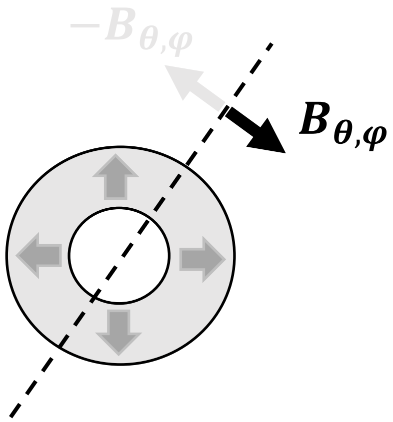
$$\mathbf{\Sigma}^{(\mathbf{f})} = \frac{a^2}{b^2}\mathbf{\Sigma}^{(\mathbf{i})}$$

B es radial:



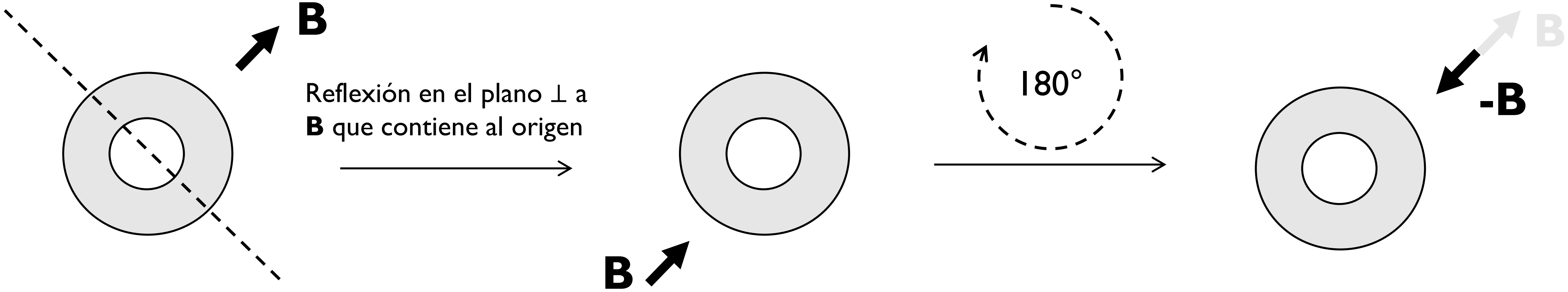
E es radial ✓

B es radial:



E es radial ✓

Como **B** es radial, $\boldsymbol{B} = 0$:



Adentro del conductor:

$$\begin{cases} \nabla \cdot \boldsymbol{E} = \frac{4\pi\rho_L}{\epsilon} \\ \nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \cdot \boldsymbol{B} = 0 \\ \nabla \times \boldsymbol{B} = \frac{4\pi}{c}\mu \boldsymbol{J_L} + \frac{\mu\epsilon}{c}\frac{\partial \boldsymbol{E}}{\partial t} \end{cases} \xrightarrow{\boldsymbol{B} = 0} \begin{cases} \nabla \cdot \boldsymbol{E} = \frac{4\pi}{\epsilon}\rho_L \\ \nabla \times \boldsymbol{E} = 0 \\ \nabla \cdot \boldsymbol{B} = 0 \\ \boxed{0 = \frac{4\pi\mu}{c}\boldsymbol{J_L} + \frac{\mu\epsilon}{c}\frac{\partial \boldsymbol{E}}{\partial t}} \end{cases}$$

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Nos vamos a enfocar en la última ecuación. Usemos la ley de Ohm:

Ley de Ohm: $\boldsymbol{J_L} = \sigma \boldsymbol{E}$

$$\left. \begin{array}{l} \boldsymbol{J_L} = \sigma \boldsymbol{E} \\ 0 = \frac{4\pi\mu}{c}\boldsymbol{J_L} + \frac{\mu\epsilon}{c}\frac{\partial \boldsymbol{E}}{\partial t} \end{array} \right\} \Rightarrow \frac{4\pi\mu}{c}\sigma\boldsymbol{E} + \frac{\mu\epsilon}{c}\frac{\partial \boldsymbol{E}}{\partial t} = 0 \Leftrightarrow \boxed{\frac{\partial \boldsymbol{E}}{\partial t} = -\frac{4\pi\sigma}{\epsilon}\boldsymbol{E}} \bigstar$$

$$\Rightarrow \boldsymbol{E}(\boldsymbol{r}, t) = \mathbb{E}(\boldsymbol{r}) \exp\left(-\frac{4\pi\sigma}{\epsilon}t\right)$$

Por simetría: $\mathbb{E}(\boldsymbol{r}) = E(r)\,\hat{\boldsymbol{r}} \Rightarrow \boldsymbol{E}(r, t) = \hat{\boldsymbol{r}} E(r) \exp\left(-\frac{4\pi\sigma}{\epsilon}t\right)$

Para sacar $E(r)$, nos fijamos la condición inicial en $t = 0$

$$\boldsymbol{E}(r,t) = \hat{\boldsymbol{r}} E(r) \exp\left(-\frac{4\pi\sigma}{\epsilon} t\right) \quad \Rightarrow \quad \boldsymbol{D}(r, t=0) = \epsilon E(r) \hat{\boldsymbol{r}}$$

A t=0, están todas las cargas en r=a

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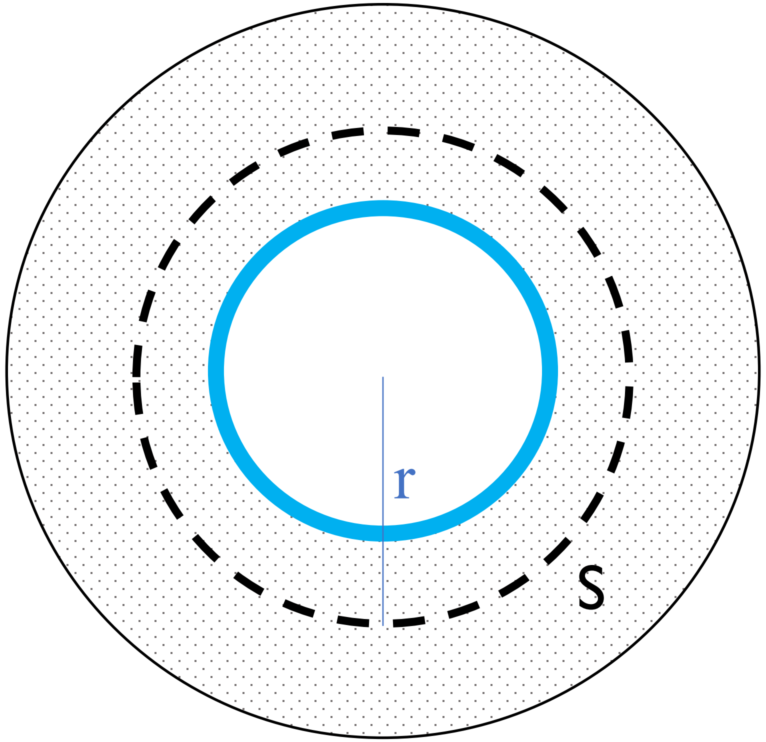

