1. Identidades vectoriales

1.1. Productos vectoriales y escalares

1.1.1. Producto escalar

Si $\overrightarrow{\mathbf{A}}$ y $\overrightarrow{\mathbf{B}}$ son vectores que en una base ortonormal $\{\overrightarrow{\mathbf{u}}_1, \overrightarrow{\mathbf{u}}_2, \overrightarrow{\mathbf{u}}_3\}$ se escriben como: $\overrightarrow{\mathbf{A}} = A_1\overrightarrow{\mathbf{u}}_1 + A_2\overrightarrow{\mathbf{u}}_2 + A_3\overrightarrow{\mathbf{u}}_3$ y $\overrightarrow{\mathbf{B}} = B_1\overrightarrow{\mathbf{u}}_1 + B_2\overrightarrow{\mathbf{u}}_2 + B_3\overrightarrow{\mathbf{u}}_3$, entonces su producto escalar se define:

$$\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}} = A_1B_1 + A_2B_2 + A_3B_3 \tag{1}$$

$$\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}} = \left|\overrightarrow{\mathbf{A}}\right| \left|\overrightarrow{\mathbf{B}}\right| \cos\theta \tag{2}$$

siendo θ el ángulo que forman dichos vectores.

Si $\overrightarrow{\mathbf{A}}$ es un vector unitario, entonces $\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}$ se corresponde con la proyección de $\overrightarrow{\mathbf{B}}$ sobre la dirección definida por el vector unitario $\overrightarrow{\mathbf{A}}$.

$$\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}} = \overrightarrow{\mathbf{B}} \cdot \overrightarrow{\mathbf{A}} \tag{3}$$

$$\overrightarrow{\mathbf{A}} \cdot \left(\overrightarrow{\mathbf{B}} + \overrightarrow{\mathbf{C}}\right) = \overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}} + \overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{C}} \tag{4}$$

1.1.2. Producto vectorial

Si $\overrightarrow{\mathbf{A}}$ y $\overrightarrow{\mathbf{B}}$ son vectores que en una base ortonormal $\{\overrightarrow{\mathbf{u}}_1, \overrightarrow{\mathbf{u}}_2, \overrightarrow{\mathbf{u}}_3\}$ se escriben como: $\overrightarrow{\mathbf{A}} = A_1\overrightarrow{\mathbf{u}}_1 + A_2\overrightarrow{\mathbf{u}}_2 + A_3\overrightarrow{\mathbf{u}}_3$ y $\overrightarrow{\mathbf{B}} = B_1\overrightarrow{\mathbf{u}}_1 + B_2\overrightarrow{\mathbf{u}}_2 + B_3\overrightarrow{\mathbf{u}}_3$, entonces su producto vectorial se define:

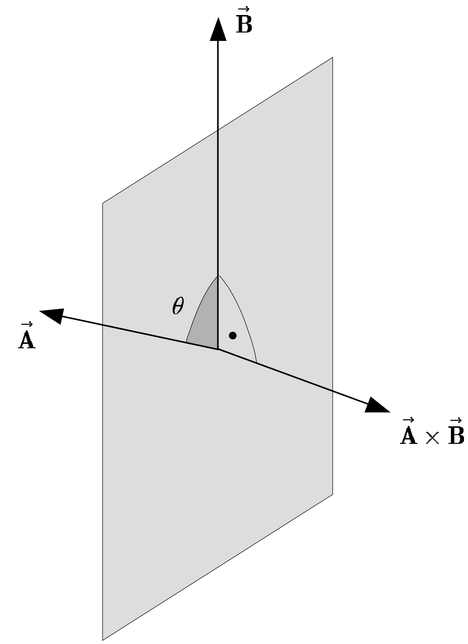
$$\begin{aligned} \overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}} &= \begin{vmatrix} \overrightarrow{\mathbf{u}}_1 & \overrightarrow{\mathbf{u}}_2 & \overrightarrow{\mathbf{u}}_3 \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} = \\ &= (A_2B_3 - A_3B_2)\,\overrightarrow{\mathbf{u}}_1 + (A_3B_1 - A_1B_3)\,\overrightarrow{\mathbf{u}}_2 + \\ &+ (A_1B_2 - A_2B_1)\,\overrightarrow{\mathbf{u}}_3 \end{aligned} \tag{5}$$

$$\left|\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}\right| = \left|\overrightarrow{\mathbf{A}}\right| \left|\overrightarrow{\mathbf{B}}\right| \operatorname{sen}\theta \tag{6}$$

