Física Teórica 1 - Práctica

Ondas planas.

Ondas planas

Temas a tratar:

- Problema 5

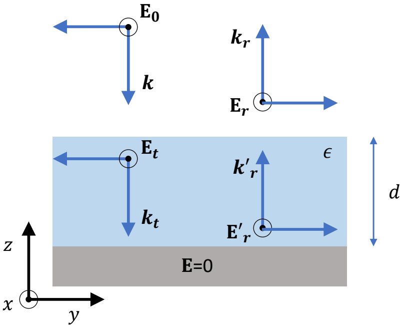
Ondas planas

Temas a tratar:

- Problema 5
- Problema 8

Problema 5

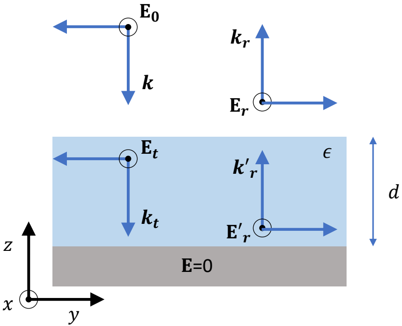
Enunciado



Problema 5

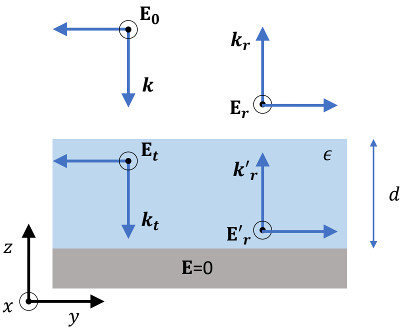
Enunciado

$$\mathrm{E}_{vac} = \mathrm{E}_0 e^{i\mathrm{k}\cdot\mathrm{x} - i\omega t} + \mathrm{E}_r e^{i\mathrm{k}_r\cdot\mathrm{x} - i\omega t}$$



Problema 5

Enunciado

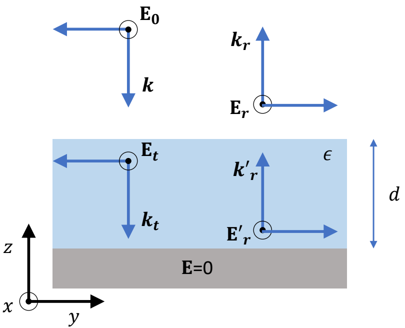


$$\mathbf{E}_{\mathrm{vac}} = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t} + \mathbf{E}_r e^{i\mathbf{k}_r\cdot\mathbf{x} - i\omega t}$$

$$\mathbf{E}_{\epsilon} = \mathbf{E}_t e^{i\mathbf{k}_t\cdot\mathbf{x} - i\omega t} + \mathbf{E}'_r e^{i\mathbf{k}'_r\cdot\mathbf{x} - i\omega t}$$

Problema 5

Enunciado



$$\mathbf{E}_{\text{vac}} = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t} + \mathbf{E}_r e^{i\mathbf{k}_r\cdot\mathbf{x}-i\omega t}$$

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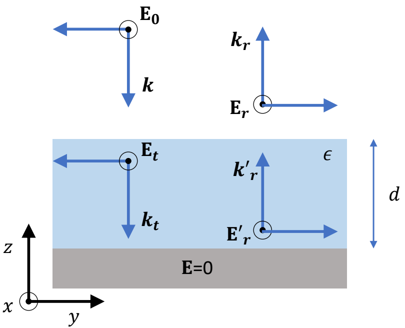
Condiciones de contorno en $z = 0$:

$$0 = \mathbf{E}_{\epsilon}\,\big|_{z=0} = \mathbf{E}_t + \mathbf{E}'_r \implies \mathbf{E}'_r = -\mathbf{E}_t$$