Def:
$Q_L(r,t)$ = la carga libre encerrada en la esfera de radio r a tiempo t

$$\oiint_{S(r)} \mathrm{d}\boldsymbol{S} \cdot \boldsymbol{D}(r,0) = 4\pi Q_L(r,0)$$

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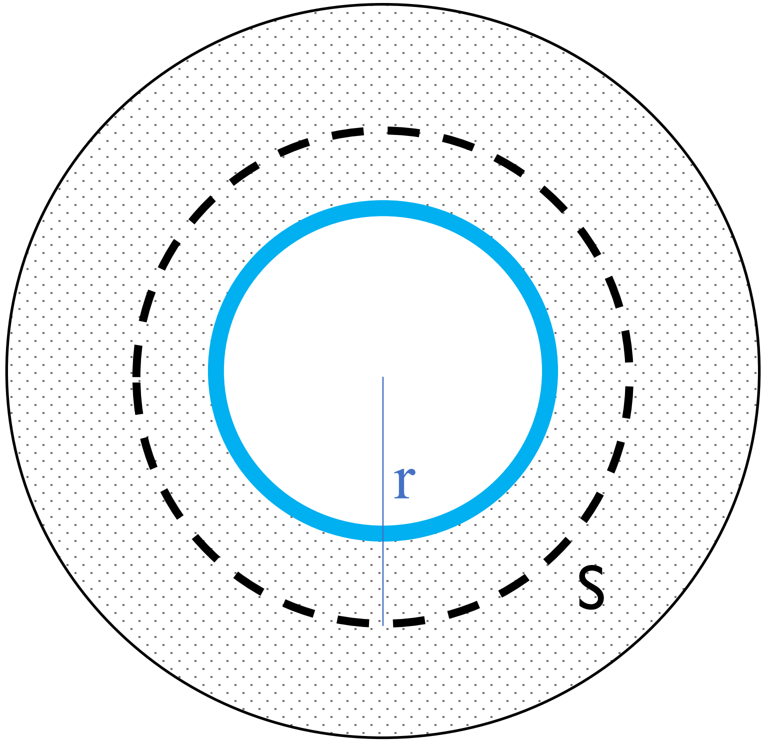

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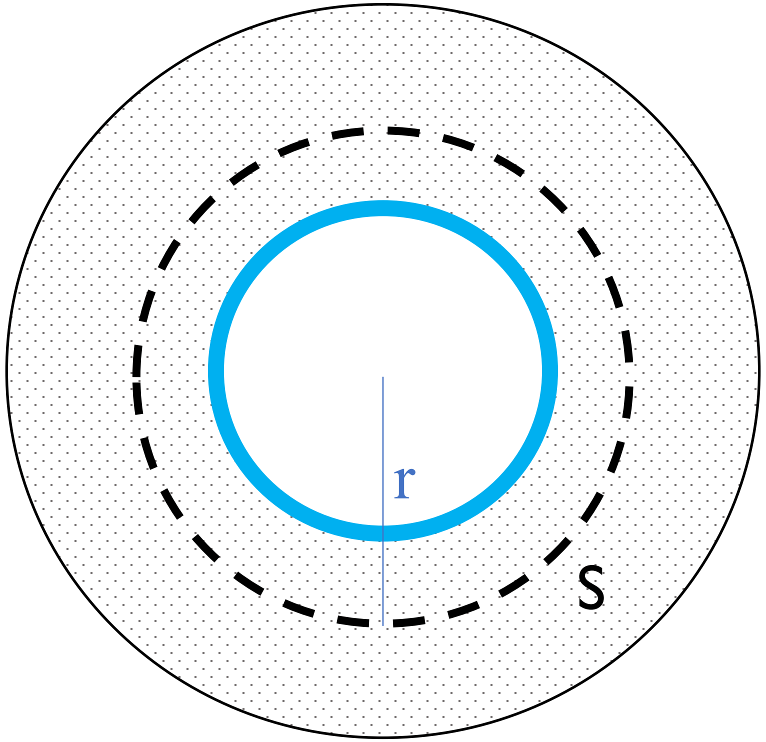

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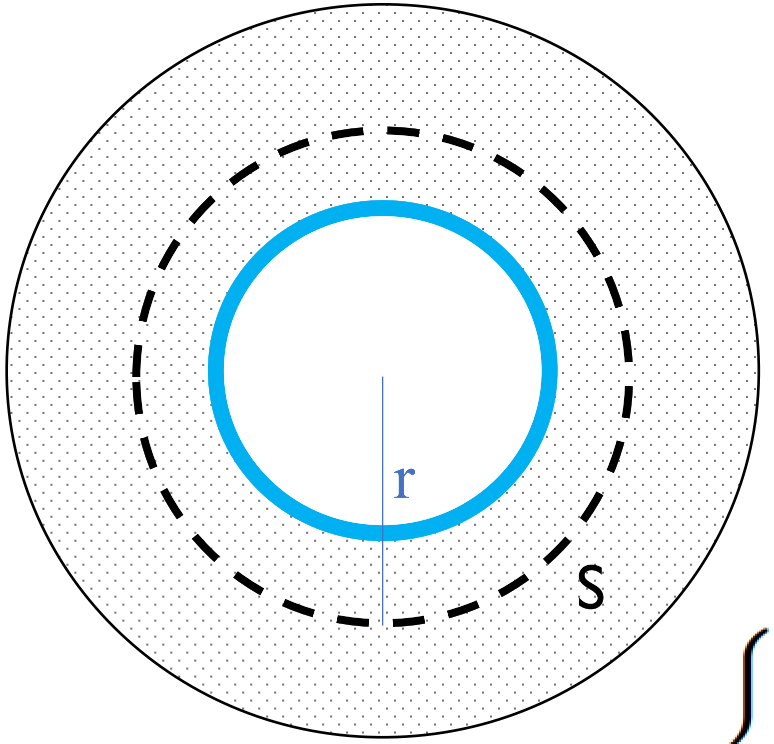