$\left|\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}\right|$ coincide con el área del paralelogramo definido por ambos vectores.

$$\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}} = -\overrightarrow{\mathbf{B}} \times \overrightarrow{\mathbf{A}} \tag{7}$$

$$\overrightarrow{\mathbf{A}} \times \left(\overrightarrow{\mathbf{B}} + \overrightarrow{\mathbf{C}}\right) = \overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}} + \overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{C}} \tag{8}$$



1.1.3. Identidades vectoriales

$$\overrightarrow{\mathbf{A}} \times \left(\overrightarrow{\mathbf{B}} \times \overrightarrow{\mathbf{C}}\right) = \overrightarrow{\mathbf{B}}\left(\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{C}}\right) - \overrightarrow{\mathbf{C}}\left(\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}}\right) \tag{9}$$

$$\left(\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}\right) \times \overrightarrow{\mathbf{C}} = \overrightarrow{\mathbf{B}}\left(\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{C}}\right) - \overrightarrow{\mathbf{A}}\left(\overrightarrow{\mathbf{B}} \cdot \overrightarrow{\mathbf{C}}\right) \tag{10}$$

$$\left(\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}\right) \cdot \left(\overrightarrow{\mathbf{C}} \times \overrightarrow{\mathbf{D}}\right) = \left(\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{C}}\right)\left(\overrightarrow{\mathbf{B}} \cdot \overrightarrow{\mathbf{D}}\right) - \left(\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{D}}\right)\left(\overrightarrow{\mathbf{B}} \cdot \overrightarrow{\mathbf{C}}\right) \tag{11}$$

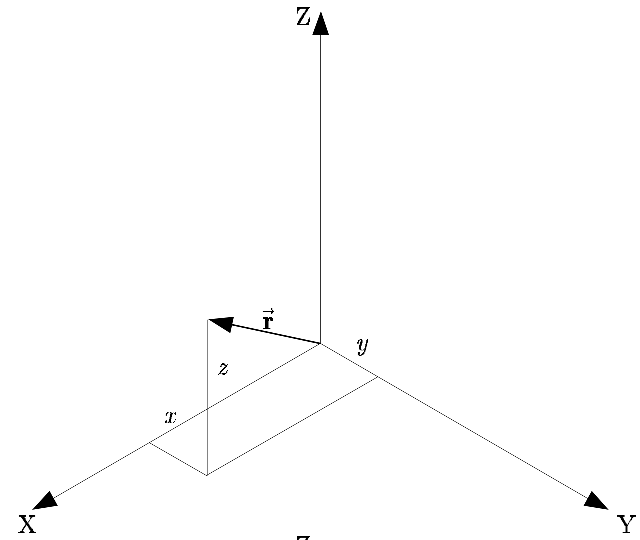
$$\begin{aligned} \left(\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}\right) &\times \left(\overrightarrow{\mathbf{C}} \times \overrightarrow{\mathbf{D}}\right) = \\ &= \overrightarrow{\mathbf{C}}\left[\overrightarrow{\mathbf{A}} \cdot \left(\overrightarrow{\mathbf{B}} \times \overrightarrow{\mathbf{D}}\right)\right] - \overrightarrow{\mathbf{D}}\left[\overrightarrow{\mathbf{A}} \cdot \left(\overrightarrow{\mathbf{B}} \times \overrightarrow{\mathbf{C}}\right)\right] = \\ &= \overrightarrow{\mathbf{B}}\left[\overrightarrow{\mathbf{A}} \cdot \left(\overrightarrow{\mathbf{C}} \times \overrightarrow{\mathbf{D}}\right)\right] - \overrightarrow{\mathbf{A}}\left[\overrightarrow{\mathbf{B}} \cdot \left(\overrightarrow{\mathbf{C}} \times \overrightarrow{\mathbf{D}}\right)\right] \end{aligned} \tag{12}$$