Problema 5

Enunciado



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Condiciones de contorno en $z = 0$:

$$0 = \mathbf{E}_{\epsilon}\,|_{z=0} = \mathbf{E}_t + \mathbf{E}'_r \implies \mathbf{E}'_r = -\mathbf{E}_t$$

$$\mathbf{E}_{vac} = [E_0 e^{-ikz-i\omega t} + E_r e^{ikz-i\omega t}]\hat{x}$$

$$\mathbf{E}_{\epsilon} = E_t[e^{-ik'z-i\omega t} - e^{ik'z-i\omega t}]\hat{x}$$

Problema 5

Condiciones de contorno en $z = d$

- $(\mathrm{D}_{\mathrm{vac}} - \mathrm{D}_{\epsilon}) \cdot \hat{z} \;|_{z=d} = 0$
- $(\mathrm{B}_{\mathrm{vac}} - \mathrm{B}_{\epsilon}) \cdot \hat{z} \;|_{z=d} = 0$
- $(\mathrm{E}_{\mathrm{vac}} - \mathrm{E}_{\epsilon}) \times \hat{z} \;|_{z=d} = 0$
- $(\mathrm{H}_{\mathrm{vac}} - \mathrm{H}_{\epsilon}) \times \hat{z} \;|_{z=d} = 0$

Problema 5

Condiciones de contorno en $z = d$

- $(\mathrm{D}_{\mathrm{vac}} - \mathrm{D}_{\epsilon}) \cdot \hat{z} \,|_{z=d} = 0$

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- $(\mathrm{E}_{\mathrm{vac}} - \mathrm{E}_{\epsilon}) \times \hat{z} \,|_{z=d} = 0$

- $(\mathrm{H}_{\mathrm{vac}} - \mathrm{H}_{\epsilon}) \times \hat{z} \,|_{z=d} = 0$

Las primeras dos se satisfacen inmediatamente porque los campos son paralelos a la superficie.

Problema 5

$$(E_{\text{vac}} - E_{\epsilon})\hat{x} \times \hat{z}|_{z=d} = 0 \quad \implies [E_0 e^{-ikd} + E_r e^{ikd}] - E_t[e^{-ik'd} - e^{ik'd}] = 0 \quad (1)$$

Problema 5

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Usando $\mathrm{H} = \sqrt{\frac{\epsilon}{\mu}}\hat{k} \times \mathrm{E}$

$$(H_{\text{vac}} - H_\epsilon)(-\hat{y}) \times \hat{z}|_{z=d} = 0 \quad \implies [E_0 e^{-ikd} - E_r e^{ikd}] - E_t\sqrt{\epsilon}[e^{-ik'd} + e^{ik'd}] = 0 \quad (2)$$

Problema 5

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Sumando (1) y (2): $\frac{E_t}{E_0} = \frac{e^{-ikd}}{\sqrt{\epsilon}\cos(kd) - i\sin(kd)}$

Problema 5

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Usando H $= \sqrt{\frac{\epsilon}{\mu}}\hat{k} \times$ E

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Reemplazando en (2) y despejando: $\frac{E_r}{E_0} = -\frac{\sqrt{\epsilon}\cos(kd) + i\sin(kd)}{\sqrt{\epsilon}\cos(kd) - i\sin(kd)} e^{-2ikd}$

Problema 5

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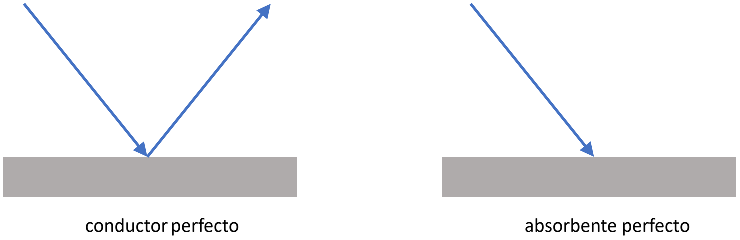
Queda tomar $\epsilon_1 = 1$ y $\epsilon_3 \to \infty$ (límite conductor) en el problema 2 y ver que da lo mismo.

