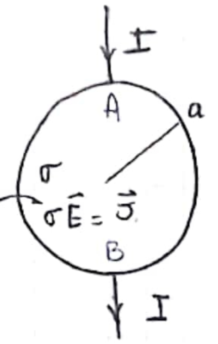
• Ec. de continuidad: $\partial_t \rho + \vec{\nabla}\cdot\vec{J} = 0$

• Régimen Estacionario $\partial_t \rho = 0 \Rightarrow \boxed{\vec{\nabla}\cdot\vec{J} = 0}$

P1)



• Ley de Ohm

$$\vec{J} = \begin{cases} \overbrace{\sigma \vec{E}\ \Theta(a-r)}^{\text{en la esfera conductora (ley de Ohm)}} + \\ \dfrac{I}{2\pi r^2}\underbrace{\left[-\delta(\cos\theta - 1) - \delta(\cos\theta + 1)\right]\Theta(r-a)}_{\text{cable exterior}}\ \hat{z} \quad \text{: corriente prescripta} \end{cases}$$

(afuera no hay conductor)

estac. $\Rightarrow \boxed{\vec{\nabla}\cdot\vec{J} = 0} \wedge \boxed{\vec{\nabla}\times\vec{E} = 0}\ (\Rightarrow \vec{E} = -\vec{\nabla}\phi)$

$$(4\pi\rho =)\ \vec{\nabla}\cdot\vec{E} = (1/\sigma)\underbrace{\vec{\nabla}\cdot\vec{J}}_{=0} + \left(\vec{\nabla}\frac{1}{\sigma}\right)\cdot\vec{J} \Rightarrow \vec{\nabla}\cdot\vec{E} = \vec{J}\cdot(\vec{\nabla}\sigma^{-1})$$

para la interfase de dos medios con σ_1 y σ_2:

• Si σ es cte en volumen $\Rightarrow \boxed{\vec{\nabla}\cdot\vec{E} = 0 \text{ : en vol.}}$

y en sup. tengo: $0 \neq (\vec{E}_2 - \vec{E}_1)\cdot\hat{n}_{12} = \left(\dfrac{\vec{J}_2}{\sigma_2} - \dfrac{\vec{J}_1}{\sigma_1}\right)\cdot\hat{n}_{12}$

ó $\boxed{(\vec{J}_2 - \vec{J}_1)\cdot\hat{n}_{12} \underset{\text{(cond.)}}{=} (\sigma_2\vec{E}_2 - \sigma_1\vec{E}_1)\,\hat{n}_{12} = 0}$

$\vec{J}_2$, $\hat{n}_{12}$, $\vec{J}_1$

$[\text{Gauss} \Leftrightarrow \vec{\nabla}\vec{J} = \vec{\nabla}(\sigma\vec{E}) = 0]$

Si $\underset{\text{(interior)}}{\sigma_1 = \sigma}$ y $\vec{J}_2$ dada $\Rightarrow (\vec{J}_2 - \vec{J}_1)\big|_a \overset{\hat{r}}{\hat{n}_{12}} = (\vec{J}_2 - \sigma\vec{E}_1)\,\hat{r}\big|_a = 0$

$$\sigma\vec{E}_1\cdot\hat{r}\big|_{a^-} = \vec{J}_2\cdot\hat{r}\big|_{a^+} \quad ; \quad \hat{r} = \hat{z}\cos\theta + \dots$$

$$\boxed{-\sigma\frac{\partial\phi}{\partial r}\bigg|_{a^-} = -\frac{I}{2\pi a^2}\left[\delta(\cos\theta+1) + \delta(\cos\theta-1)\right]\cdot\cos\theta}$$

y $\nabla^2\phi = 0$ en vol. int y est. $\Rightarrow$ Uso sol. Laplace:

$$\phi = \sum_{l=0}^{+\infty} P_l(\cos\theta)\cdot A_l\cdot\frac{r_<^l}{r_>^{l+1}} \quad \text{(por continuidad)}.$$

$$\therefore\ -\sigma\sum_{l\geq 1}^{+\infty} P_l(\cos\theta)\cdot A_l\cdot l\cdot\frac{a^{l-1}}{a^{l+1}} = \frac{-I}{2\pi a^2}\left[\delta(\cos\theta+1) + \delta(\cos\theta-1)\right]\cdot\cos\theta$$

$\int P_{l'}(\cos\theta)\,d(\cos\theta) \times$:

(ortogonalización)

$$\cancel{X}\sigma A_l\cdot l\cdot\frac{1}{\cancel{a^2}}\frac{2}{2l+1} = \frac{I}{2\pi\cancel{a^2}}\cancel{(-1)}\left[P_l(-1)\overset{(-1)}{\cancel{(-1)}} + 1\cdot P_l(1)\right]$$

$$\boxed{A_l = \frac{2l+1}{2l}\cdot\frac{I}{2\pi\sigma}\left[\underset{=1}{P_l(1)} - \underset{(-1)^{l+1}}{P_l(-1)}\right]} = \begin{cases} \dfrac{2l+1}{2l}\,\dfrac{I}{\sigma}\cdot\dfrac{2}{2\pi}, & l \text{ impar} \\ 0, & \text{si } l \text{ par} \geq 2 \end{cases}$$

Elegimos $A_0 = 0$ (esfera descargada)

$$\phi = \sum_{l}^{\text{impar}} P_l(\cos\theta) \cdot A_l \cdot \frac{r_<^l}{r_>^{l+1}} \; ; \quad A_l = \frac{2l+1}{l} \cdot \frac{I}{\sigma\, 2\pi}$$

$$\phi = \frac{1}{2\pi}\frac{I}{\sigma} \sum_{l}^{\text{impar}} P_l(y) \cdot \frac{2l+1}{l} \cdot \frac{r_<^l}{r_>^{l+1}} \; ; \qquad y = \cos\theta$$

$x = r/a$

$$\phi_{\text{adentro}} = \frac{I}{\sigma} \sum_{l}^{\text{impar}} P_l(y) \cdot \frac{2l+1}{l} \cdot \underbrace{\frac{r^l}{a^{l+1}}} = \frac{I}{a\sigma} \sum P_l(y) \cdot x^l \left(\frac{2l+1}{l}\right)$$

re-escribo para usar ayuda del enunciado.

$$\otimes \quad \underbrace{\frac{1}{l+1} \cdot \frac{d}{dr} (r/a)^{l+1}} = \frac{r^l}{a^{l+1}} \cancel{(l+1)}$$

$$\phi_{\text{adentro}} = \frac{I}{2\pi\sigma a} \sum_{l=0}^{+\infty} P_l(\cos\theta) \, \frac{2l+1}{2l} \Big[\underbrace{P_l(1)}_{=1} - \underbrace{P_l(-1)}_{(-1)^l}\Big] \cdot \left(\frac{r}{a}\right)^l$$

$$\boxed{\phi_{\text{adentro}}} = \phi_+ + \phi_- + \tilde{\phi}_+ + \tilde{\phi}_- \, , \quad \text{con:}$$

$$\phi_\pm = \pm \frac{I}{2\pi\sigma a} \sum_{l=0}^{+\infty} P_l(\cos) \cdot \left(\frac{\pm r}{a}\right)^l = \pm \frac{I}{2\pi\sigma a} \frac{1}{\sqrt{1 + (r/a)^2 \mp 2\cos\theta\, r/a}}$$

$$\tilde{\phi}_\pm = \frac{I}{2\pi\sigma a} \frac{1}{2} \sum_{l=0}^{+\infty} P_l(\cos) \Big[\underbrace{\frac{\pm(\pm r/a)^l}{l}} \Big] = \frac{I}{4\pi\sigma a} \int_0^x dx' \sum_{l=0}^{+\infty} P_l(y) \cdot (\pm x')^l \left(\frac{1}{\pm x'}\right)$$

$$\frac{\pm(\pm x)^l}{l} = \int_0^x dx\, (\pm x)^{l-1} \; ; \; (x = r/a)$$

$$= \frac{I}{\pi\sigma a} \cdot \frac{1}{2} \int_0^x dx' \cdot (\pm x')^{-1} \cdot \overbrace{\left[1 + x'^2 \pm 2x'y\right]^{-1/2}}^{1/R_\pm}$$

$$\tilde{\phi}_\pm = \pm \frac{I}{2\pi\sigma a} \frac{1}{2} \int_0^x \frac{1}{R_\pm} \cdot \frac{dx'}{x'} = \frac{\pm I}{2\pi\sigma a} \cdot \frac{1}{2} \, \mathrm{sh}^{-1}\left(\frac{a \pm r\cos\theta}{r \cdot \sin\theta}\right)$$