2. Coordenadas Cartesianas

2.1. Transformación de Coordenadas

2.1.1. Desde Coordenadas Cilíndricas

$$x = \rho \cos\varphi \tag{13}$$
$$y = \rho \operatorname{sen}\varphi \tag{14}$$
$$z = z \tag{15}$$



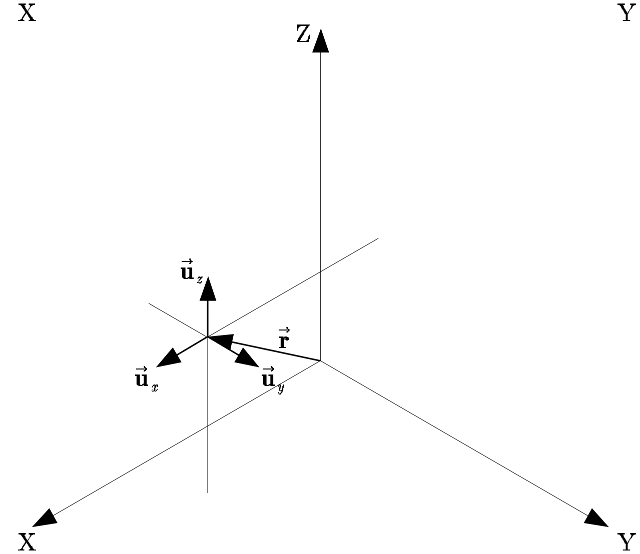
2.1.2. Desde Coordenadas Esféricas

$$x = r \operatorname{sen}\theta \cos\varphi \tag{16}$$
$$y = r \operatorname{sen}\theta \operatorname{sen}\varphi \tag{17}$$
$$z = r \cos\theta \tag{18}$$

2.2. Transformación de Vectores

2.2.1. Desde Coordenadas Cilíndricas

$$A_x = A_\rho \cos\varphi - A_\varphi \operatorname{sen}\varphi \tag{19}$$
$$A_y = A_\rho \operatorname{sen}\varphi + A_\varphi \cos\varphi \tag{20}$$
$$A_z = A_z \tag{21}$$



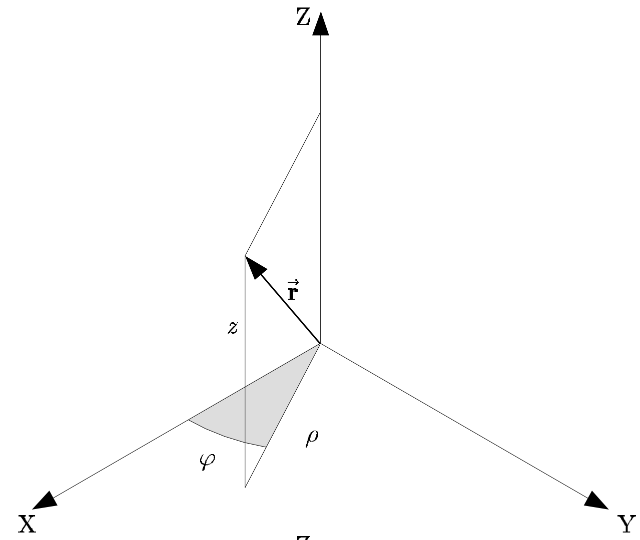
2.2.2. Desde Coordenadas Esféricas

$$A_x = A_r \operatorname{sen}\theta \cos\varphi - A_\varphi \operatorname{sen}\varphi + A_\theta \cos\theta \cos\varphi \tag{22}$$
$$A_y = A_r \operatorname{sen}\theta \cos\varphi + A_\varphi \cos\varphi + A_\theta \cos\theta \cos\varphi \tag{23}$$
$$A_z = A_r \cos\theta - A_\theta \operatorname{sen}\theta \tag{24}$$