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$$\begin{cases} \tau \overset{\text{def}}{=} \dfrac{\epsilon}{4\pi\sigma} \\ Q \overset{\text{def}}{=} Q_L(a,0) \end{cases} \Rightarrow \quad \boldsymbol{E}(r,t) = \frac{Q}{\epsilon r^2} e^{-t/\tau}\, \hat{\boldsymbol{r}}$$

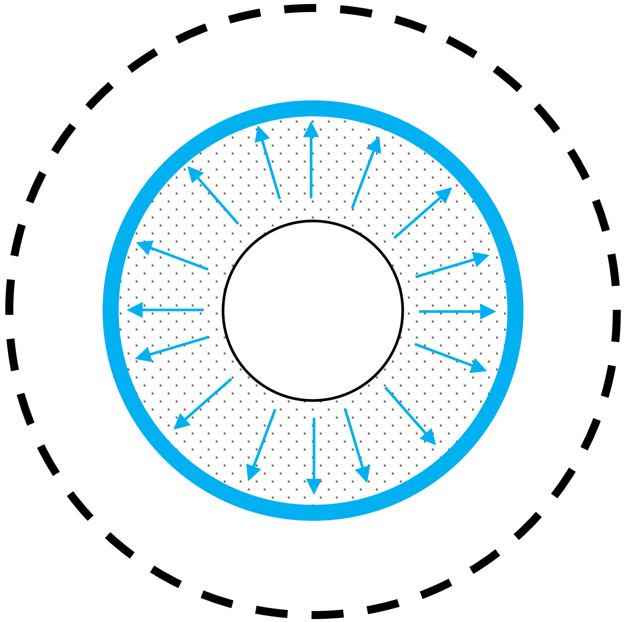
Afuera del conductor:

$$\begin{cases} \boldsymbol{\nabla} \cdot \boldsymbol{E} = 0 & \leftarrow \text{Vacío} \quad \longrightarrow \quad \mathbf{E = D} \\ \boldsymbol{\nabla} \times \boldsymbol{E} = 0 & \leftarrow \boldsymbol{B} = 0 \\ \frac{\partial \boldsymbol{E}}{\partial t} = 0 & \leftarrow \text{Vacío} + \boldsymbol{B} = 0 \end{cases}$$

Afuera del conductor:

$$\begin{cases} \boldsymbol{\nabla} \cdot \boldsymbol{E} = 0 & \leftarrow \text{Vacío} \\ \boldsymbol{\nabla} \times \boldsymbol{E} = 0 & \leftarrow \boldsymbol{B} = 0 \\ \frac{\partial \mathbf{E}}{\partial t} = 0 & \leftarrow \text{Vacío} + \boldsymbol{B} = 0 \end{cases}$$

$\longrightarrow$ **E = D**



$$\boldsymbol{D} = D(r,t)\hat{\boldsymbol{r}}$$

$$\oiint_S \mathrm{d}\boldsymbol{S} \cdot \hat{\boldsymbol{r}} D(r,t) = 4\pi r^2 D(r,t) = 4\pi Q$$

$$\Longrightarrow D(r,t) = \frac{Q}{r^2}, \forall t \quad \Longrightarrow \boldsymbol{D}(r,t) = \boldsymbol{E}(r,t) = \frac{Q}{r^2}\hat{\boldsymbol{r}}$$

El campo afuera no depende del tiempo. Todo el tiempo ve una carga Q, distribuida simétricamente en la zona en donde está la esfera.

Ahora que ya tenemos todos los campos, podemos ver qué pasa con la energía del sistema:

$$u = \frac{1}{8\pi}\left(\boldsymbol{E}\cdot\boldsymbol{D} + \cancelto{0}{\boldsymbol{B}\cdot\boldsymbol{H}}\right) = \frac{1}{8\pi}\boldsymbol{E}\cdot\boldsymbol{D}$$

Sólo queda la energía debida al campo eléctrico (no hay energía magnética)

Adentro $(\boldsymbol{a} < \boldsymbol{r} < \boldsymbol{b})$

$$\boldsymbol{E}(r,t) = \frac{Q}{\epsilon r^2} e^{-t/\tau}\hat{\boldsymbol{r}}$$

$$\boldsymbol{D}(r,t) = \frac{Q}{r^2} e^{-t/\tau}\hat{\boldsymbol{r}}$$

$$\boldsymbol{J_L}(r,t) = \frac{\sigma Q}{\epsilon r^2} e^{-t/\tau}\hat{\boldsymbol{r}}$$

Con:

$$\tau = \frac{\epsilon}{4\pi\sigma}$$

Afuera $(\boldsymbol{b} < \boldsymbol{r})$

$$\boldsymbol{E}(r,t) = \frac{Q}{r^2}\hat{\boldsymbol{r}}$$

$$\boldsymbol{D}(r,t) = \frac{Q}{r^2}\hat{\boldsymbol{r}}$$

$$\boldsymbol{J_L}(r,t) = 0$$

Los campos afuera no cambian nunca ⇒ No hace falta considerarlos para ver la diferencia de energía en el tiempo

$$U|_{t=0} = \overbrace{\int d\Omega \int_a^b dr r^2 \left(\frac{1}{8\pi}\boldsymbol{E}\cdot\boldsymbol{D}\right)\Big|_{t=0}}^{U_{\text{int}}|_{t=0}} + U_{\text{ext}}$$

$$U_{\text{int}}|_{t=0} = \underbrace{\int d\Omega}_{4\pi} \int_a^b dr r^2 \frac{1}{8\pi}\frac{Q^2}{\epsilon r^4} = \frac{Q^2}{2\epsilon}\int_a^b dr \frac{1}{r^2} = \frac{Q^2}{2\epsilon}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$\sim U_{EM}$

$$\boldsymbol{S} = \frac{c}{4\pi}\boldsymbol{E}\times\boldsymbol{H}^{\nearrow 0} = 0$$