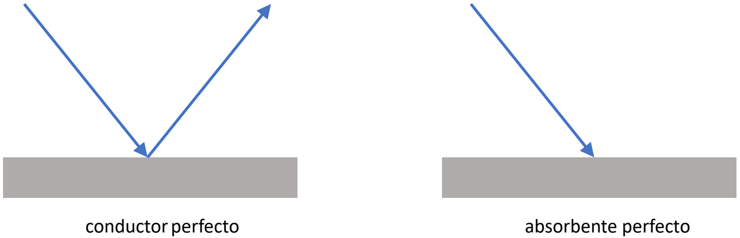
Problema 8

Enunciado



Problema 8

Enunciado

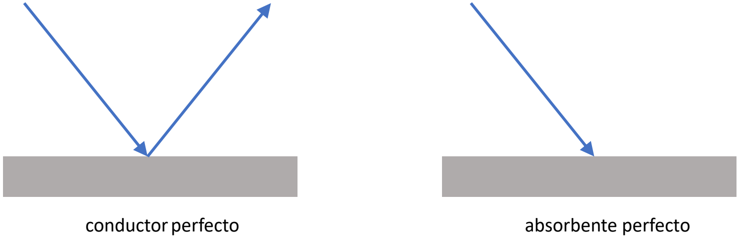


$$\frac{d\mathrm{P}_{\mathrm{mec}}}{dt} + \frac{d\mathrm{P}_{\mathrm{EM}}}{dt} = \mathrm{F}_0 + \frac{1}{4\pi}\oint_S d^2 r \mathrm{T}\cdot \mathrm{n}$$

$\mathrm{P}_{\mathrm{EM}} := \frac{1}{4\pi c}\int_V d^3 r \mathrm{E}\times\mathrm{B}$

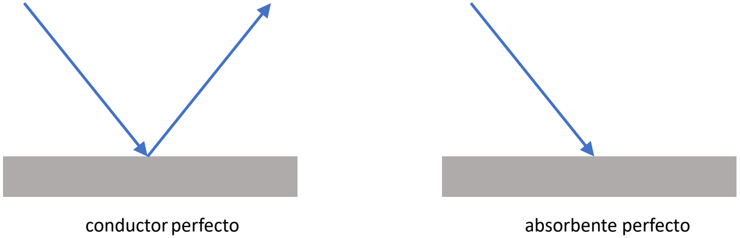
Problema 8

Enunciado



Problema 8

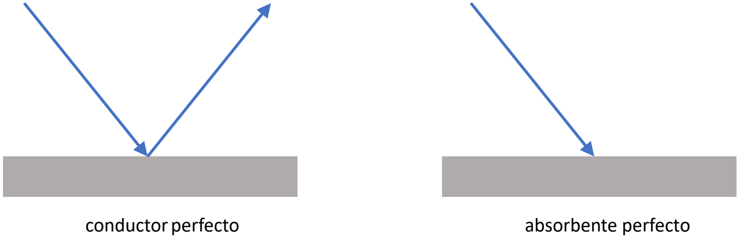
Enunciado



$$\frac{1}{4\pi}\mathsf{T}\cdot\mathsf{n} = \frac{1}{4\pi}[(\mathsf{E}\cdot\mathsf{n})\mathsf{E} + (\mathsf{B}\cdot\mathsf{n})\mathsf{B} - \frac{1}{2}(E^2 + B^2)\mathsf{n}]$$

Problema 8

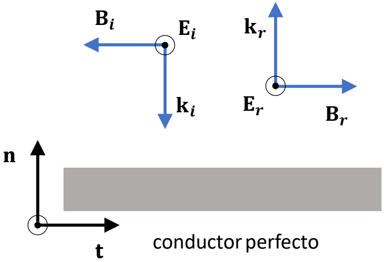
Enunciado



$$\frac{1}{4\pi}\mathsf{T}\cdot\mathsf{n} = \frac{1}{4\pi}[(\mathsf{E}\cdot\mathsf{n})\mathsf{E} + (\mathsf{B}\cdot\mathsf{n})\mathsf{B} - \frac{1}{2}(E^2 + B^2)\mathsf{n}]$$