3. Coordenadas Cilíndricas

3.1. Transformación de Coordenadas

3.1.1. Desde Coordenadas Cartesianas



$$\rho = \sqrt{x^2+y^2} \tag{25}$$

$$\operatorname{tg}\varphi = \frac{y}{x} \tag{26}$$

$$z = z \tag{27}$$

3.1.2. Desde Coordenadas Esféricas

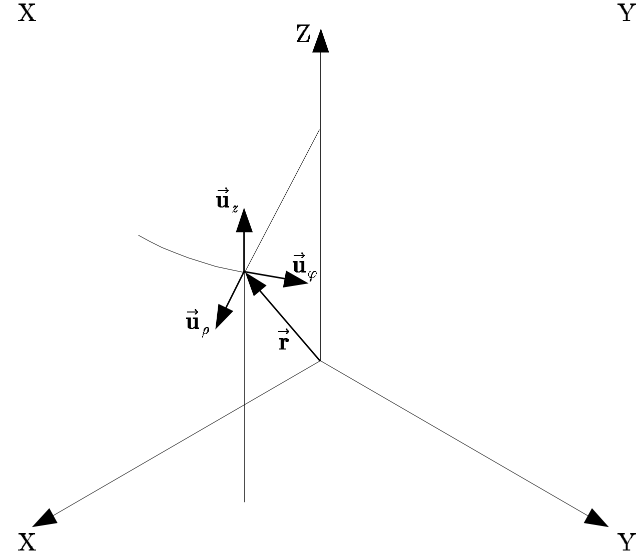
$$\rho = r \operatorname{sen}\theta \tag{28}$$

$$\varphi = \varphi \tag{29}$$

$$z = r\cos\theta \tag{30}$$

3.2. Transformación de Vectores

3.2.1. Desde Coordenadas Cartesianas



$$A_\rho = A_x\frac{x}{\sqrt{x^2+y^2}} + A_y\frac{y}{\sqrt{x^2+y^2}} \tag{31}$$

$$A_\varphi = -A_x\frac{y}{\sqrt{x^2+y^2}} + A_y\frac{x}{\sqrt{x^2+y^2}} \tag{32}$$

$$A_z = A_z \tag{33}$$

3.2.2. Desde Coordenadas Esféricas

$$A_\rho = A_r\operatorname{sen}\theta + A_\theta\cos\theta \tag{34}$$

$$A_\varphi = A_\varphi \tag{35}$$

$$A_z = A_r\cos\theta - A_\theta\operatorname{sen}\theta \tag{36}$$

4. Coordenadas Esféricas

4.1. Transformación de Coordenadas

4.1.1. Desde Coordenadas Cartesianas

$$r = \sqrt{x^2+y^2+z^2} \tag{37}$$

$$\operatorname{tg}\theta = \frac{\sqrt{x^2+y^2}}{z} \tag{38}$$

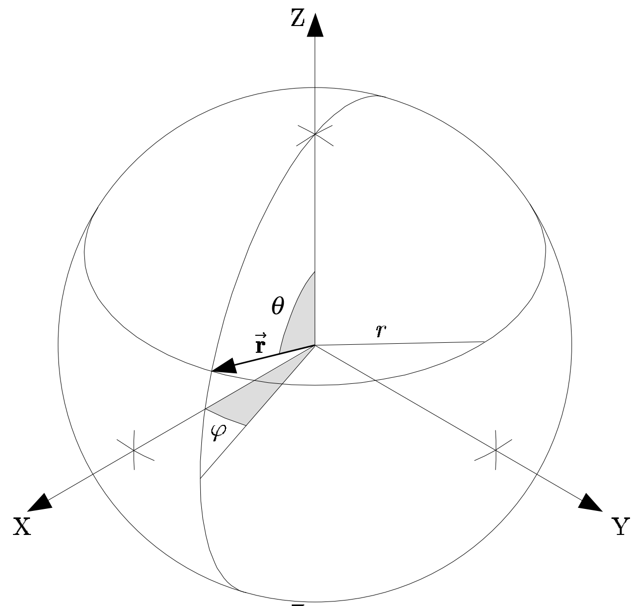
$$\operatorname{tg}\varphi = \frac{y}{x} \tag{39}$$