$\boldsymbol{J_L}\cdot\boldsymbol{E}$ = energía por unidad de tiempo y unidad de volumen que entregan los campos a las cargas y corrientes libres:

$$\underbrace{\boldsymbol{J_L}\cdot\boldsymbol{E}}_{\partial u_{\text{mec}}/\partial t} + \underbrace{\frac{\partial}{\partial t}\left[\frac{1}{8\pi}(\boldsymbol{D}\cdot\boldsymbol{E} + \boldsymbol{B}\cdot\boldsymbol{H})\right]}_{\partial u_{\text{EM}}/\partial t} + \nabla\cdot\boldsymbol{S}^{\nearrow 0} = 0$$

$$\int_0^{+\infty} dt \int d\Omega \int_a^b dr r^2 \boldsymbol{J_L}\cdot\boldsymbol{E} = 4\pi\int_0^{\infty} dt \int_a^b dr r^2 \frac{\sigma Q^2}{\epsilon^2 r^4} e^{-2t/\tau}$$

$$= \frac{4\pi\sigma Q^2}{\epsilon^2}\left(\int_0^{\infty} dt e^{-2t/\tau}\right)\left(\int_a^b dr \frac{1}{r^2}\right)$$

$$= \frac{4\pi\sigma Q^2}{\epsilon^2}\left[-\frac{\tau}{2}e^{-2t/\tau}\right]_0^{\infty}\left[-\frac{1}{r}\right]_a^b$$

$$= \frac{4\pi\sigma Q^2}{\epsilon^2}\frac{1}{2}\underbrace{\frac{\epsilon}{4\pi\sigma}}_{\tau}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$= \frac{Q^2}{2\epsilon}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$\sim U_{mec}$

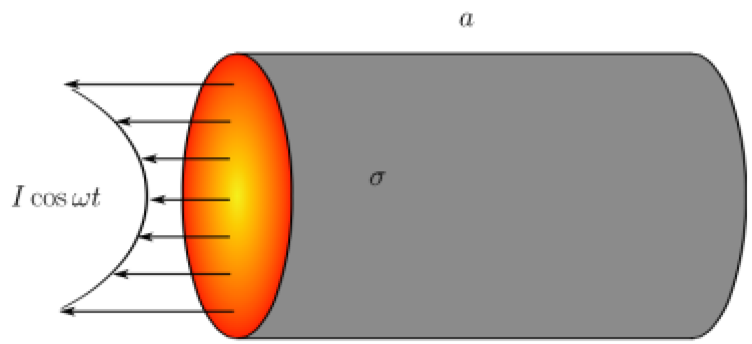
Por un conductor cilíndrico, macizo, de radio a, $\mu = \epsilon = 1$ y conductividad σ circula una corriente alterna del tipo $I = I_0 \cos(\omega t)$. La distribución de la corriente dentro del conductor **no** puede asumirse conocida, sino que debe encontrarse de manera consistente con las ecuaciones de Maxwell. Bajo la **aproximación cuasiestacionaria** (i.e. despreciar el término de corriente de desplazamiento), calcular:

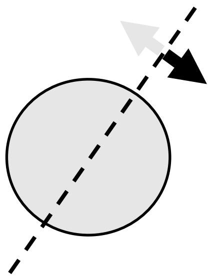
(a) Los campos $\mathbf{E}$ y $\mathbf{B}$ en el interior del conductor.

(b) Estudie los casos límites de la distribución de $\mathbf{j}(\mathbf{r})$ cuando $\delta/a \gg 1$ y $\delta/a \ll 1$, donde δ es el espesor pelicular o "skin depth".

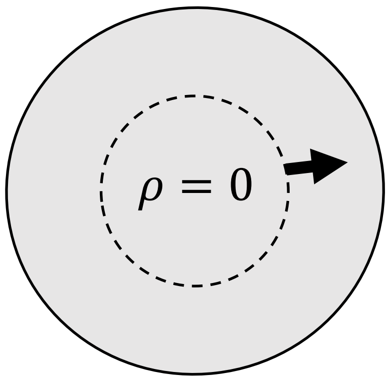
(c) Encontrar la potencia media disipada y la resistencia efectiva en los casos límites, y, en el caso del espesor pelicular mucho menor que a, la corriente superficial efectiva.

(d) Calcular numéricamente y graficar la resistencia vs. la frecuencia en el caso general.





$\Rightarrow E_\varphi = 0$



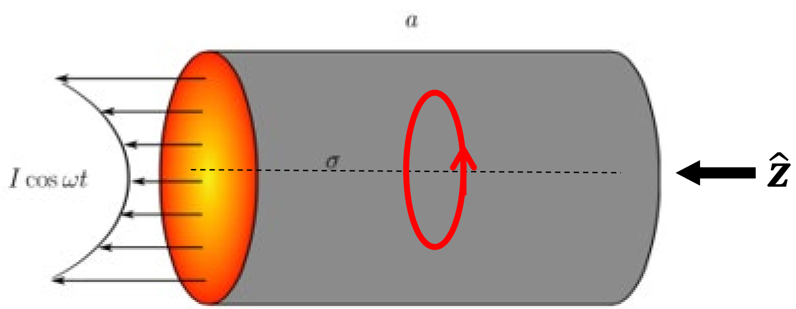
$E_\rho = 0$ (r<a)

Por simetría no puede depender de (φ, z)

$$\Longrightarrow \quad \boldsymbol{E}(\boldsymbol{r}) = E(\rho)\, \hat{\boldsymbol{z}}$$

Método orden a orden

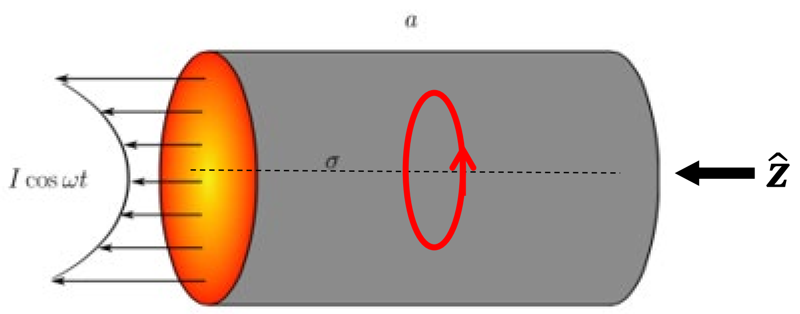
$$\boldsymbol{B} = B(\rho)\,\hat{\varphi}e^{-i\omega t}$$
$$\boldsymbol{E} = E(\rho)\,\hat{z}e^{-i\omega t}$$



Método orden a orden

$$\boldsymbol{B} = B(\rho)\,\hat{\varphi} e^{-i\omega t}$$
$$\boldsymbol{E} = E(\rho)\,\hat{z} e^{-i\omega t}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{E}^{(0)} = 0 \quad \boldsymbol{\nabla}\cdot\boldsymbol{B}^{(0)} = 0$$
$$\boldsymbol{\nabla}\times\boldsymbol{E}^{(0)} = 0 \quad \boldsymbol{\nabla}\times\boldsymbol{B}^{(0)} = \frac{4\pi}{c}\boldsymbol{J}^{(0)} = \frac{4\pi\sigma}{c}E_0\hat{\boldsymbol{z}}$$