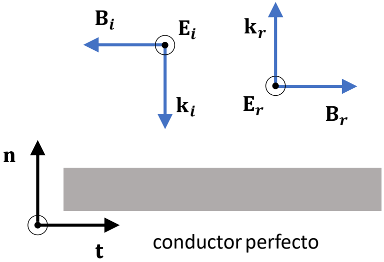
$$\frac{1}{4\pi}\langle\mathsf{T}\cdot\mathsf{n}\rangle = \frac{1}{8\pi}\mathrm{Re}[(\mathsf{E}^*\cdot\mathsf{n})\mathsf{E} + (\mathsf{B}^*\cdot\mathsf{n})\mathsf{B} - \frac{1}{2}(|\mathsf{E}|^2 + |\mathsf{B}|^2)\mathsf{n}]$$

Problema 8



$\mathbf{B}_i$
$\mathbf{E}_i$
$\mathbf{k}_r$
$\mathbf{k}_i$
$\mathbf{E}_r$
$\mathbf{B}_r$
$\mathbf{n}$
$\mathbf{t}$
conductor perfecto

Problema 8

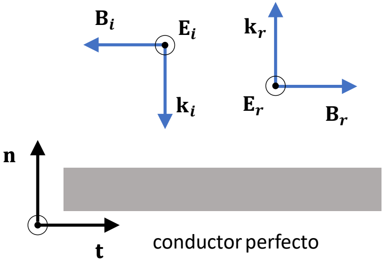


<u>Condiciones de contorno</u>

$\mathrm{E} = \mathrm{E}_i + \mathrm{E}_r = 0$

$\mathrm{B} = \mathrm{B}_i + \mathrm{B}_r = -2E_i\mathrm{t}$

Problema 8



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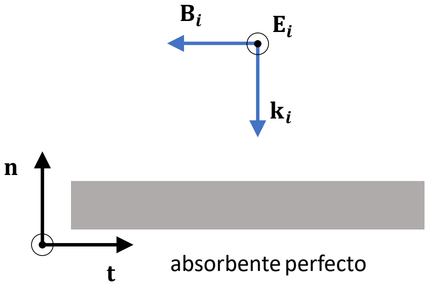
Reemplazando en el tensor de Maxwell sobre la superficie

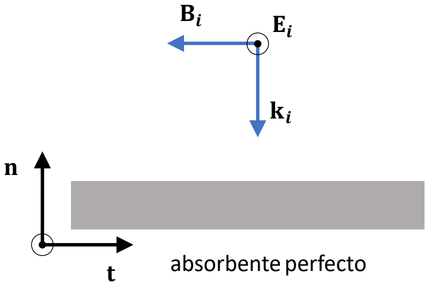
$\frac{1}{4\pi}\langle \mathrm{T}\cdot\mathrm{n}\rangle = \frac{1}{8\pi}[(\mathrm{B}^*\cdot\mathrm{n})\mathrm{B} - \frac{1}{2}(|\mathrm{E}|^2 + |\mathrm{B}|^2)\mathrm{n}] = -\frac{1}{4\pi}|E_i|^2\mathrm{n}$

Problema 8



$\mathbf{B}_i$
$\mathbf{E}_i$
$\mathbf{k}_i$
$\mathbf{n}$
$\mathbf{t}$
absorbente perfecto

Problema 8

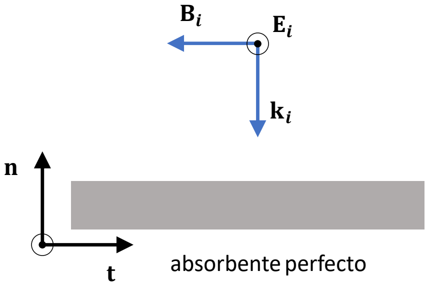


<u>Condiciones de contorno</u>

$\mathbf{E} = \mathbf{E}_i$

$\mathbf{B} = \mathbf{B}_i = -E_i \mathbf{t}$

Problema 8



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Reemplazando en el tensor de Maxwell sobre la superficie

$\frac{1}{4\pi}\langle \mathbf{T} \cdot \mathbf{n}\rangle = -\frac{1}{16\pi}[(|E_i|^2 + |B_i|^2)\mathbf{n}] = -\frac{1}{8\pi}|E_i|^2\mathbf{n}$

Problema 8

Si la incidencia es normal $\mathsf{E} \cdot \mathsf{n} = \mathsf{B} \cdot \mathsf{n} = 0$.