4.1.2. Desde Coordenadas Cilíndricas

$$r = \sqrt{\rho^2+z^2} \tag{40}$$

$$\operatorname{tg}\theta = \frac{\rho}{z} \tag{41}$$

$$\varphi = \varphi \tag{42}$$



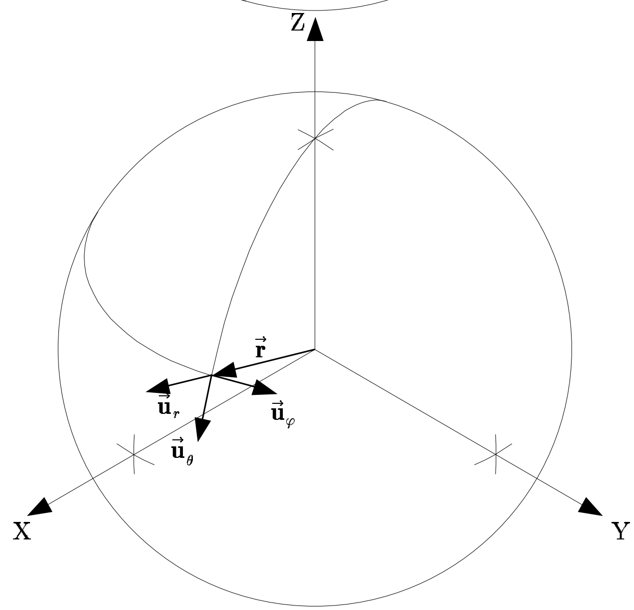
4.2. Transformación de Vectores

4.2.1. Desde Coordenadas Cartesianas

$$\begin{aligned} A_r &= A_x \frac{x}{\sqrt{x^2+y^2+z^2}} + \\ &+ A_y \frac{y}{\sqrt{x^2+y^2+z^2}} + A_z \frac{z}{\sqrt{x^2+y^2+z^2}} \end{aligned} \tag{43}$$

$$A_\varphi = -A_x \frac{y}{\sqrt{x^2+y^2}} + A_y \frac{x}{\sqrt{x^2+y^2}} \tag{44}$$

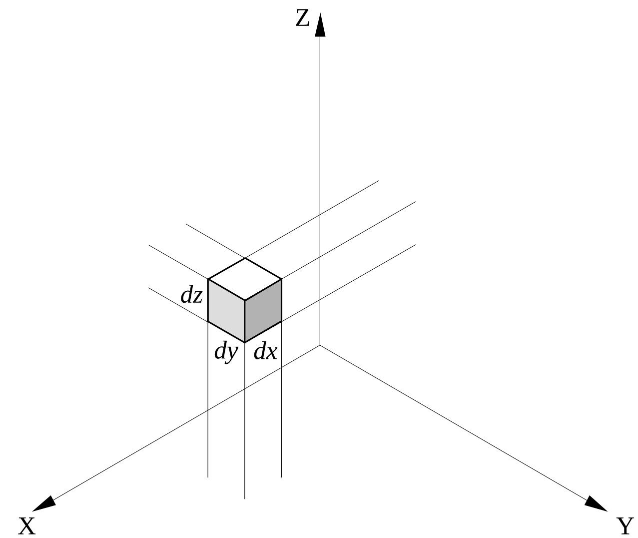
$$\begin{aligned} A_\theta &= A_x \frac{xz}{\sqrt{(x^2+y^2+z^2)\,(x^2+y^2)}} + \\ &+ A_y \frac{yz}{\sqrt{(x^2+y^2+z^2)\,(x^2+y^2)}} - A_z \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}} \end{aligned} \tag{45}$$



4.2.2. Desde Coordenadas Cilíndricas

$$A_r = A_\rho \frac{\rho}{\sqrt{\rho^2+z^2}} + A_z \frac{z}{\sqrt{\rho^2+z^2}} \tag{46}$$

$$
\begin{aligned}
A_\varphi &= A_\varphi && (47)\\
A_\theta &= A_\rho \frac{z}{\sqrt{\rho^2+z^2}} - A_z \frac{\rho}{\sqrt{\rho^2+z^2}} && (48)
\end{aligned}
$$