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$$\boldsymbol{B} = B(\rho)\,\hat{\varphi} e^{-i\omega t}$$

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$$\nabla \cdot \boldsymbol{E}^{(0)} = 0 \quad \nabla \cdot \boldsymbol{B}^{(0)} = 0$$

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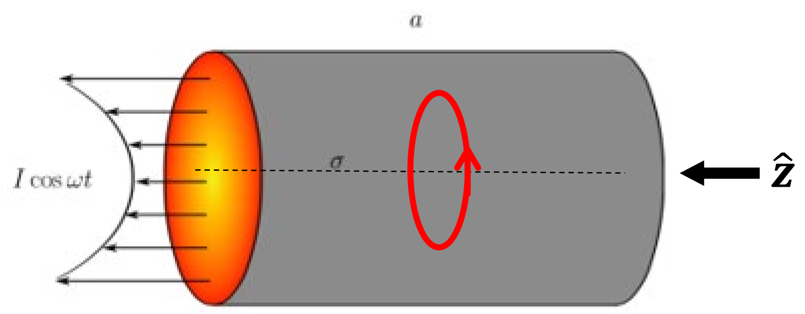
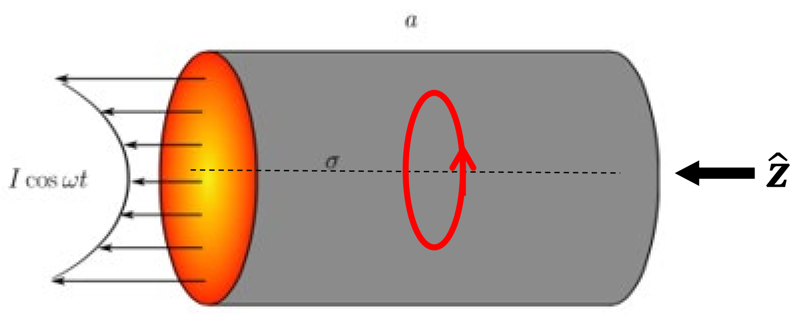
$$\boldsymbol{E}^{(0)} = \boldsymbol{cte} = E_0\hat{z}$$



Método orden a orden



$$\boldsymbol{B} = B(\rho)\,\hat{\varphi}e^{-i\omega t}$$
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$$\nabla\cdot\boldsymbol{E}^{(0)} = 0 \quad \nabla\cdot\boldsymbol{B}^{(0)} = 0$$
$$\nabla\times\boldsymbol{E}^{(0)} = 0 \quad \nabla\times\boldsymbol{B}^{(0)} = \frac{4\pi}{c}\boldsymbol{J}^{(0)} = \frac{4\pi\sigma}{c}E_0\hat{z}$$
$$\boldsymbol{E}^{(0)} = \boldsymbol{cte} = E_0\hat{z}$$

$$\Longrightarrow \quad 2\pi\rho B_0(\rho) = \frac{4\pi\sigma}{c}\pi\rho^2 E_0 \qquad \boldsymbol{B}^{(0)} = \frac{2\pi\sigma}{c}E_0\rho\hat{\varphi}$$

Método orden a orden

$$\boldsymbol{B} = B(\rho)\,\hat{\varphi} e^{-i\omega t}$$
$$\boldsymbol{E} = E(\rho)\,\hat{z} e^{-i\omega t}$$

$$\nabla \cdot \boldsymbol{E}^{(0)} = 0 \qquad \nabla \cdot \boldsymbol{B}^{(0)} = 0$$
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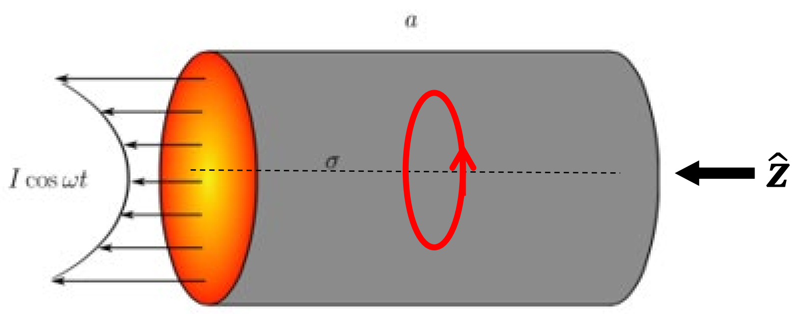
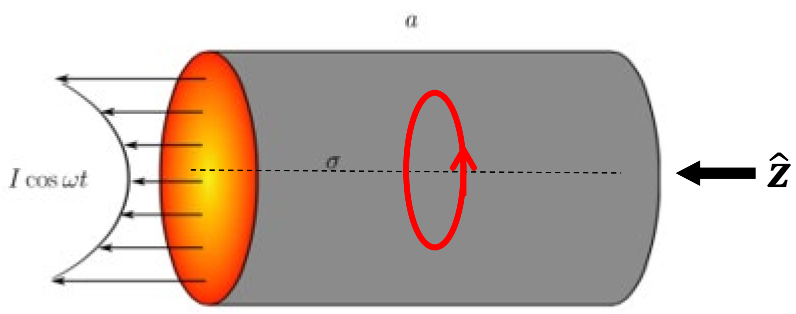


$$\Longrightarrow \quad 2\pi\rho B_0(\rho) = \frac{4\pi\sigma}{c}\pi\rho^2 E_0 \qquad \boxed{\boldsymbol{B}^{(0)} = \frac{2\pi\sigma}{c}E_0\rho\hat{\varphi}}$$

Método orden a orden



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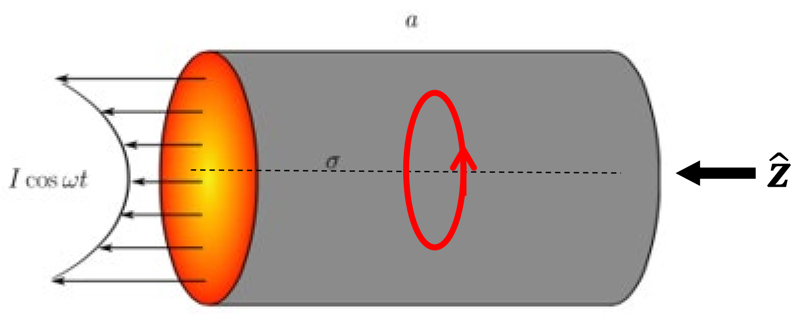