Problema 8

Si la incidencia es normal $\mathsf{E}\cdot\mathsf{n}=\mathsf{B}\cdot\mathsf{n}=0$.

La presión estrictamente es la fuerza normal a la superficie

$$
\begin{aligned}
P =& -\mathsf{n}\cdot\frac{1}{4\pi}\langle \mathsf{T}\cdot\mathsf{n}\rangle = -\mathsf{n}\cdot\frac{1}{8\pi}\mathrm{Re}[(\mathsf{E}^*\cdot\mathsf{n})\mathsf{E}+(\mathsf{B}^*\cdot\mathsf{n})\mathsf{B}-\frac{1}{2}(|\mathsf{E}|^2+|\mathsf{B}|^2)\mathsf{n}] \\
=& \frac{1}{16\pi}(|\mathsf{E}|^2+|\mathsf{B}|^2) = \langle u\rangle
\end{aligned}
$$

Problema 8

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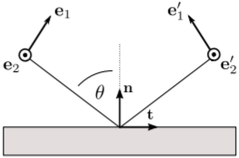
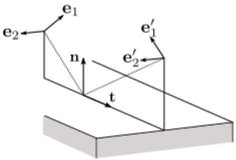
$$\begin{aligned} P =& -\mathsf{n}\cdot\frac{1}{4\pi}\langle \mathsf{T}\cdot\mathsf{n}\rangle = -\mathsf{n}\cdot\frac{1}{8\pi}\mathrm{Re}[(\mathsf{E}^*\cdot\mathsf{n})\mathsf{E} + (\mathsf{B}^*\cdot\mathsf{n})\mathsf{B} - \frac{1}{2}(|\mathsf{E}|^2 + |\mathsf{B}|^2)\mathsf{n}] \\ =& \frac{1}{16\pi}(|\mathsf{E}|^2 + |\mathsf{B}|^2) = \langle u\rangle \end{aligned}$$

$P = \frac{F}{A} = \frac{1}{A}\frac{\Delta p}{\Delta t} = \frac{U}{Ac\Delta t} = \frac{U}{V} = u$

Problema 8

Si la incidencia es oblicua tenemos





Problema 8

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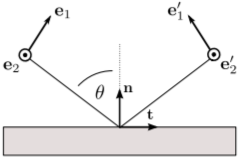

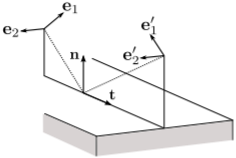

$$\mathrm{E}_i = (E_1\mathrm{e}_1 + E_2\mathrm{e}_2)e^{i(\mathrm{k}_i\cdot\mathrm{r}-\omega t)},$$

$$\mathrm{E}_r = (E'_1\mathrm{e}'_1 + E'_2\mathrm{e}'_2)e^{i(\mathrm{k}_r\cdot\mathrm{r}-\omega t)},$$

Problema 8

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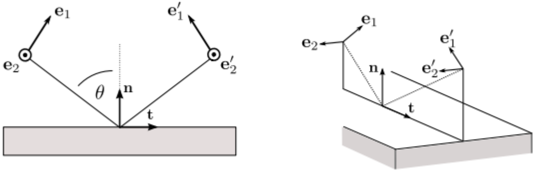

$$\mathsf{E}_i = (E_1\mathsf{e}_1 + E_2\mathsf{e}_2)e^{i(\mathsf{k}_i\cdot\mathsf{r}-\omega t)},$$

$$\mathsf{E}_r = (E_1'\mathsf{e}_1' + E_2'\mathsf{e}_2')e^{i(\mathsf{k}_r\cdot\mathsf{r}-\omega t)},$$

$$\mathsf{B}_i = (-E_2\mathsf{e}_1 + E_1\mathsf{e}_2)e^{i(\mathsf{k}_i\cdot\mathsf{r}-\omega t)},$$

$$\mathsf{B}_r = (-E_1\mathsf{e}_1' + E_2\mathsf{e}_2)e^{i(\mathsf{k}_r\cdot\mathsf{r}-\omega t)},$$

donde $\mathsf{e}_1 = \sin\theta\ \mathsf{n} + \cos\theta\ \mathsf{t}, \quad \mathsf{e}_1' = \sin\theta\ \mathsf{n} - \cos\theta\ \mathsf{t}, \quad \mathsf{e}_2 = \mathsf{t}\times\mathsf{n}$