5. Fórmulas de análisis vectorial

5.1. Coordenadas Cartesianas

5.1.1. Elemento de longitud

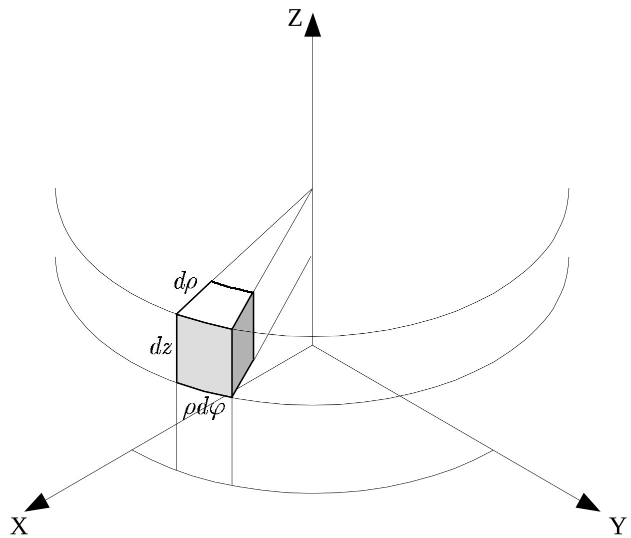
$$
\begin{aligned}
d\overrightarrow{\mathbf{r}} &= dx\overrightarrow{\mathbf{u}}_x + dy\overrightarrow{\mathbf{u}}_y + dz\overrightarrow{\mathbf{u}}_z && (49)\\
dr &= \sqrt{dx^2+dy^2+dz^2} && (50)
\end{aligned}
$$

5.1.2. Elementos de superficie

$$
\begin{aligned}
d\overrightarrow{\mathbf{s}}_x &= dydz\overrightarrow{\mathbf{u}}_x && (51)\\
d\overrightarrow{\mathbf{s}}_y &= dzdx\overrightarrow{\mathbf{u}}_y && (52)\\
d\overrightarrow{\mathbf{s}}_z &= dxdy\overrightarrow{\mathbf{u}}_z && (53)
\end{aligned}
$$

5.1.3. Elemento de volumen

$$dv = dxdydz \tag{54}$$



5.2. Coordenadas Cilíndricas

5.2.1. Elemento de longitud

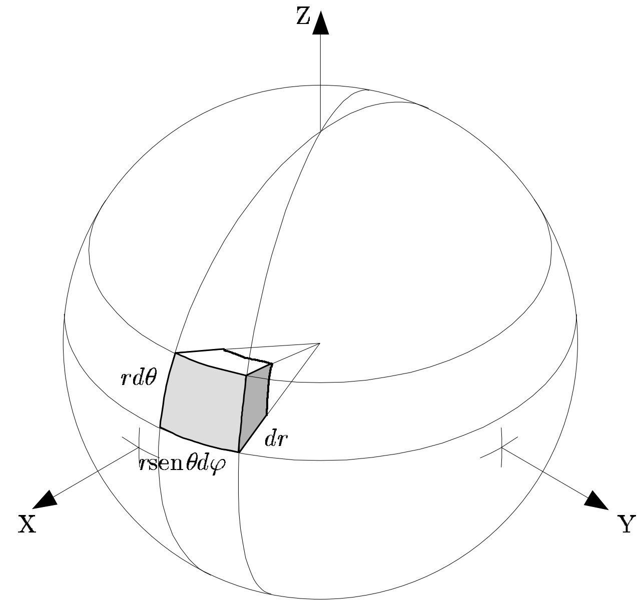
$$d\vec{\mathbf{r}} = d\rho\vec{\mathbf{u}}_\rho + \rho d\varphi\vec{\mathbf{u}}_\varphi + dz\vec{\mathbf{u}}_z \tag{55}$$
$$dr = \sqrt{d\rho^2 + \rho^2 d\varphi^2 + dz^2} \tag{56}$$

5.2.2. Elementos de superficie

$$d\vec{\mathbf{s}}_\rho = \rho d\varphi dz\vec{\mathbf{u}}_\rho \tag{57}$$
$$d\vec{\mathbf{s}}_\varphi = d\rho dz\vec{\mathbf{u}}_\varphi \tag{58}$$
$$d\vec{\mathbf{s}}_z = \rho d\rho d\varphi\vec{\mathbf{u}}_z \tag{59}$$