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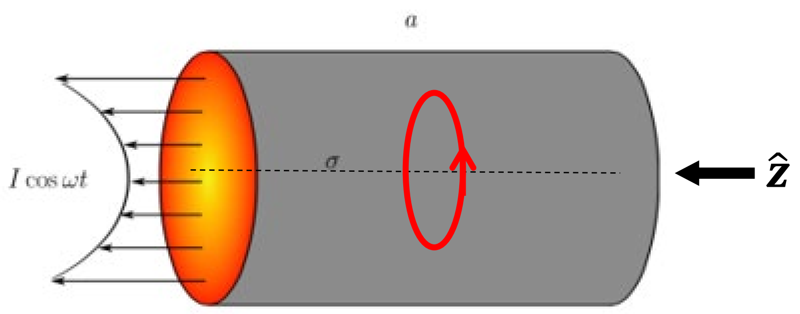
Método orden a orden



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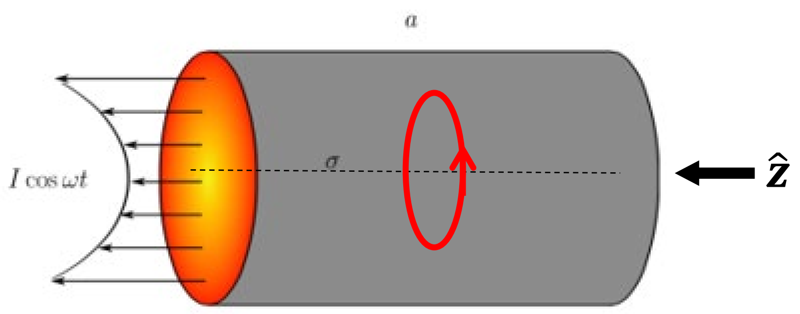
Método orden a orden



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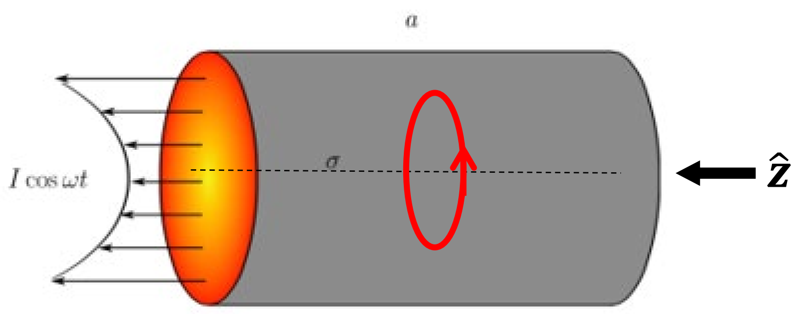
Método orden a orden



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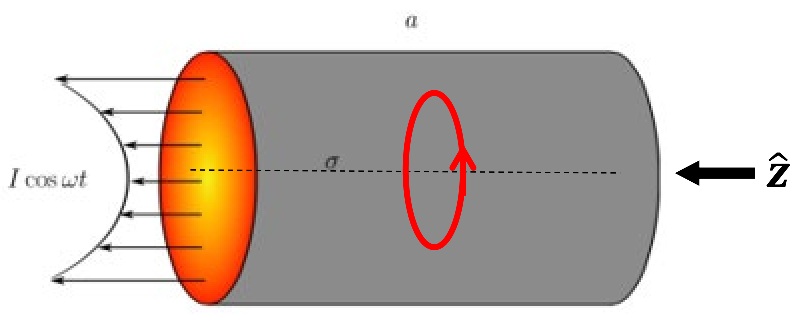
Método orden a orden



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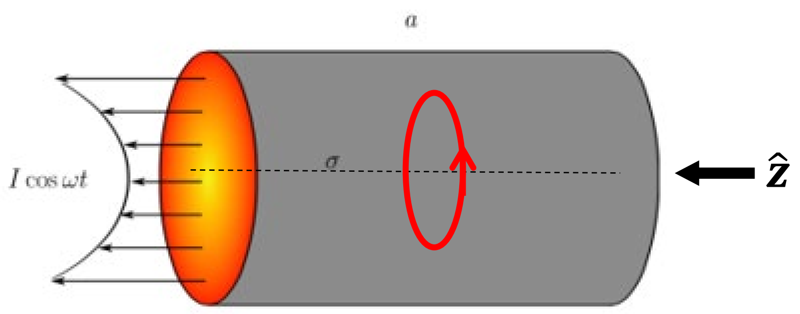
Método orden a orden



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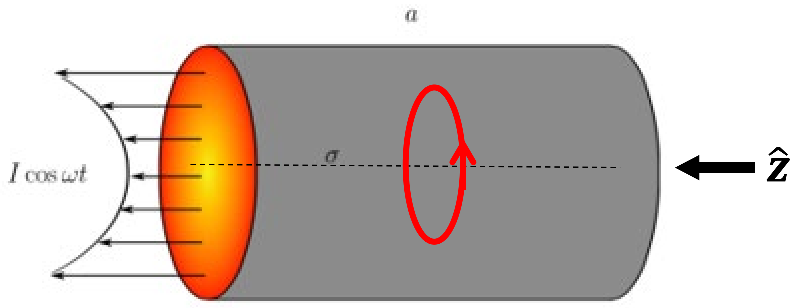
Método orden a orden



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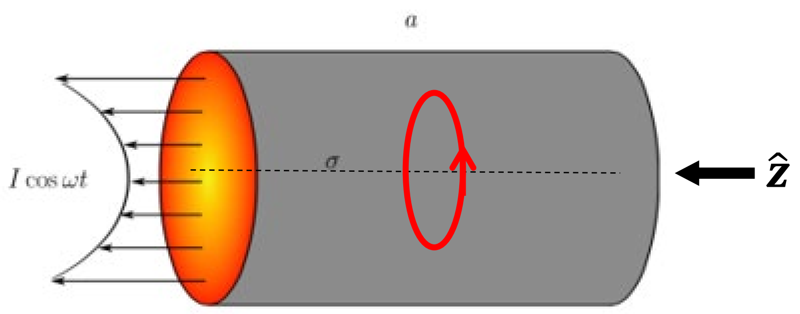
Método orden a orden