Problema 8

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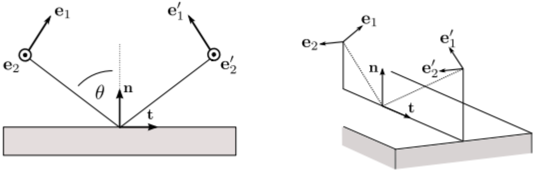

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$E_1' = \alpha E_1, \quad E_2' = -\alpha E_2 \quad$ (conductor $\alpha = 1$, absorbente $\alpha = 0$)

Problema 8

Los campos totales son

$$\mathsf{E} = E_1[(1+\alpha)\sin\theta\,\mathsf{n} + (1-\alpha)\cos\theta\,\mathsf{t}] + (1-\alpha)E_2\mathsf{e}_2$$

$$\mathsf{B} = -E_1[(1-\alpha)\sin\theta\,\mathsf{n} + (1+\alpha)\cos\theta\,\mathsf{t}] + (1+\alpha)E_1\mathsf{e}_2$$

Problema 8

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Reemplazando en el tensor de Maxwell tenemos que la fuerza por unidad de superficie es

$\mathsf{f} = \frac{1}{4\pi}\langle \mathsf{T}\cdot\mathsf{n}\rangle = -\frac{1}{8\pi}(|E_1|^2 + |E_2|^2)[(1+|\alpha|^2)\cos^2\theta\mathsf{n} - (1-|\alpha|^2)\cos\theta\sin\theta\ \mathsf{t}] \quad (3)$

Problema 8

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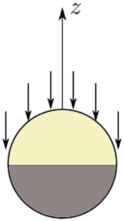
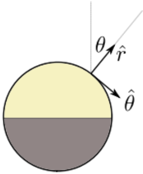
La presión es

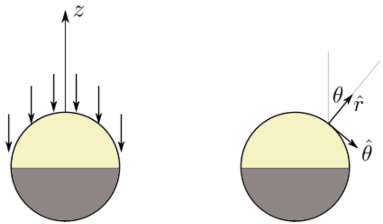
$$P = -\mathsf{n}\cdot\frac{1}{4\pi}\langle \mathsf{T}\cdot\mathsf{n}\rangle = -\frac{1}{8\pi}(|E_1|^2 + |E_2|^2)(1+|\alpha|^2)\cos^2\theta$$

Problema 8



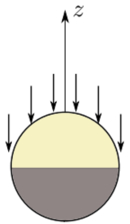


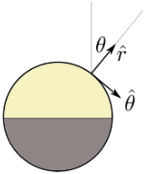
Problema 8



Tierra absorbente $\alpha = 0$

Problema 8





Tierra absorbente $\alpha = 0$

$F_z = \int_{SE} \frac{1}{4\pi} \langle \mathrm{T} \cdot \mathrm{n} \rangle \cdot \hat{z} ds$

Usando (3) con $\mathrm{n} = \hat{r}$ y $\mathrm{t} = \hat{\theta}$

$F_z = -\int_0^{2\pi} d\varphi \int_0^1 d(\cos\theta) a^2 \frac{1}{4\pi} |E|^2 (\cos^2\theta \cos\theta + \cos\theta \sin^2\theta) = -\frac{a^2|E|^2}{8}$

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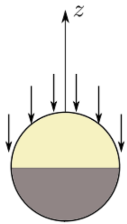