5.2.3. Elemento de volumen

$$dv = \rho d\rho d\varphi dz \tag{60}$$



5.3. Coordenadas Esféricas

5.3.1. Elemento de longitud

$$d\vec{\mathbf{r}} = dr\vec{\mathbf{u}}_r + r\operatorname{sen}\theta d\varphi\vec{\mathbf{u}}_\varphi + rd\theta\vec{\mathbf{u}}_\theta \tag{61}$$
$$dr = \sqrt{dr^2 + r^2\operatorname{sen}^2\theta d\varphi^2 + r^2 d\theta^2} \tag{62}$$

5.3.2. Elementos de superficie

$$d\vec{\mathbf{s}}_r = r^2\operatorname{sen}\theta d\varphi d\theta\vec{\mathbf{u}}_r \tag{63}$$
$$d\vec{\mathbf{s}}_\theta = r\operatorname{sen}\theta dr d\varphi\vec{\mathbf{u}}_\theta \tag{64}$$
$$d\vec{\mathbf{s}}_\varphi = r dr d\theta\vec{\mathbf{u}}_\varphi \tag{65}$$

5.3.3. Elemento de volumen

$$dv = r^2\operatorname{sen}\theta dr d\theta d\varphi \tag{66}$$

5.4. Gradiente, Divergencia y Rotacional

5.4.1. Coordenadas Cartesianas

$$\vec{\nabla}\Phi = \frac{\partial\Phi}{\partial x}\vec{\mathbf{u}}_x + \frac{\partial\Phi}{\partial y}\vec{\mathbf{u}}_y + \frac{\partial\Phi}{\partial z}\vec{\mathbf{u}}_z \quad (67)$$

$$\vec{\nabla}\cdot\vec{\mathbf{A}} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \quad (68)$$

$$\begin{aligned}\vec{\nabla}\times\vec{\mathbf{A}} &= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{\mathbf{u}}_x + \quad (69)\\ &+ \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{\mathbf{u}}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{\mathbf{u}}_z\end{aligned}$$

5.4.2. Coordenadas Cilíndricas

$$\vec{\nabla}\Phi = \frac{\partial\Phi}{\partial\rho}\vec{\mathbf{u}}_\rho + \frac{1}{\rho}\frac{\partial\Phi}{\partial\varphi}\vec{\mathbf{u}}_\varphi + \frac{\partial\Phi}{\partial z}\vec{\mathbf{u}}_z \quad (70)$$

$$\vec{\nabla}\cdot\vec{\mathbf{A}} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z} \quad (71)$$

$$\begin{aligned}\vec{\nabla}\times\vec{\mathbf{A}} &= \left(\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right)\vec{\mathbf{u}}_\rho + \quad (72)\\ &+ \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right)\vec{\mathbf{u}}_\varphi + \left[\frac{1}{\rho}\frac{\partial(\rho A_\varphi)}{\partial\rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial\varphi}\right]\vec{\mathbf{u}}_z\end{aligned}$$

5.4.3. Coordenadas Esféricas

$$\vec{\nabla}\Phi = \frac{\partial\Phi}{\partial r}\vec{\mathbf{u}}_r + \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\vec{\mathbf{u}}_\theta + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial\Phi}{\partial\varphi}\vec{\mathbf{u}}_\varphi \quad (73)$$

$$\vec{\nabla}\cdot\vec{\mathbf{A}} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial(\mathrm{sen}\,\theta A_\theta)}{\partial\theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial A_\phi}{\partial\varphi} \quad (74)$$

$$\begin{aligned}\vec{\nabla}\times\vec{\mathbf{A}} &= \left\{\frac{1}{r\,\mathrm{sen}\,\theta}\left[\frac{\partial(\mathrm{sen}\,\theta A_\varphi)}{\partial\theta} - \frac{\partial A_\theta}{\partial\varphi}\right]\right\}\vec{\mathbf{u}}_r + \quad (75)\\ &+ \left[\frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial A_r}{\partial\varphi} - \frac{1}{r}\frac{\partial(rA_\varphi)}{\partial r}\right]\vec{\mathbf{u}}_\theta + \\ &+ \left\{\frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]\right\}\vec{\mathbf{u}}_\varphi\end{aligned}$$