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$$B_0(\rho) + B_1(\rho) = \frac{2\pi\sigma}{c}E_0\rho - \cancel{\frac{i\omega}{c}E_0\frac{\rho}{2}} - \frac{i\omega}{c}E_0\frac{\pi^2\sigma^2}{c^2}\rho^3 \approx \frac{2\pi\sigma}{ck}E_0\left[(k\rho) - \frac{(k\rho)^3}{8}\right]$$

$$k^2 \equiv 4\pi i\frac{\omega\sigma}{c^2}$$

$$J_1(k\rho) = \sum_{m=0}^{\infty}\frac{(-1)^m}{m!\,(m+1)!}\left(\frac{k\rho}{2}\right)^{2m+1} = \frac{1}{2}\left[(k\rho) - \frac{(k\rho)}{8} + \cdots\right]$$

Método "tirar la corriente de desplazamiento":

$\rho_L = 0$

$$\begin{cases} \nabla \cdot \boldsymbol{D} = 4\pi \rho_L \\ \nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \cdot \boldsymbol{B} = 0 \\ \nabla \times \boldsymbol{H} = \frac{4\pi}{c}\boldsymbol{J_L} + \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t} \end{cases}$$

$\partial_t = -i\omega$

Lo tiramos. Es muy chico en comparación a **J**

$\mu = 1 \Rightarrow \boldsymbol{B} = \boldsymbol{H}$

$\boldsymbol{I} = I_0 \cos(\omega t)\, \hat{\boldsymbol{z}} \longrightarrow I_0$ es dato, pero $\boldsymbol{J}$ no

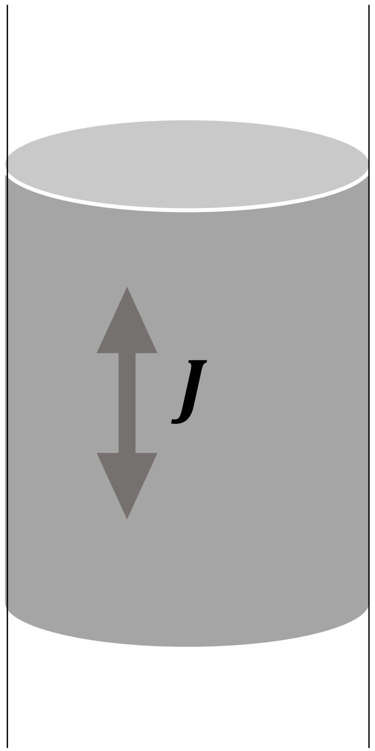
$$\begin{cases} \nabla \cdot \boldsymbol{E} = 0 & \nabla \cdot \boldsymbol{B} = 0 \\ \nabla \times \boldsymbol{E} = \frac{i\omega}{c}\boldsymbol{B} & \nabla \times \boldsymbol{B} = \frac{4\pi}{c}\sigma \boldsymbol{E} \end{cases}$$

Dentro del conductor:

$$\boldsymbol{\nabla} \times \boldsymbol{E} = \frac{i\omega}{c}\boldsymbol{B} \xrightarrow{\boldsymbol{\nabla}\times} \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{E} = \frac{i\omega}{c}\boldsymbol{\nabla} \times \boldsymbol{B}$$

$$\boldsymbol{\nabla}\underbrace{\cancel{(\boldsymbol{\nabla}\cdot\boldsymbol{E})}}_{0} - \boldsymbol{\nabla}^2\boldsymbol{E} = \frac{i\omega}{c}\frac{4\pi}{c}\sigma\boldsymbol{E} \implies \boxed{\boldsymbol{\nabla}^2\boldsymbol{E} + \frac{4\pi}{c^2}i\omega\sigma\boldsymbol{E} = 0} \star$$

La i reemplazó la derivada temporal.
⇒ $\boldsymbol{E}$ ya no depende de t en esta ecuación!



En cilíndricas la ecuación ⭐ $\boldsymbol{\nabla}^2 \boldsymbol{E} + \frac{4\pi}{c^2} i\omega\sigma \boldsymbol{E} = 0$

toma la forma: $\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{4\pi}{c^2} i\omega\sigma\right] E(\rho) = 0$

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Bessel:

Y esta es la ecuación de Bessel con $\nu=0$ y $k^2=\frac{4\pi}{c^2}i\omega\sigma$ $\longrightarrow$ $$\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial R}{\partial\rho}\right)+\left(k^2-\frac{\nu^2}{\rho^2}\right)R=0\right)$$

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Y esta es la ecuación de Bessel con $\nu = 0$ y $k^2 = \frac{4\pi}{c^2} i\omega\sigma$ ⟶ $\left\{\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial R}{\partial \rho}\right) + \left(k^2 - \frac{\nu^2}{\rho^2}\right) R = 0\right\}$

$$\Longrightarrow \quad E(\rho) = E_0 J_0(k\rho) \qquad \text{con: } k = \frac{\sqrt{2\pi\omega\sigma}}{c}(1+i)$$

(No elegimos N_0 porque explota en el centro)

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Y esta es la ecuación de Bessel con $\nu = 0$ y $k^2 = \frac{4\pi}{c^2} i\omega\sigma$ ⟶

Bessel:

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(No elegimos N_0 porque explota en el centro)

Con el rotor de **E** podemos calcular **B**:

$$\frac{i\omega}{c}\boldsymbol{B} = \nabla \times \boldsymbol{E} = \frac{1}{\rho}\begin{vmatrix} \hat{\boldsymbol{\rho}} & \rho\hat{\boldsymbol{\varphi}} & \hat{\boldsymbol{z}} \\ \partial_\rho & \partial_\varphi & \partial_z \\ E_\rho & \rho E_\varphi & E_z \end{vmatrix} = \frac{1}{\rho}\begin{vmatrix} \hat{\boldsymbol{\rho}} & \rho\hat{\boldsymbol{\varphi}} & \hat{\boldsymbol{z}} \\ \partial_\rho & 0 & 0 \\ 0 & 0 & E_z \end{vmatrix} = -\frac{\partial E_z}{\partial\rho}\hat{\boldsymbol{\varphi}}$$

En cilíndricas la ecuación ★ $\nabla^2 \boldsymbol{E} + \frac{4\pi}{c^2} i\omega\sigma \boldsymbol{E} = 0$

toma la forma: $\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{4\pi}{c^2} i\omega\sigma\right] E(\rho) = 0$

Bessel:

Y esta es la ecuación de Bessel con $\nu = 0$ y $k^2 = \frac{4\pi}{c^2} i\omega\sigma$ $\longrightarrow$ $\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial R}{\partial\rho}\right) + \left(k^2 - \frac{\nu^2}{\rho^2}\right) R = 0\right]$

$$\Longrightarrow \quad E(\rho) = E_0 J_0(k\rho) \qquad \text{con: } k = \frac{\sqrt{2\pi\omega\sigma}}{c}(1+i)$$ (No elegimos N_0 porque explota en el centro)