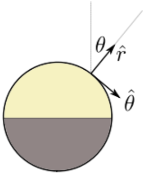

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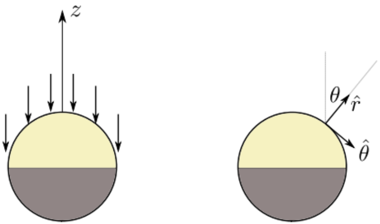

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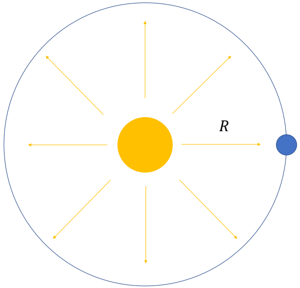
$\frac{dU_{mec}}{dt} + \frac{dU_{em}}{dt} = -\frac{c}{4\pi} \int_{S_R} d^2 r \langle \mathrm{E} \times \mathrm{H} \rangle$



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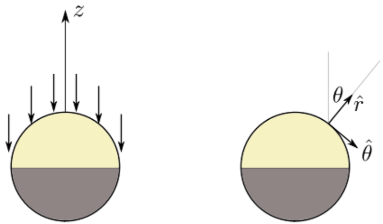

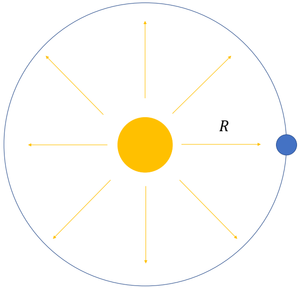

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$Pot = \frac{c}{4\pi} \int_{S_R} d^2 r |\langle \mathrm{E} \times \mathrm{H} \rangle| = \frac{c}{4\pi} |\langle \mathrm{E} \times \mathrm{H} \rangle| 4\pi R^2 = \frac{cR^2}{2} |E|^2$

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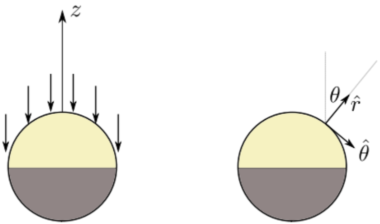

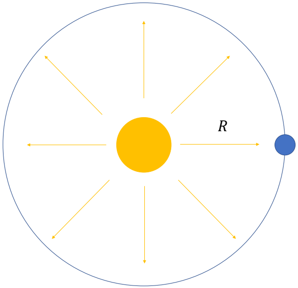

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$F_z + \frac{GM_{\mathrm{Sol}} m}{R^2} = 0$

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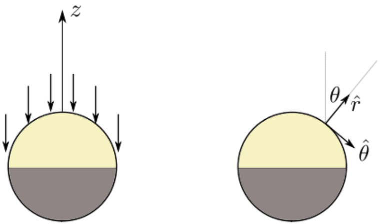

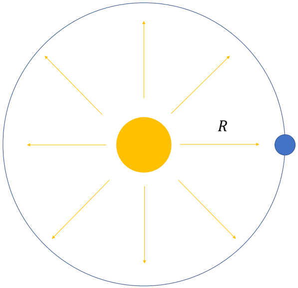

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$$F_z + \frac{G M_{\mathrm{Sol}} m}{R^2} = 0$$

Usando que $m = \rho \frac{4}{3} \pi a^3$ tenemos

$$a = \frac{3}{16\pi} \frac{Pot}{c G M_{\mathrm{Sol}} \rho}$$

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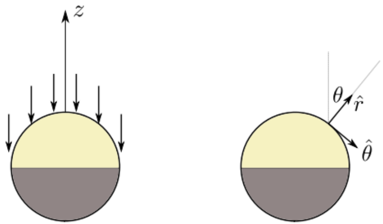

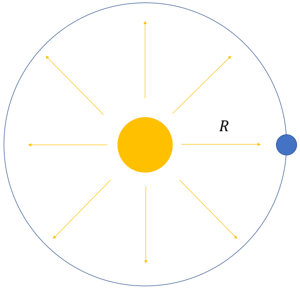

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Para $\rho = 1\ g\ cm^{-3}$, $Pot = 4 \times 10^{26} W$

$\implies \ a \approx 600\ nm.$