5.4.4. Identidades

$$\vec{\nabla}\times\vec{\nabla}\Phi = \vec{\mathbf{0}} \quad (76)$$

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{\mathbf{A}}\right) = 0 \quad (77)$$

$$\vec{\nabla}\cdot\left(\vec{\nabla}\Phi\right) = \nabla^2\Phi \quad (78)$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{\mathbf{A}}\right) = \vec{\nabla}\left(\vec{\nabla}\cdot\vec{\mathbf{A}}\right) - \nabla^2\vec{\mathbf{A}} \quad (79)$$

$$\vec{\nabla}(\Phi+\Psi) = \vec{\nabla}\Phi + \vec{\nabla}\Psi \quad (80)$$

$$\vec{\nabla}\cdot\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}\right) = \vec{\nabla}\cdot\vec{\mathbf{A}} + \vec{\nabla}\cdot\vec{\mathbf{B}} \quad (81)$$

$$\vec{\nabla}\times\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}\right) = \vec{\nabla}\times\vec{\mathbf{A}} + \vec{\nabla}\times\vec{\mathbf{B}} \quad (82)$$

$$\vec{\nabla}(\Phi\Psi) = \Psi\vec{\nabla}\Phi + \Phi\vec{\nabla}\Psi \quad (83)$$

$$\vec{\nabla}\cdot\left(\Phi\vec{\mathbf{A}}\right) = \vec{\mathbf{A}}\cdot\vec{\nabla}\Phi + \Phi\vec{\nabla}\cdot\vec{\mathbf{A}} \quad (84)$$

$$\begin{aligned}\vec{\nabla}\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}\right) &= \left(\vec{\mathbf{A}}\cdot\vec{\nabla}\right)\vec{\mathbf{B}} + \left(\vec{\mathbf{B}}\cdot\vec{\nabla}\right)\vec{\mathbf{A}} + \quad (85)\\ &+ \vec{\mathbf{A}}\times\left(\vec{\nabla}\times\vec{\mathbf{B}}\right) + \vec{\mathbf{B}}\times\left(\vec{\nabla}\times\vec{\mathbf{A}}\right)\end{aligned}$$

$$\vec{\nabla}\cdot\left(\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right) = \vec{\mathbf{B}}\vec{\nabla}\times\vec{\mathbf{A}} + \vec{\mathbf{A}}\vec{\nabla}\times\vec{\mathbf{B}} \quad (86)$$

$$\vec{\nabla}\times\left(\Phi\vec{\mathbf{A}}\right) = \vec{\nabla}\Phi\times\vec{\mathbf{A}} + \Phi\vec{\nabla}\times\vec{\mathbf{A}} \quad (87)$$

$$\begin{aligned}\vec{\nabla}\times\left(\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right) &= \vec{\mathbf{A}}\vec{\nabla}\cdot\vec{\mathbf{B}} - \vec{\mathbf{B}}\vec{\nabla}\cdot\vec{\mathbf{A}} + \quad (88)\\ &+ \left(\vec{\mathbf{B}}\cdot\vec{\nabla}\right)\vec{\mathbf{A}} - \left(\vec{\mathbf{A}}\cdot\vec{\nabla}\right)\vec{\mathbf{B}}\end{aligned}$$

5.4.5. Teoremas

$$\int_v \left(\vec{\nabla}\cdot\vec{\mathbf{A}}\right)dv = \oint_s \vec{\mathbf{A}}\cdot d\vec{\mathbf{s}} \quad (89)$$

$$\int_s \left(\vec{\nabla}\times\vec{\mathbf{A}}\right)d\vec{\mathbf{s}} = \oint_c \vec{\mathbf{A}}\cdot d\vec{\mathbf{r}} \quad (90)$$

$$\int_v \left(\vec{\nabla}\times\vec{\mathbf{A}}\right)dv = \oint_s d\vec{\mathbf{s}}\times\vec{\mathbf{A}} \quad (91)$$

$$\int_s d\vec{\mathbf{s}}\times\vec{\nabla}\Phi = \oint_c \Phi d\vec{\mathbf{r}} \quad (92)$$