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Algunas propiedades de las J de Bessel:

$$\left.\begin{aligned} &\frac{\mathrm{d}J_\nu}{\mathrm{d}x}(x) = \frac{1}{2}\left[J_{\nu-1}(x) - J_{\nu+1}(x)\right] \\ &\nu \in \mathbb{Z} \implies J_{-\nu}(x) = (-1)^\nu J_\nu(x) \end{aligned}\right\} \quad \frac{\mathrm{d}J_0}{\mathrm{d}x}(x) = -J_1(x)$$

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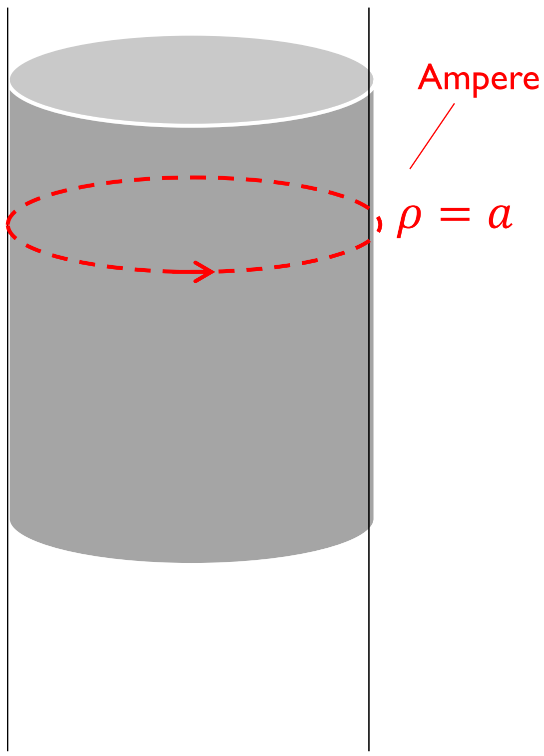
$$\Longrightarrow \quad \boldsymbol{B}(\boldsymbol{r}) = \frac{kc}{i\omega}E_0 J_1(k\rho)\,\hat{\boldsymbol{\varphi}}$$

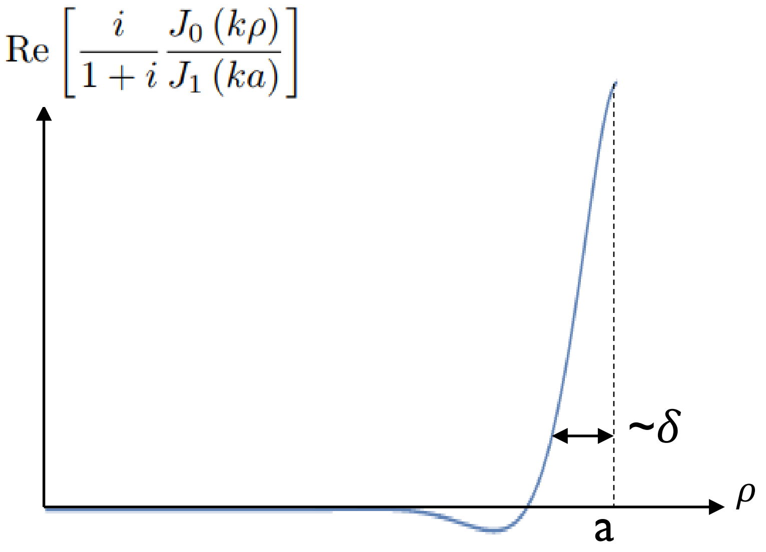


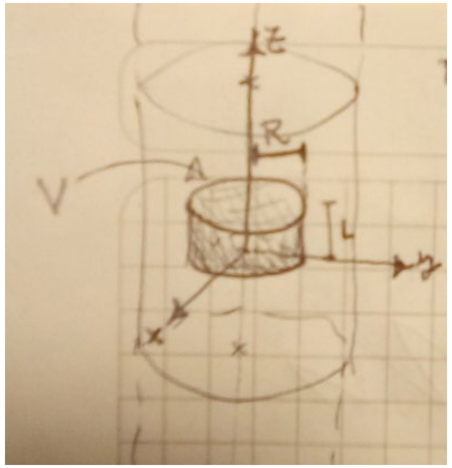
$$\frac{4\pi}{c} I_0 = \oint_{\rho=a} \mathrm{d}\boldsymbol{l} \cdot \boldsymbol{B} = 2\pi a \frac{kc}{i\omega} E_0 J_1(ka) \implies E_0 = \frac{2i\omega}{c^2 ka} \frac{I_0}{J_1(ka)}$$

$$\begin{cases} \boldsymbol{E}(\boldsymbol{r},t) = \dfrac{2i\omega I_0}{c^2 ka} \dfrac{J_0(k\rho)}{J_1(ka)} e^{-i\omega t} \hat{\boldsymbol{z}} \\ \boldsymbol{B}(\boldsymbol{r},t) = \dfrac{2I_0}{ca} \dfrac{J_1(k\rho)}{J_1(ka)} e^{-i\omega t} \hat{\boldsymbol{\varphi}} \\ \boldsymbol{J}(\boldsymbol{r},t) = \sigma \boldsymbol{E}(\boldsymbol{r},t) \end{cases}$$

$$k = \frac{1+i}{\delta} \qquad \delta = \frac{c}{\sqrt{2\pi\omega\sigma}}$$

Para tener los campos en su forma final, hay que tomarles la parte real





$$\boldsymbol{E} = \mathbb{E}(\rho)\,\hat{\boldsymbol{z}} e^{-i\omega t}$$

$$\boldsymbol{B} = \mathbb{B}(\rho)\,\hat{\boldsymbol{\varphi}} e^{-i\omega t}$$

$$\begin{aligned}
\int_V \mathrm{d}V \, \langle \boldsymbol{J}_L \cdot \boldsymbol{E} \rangle &= \int_V \mathrm{d}V \frac{\sigma}{2} \mathfrak{Re}\left\{\mathbb{E}^* \mathbb{E}\right\} \\
&= \frac{\sigma}{2} \int_V \mathrm{d}V \, |\mathbb{E}|^2 \\
&= \frac{\sigma}{2} 2\pi R L \int_0^R \mathrm{d}\rho \rho \, |\mathbb{E}|^2 \\
&= \left(\frac{I_0}{ac}\right)^2 L R \omega \, |J_1(ka)|^2 \int_0^R \mathrm{d}\rho \rho \, |J_0(k\rho)|^2 > 0
\end{aligned}$$