Resumen de formulación covariante

Métrica de Minkowski

$\eta^{\mu\nu}$ (contra-variante), $\eta_{\mu\nu}$ (co-variante)

$$\eta^{\alpha\beta} = \eta_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{1}$$

donde $\eta_{\alpha\gamma}\eta^{\gamma\beta} = \delta_\alpha^{\ \beta}$, $\eta^{\alpha\gamma}\eta_{\gamma\beta} = \delta^\alpha_{\ \beta}$ (delta de Kronecker, 1 si $\alpha = \beta$ o 0 si $\alpha \neq \beta$)

Cuadrivectores (rango 1)

$v^\alpha = (v^0, \mathbf{v})$ (contra-variante) , $u_\alpha = (u_0, \mathbf{u})$ (co-variante)

(a) $v_\alpha = \eta_{\alpha\beta} v^\beta = (v^0, -\mathbf{v})$

(b) $u^\alpha = \eta^{\alpha\beta} u_\beta = (u_0, -\mathbf{u})$

(c) $u_\alpha v^\alpha = u^\alpha v_\alpha = u_0 v^0 - \mathbf{u} \cdot \mathbf{v}$

(d) $dv^\alpha = (dv^0, d\mathbf{v})$

(e) $\partial_\alpha = \dfrac{\partial}{\partial v^\alpha} = \left(\dfrac{\partial}{\partial v^0}, \nabla\right)$

(f) $\partial^\alpha u_\alpha = \dfrac{\partial u_\alpha}{\partial v_\alpha} = \dfrac{\partial u_0}{\partial v_0} + \nabla \cdot \mathbf{u}$

(g) $\partial_\alpha u^\alpha = \dfrac{\partial u^\alpha}{\partial v^\alpha} = \dfrac{\partial u^0}{\partial v^0} + \nabla \cdot \mathbf{u}$

(h) $\partial_\alpha \partial^\alpha = \dfrac{\partial^2}{\partial v^0 \partial v_0} - \nabla^2$

Cuadritensores (rango 2)

$T^{\alpha\beta}$ (contra-variante) , $T_{\alpha\beta}$ (co-variante) , $T_\alpha^{\ \beta}$ (mixto) , $T^\alpha_{\ \beta}$ (mixto)

(a) $T^{\alpha\beta} = \begin{pmatrix} T^{00} & T^{0j} \\ T^{i0} & T^{ij} \end{pmatrix}$

(b) $T^\alpha_{\ \beta} = T^{\alpha\gamma}\eta_{\gamma\beta} = \begin{pmatrix} T^{00} & -T^{0j} \\ T^{i0} & -T^{ij} \end{pmatrix}$

(c) $T_\alpha^{\ \beta} = \eta_{\alpha\gamma}T^{\gamma\beta} = \begin{pmatrix} T^{00} & T^{0j} \\ -T^{i0} & -T^{ij} \end{pmatrix}$

(d) $T_{\alpha\beta} = \eta_{\alpha\gamma}T^{\gamma\delta}\eta_{\delta\beta} = \begin{pmatrix} T^{00} & -T^{0j} \\ -T^{i0} & T^{ij} \end{pmatrix}$

(e) $T_{\alpha\beta} = \begin{pmatrix} T_{00} & T_{0j} \\ T_{i0} & T_{ij} \end{pmatrix}$

(f) $T^\alpha_{\ \beta} = \eta^{\alpha\gamma}T_{\gamma\beta} = \begin{pmatrix} T_{00} & T_{0j} \\ -T_{i0} & -T_{ij} \end{pmatrix}$

(g) $T_\alpha^{\ \beta} = T_{\alpha\gamma}\eta^{\gamma\beta} = \begin{pmatrix} T_{00} & -T_{0j} \\ T_{i0} & -T_{ij} \end{pmatrix}$

(h) $T^{\alpha\beta} = \eta^{\alpha\gamma}T_{\gamma\delta}\eta^{\delta\beta} = \begin{pmatrix} T_{00} & -T_{0j} \\ -T_{i0} & T_{ij} \end{pmatrix}$

(i) $v^\alpha = T^\alpha_{\ \beta}u^\beta$

(j) $u_\alpha = T_\alpha^{\ \beta}u_\beta$

Ejemplos

Rango	Ejemplos	
1	$x^\alpha = (ct, x, y, z)$	(posición)
	$u^\alpha = \gamma_u\,(c, u_x, u_y, u_z)$	(velocidad)
	$a^\alpha = \gamma_u^2\,(\gamma_u^2(\mathbf{a}\cdot\mathbf{u})/c, \mathbf{a} + \gamma_u^2(\mathbf{a}\cdot\mathbf{u})\mathbf{u}/c^2)$	(aceleración)
	$p^\alpha = m\,u^\alpha = (e/c, p_x, p_y, p_z)$	(momento)
	$f^\alpha = m\,a^\alpha$	(fuerza de Minkowski)
	$f^\alpha = \gamma_u\,(q\mathbf{E}\cdot\mathbf{u}/c, q\mathbf{E} + q(\mathbf{u}\times\mathbf{B})/c)$	(fuerza de Lorentz)
	$\nu^\alpha = (1, n_x, n_y, n_z)$	(frecuencia)
	$k^\alpha = (\omega/c, k_x, k_y, k_z)$	(número de onda)
	$J^\alpha = (c\rho, J_x, J_y, J_z)$	(densidad de corriente)
	$A^\alpha = (\phi, A_x, A_y, A_z)$	(potencial)
2	$F^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$	(intensidad de campo)
	$\mathcal{F}^{\alpha\beta} = \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z & -E_y \\ B_y & -E_z & 0 & E_x \\ B_z & E_y & -E_x & 0 \end{pmatrix}$	(intensidad de campo dual)

Boost de Lorentz

$$\Lambda^\alpha{}_\beta = \left(\begin{array}{c|c} \gamma & -\gamma\beta_j \\ \hline -\gamma\beta_i & \delta_{ij} + (\gamma-1)\beta_i\beta_j/\beta^2 \end{array}\right)$$

(a) $v^\alpha = \Lambda^\alpha{}_\beta u^\beta$

(b) $\mathcal{T}^{\alpha\beta} = \Lambda^\alpha{}_\gamma \Lambda^\beta{}_\delta T^{\gamma\delta} \equiv ATA'$