Física Teórica 1 - Práctica

Ondas planas.

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Temas a tratar:

- Problema 3

Ondas planas

Temas a tratar:

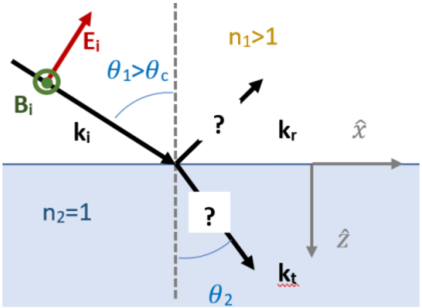
- Problema 3
- Problema 4

Ondas planas

Temas a tratar:

- Problema 3
- Problema 4
- Problema 5

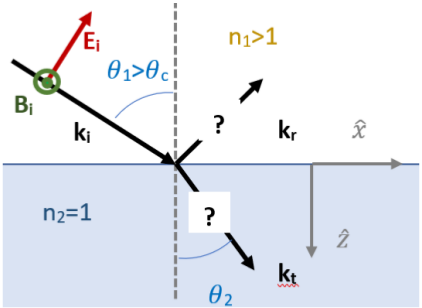
Problema 3



a) Por la ley de Snell tenemos que el ángulo de la onda en la segunda interface viene dado por

$$n \sin\theta = \sin\theta', \tag{1}$$

Problema 3



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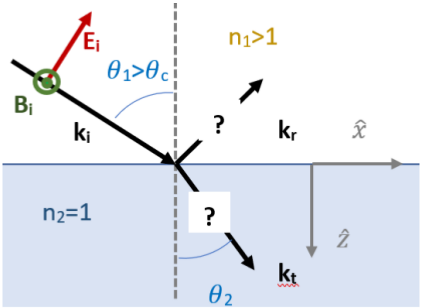

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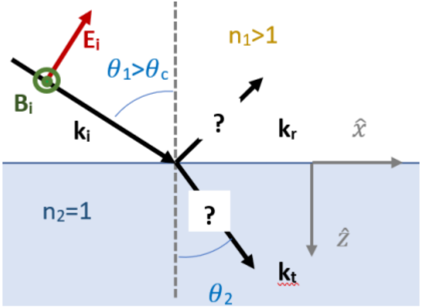

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entonces este ángulo no puede ser real.

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$$\mathbf{k}_2 = k_t(\sin\theta'\hat{x} + \cos\theta'\hat{z}) \tag{3}$$

donde $\cos\theta' = \sqrt{1-\sin^2\theta'} = \sqrt{1-n^2\sin^2\theta} = i\sqrt{n^2\sin^2\theta - 1} = ni\kappa$ y $k_t = \frac{\omega}{c}$.

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$$\mathbf{k}_2 = \frac{\omega}{c}(n\sin\theta\hat{x} + in\kappa\hat{z}) = k_i(\sin\theta\hat{x} + i\kappa\hat{z}) \tag{4}$$

donde $k_i = n\frac{\omega}{c}$ y los campos son proporcionales a

$$\mathbf{E}(\mathbf{r},t),\, \mathbf{B}(\mathbf{r},t) \propto e^{i\mathbf{k}_2\cdot\mathbf{x}} = e^{ik_i x\sin\theta}e^{ik_i i\kappa z} = e^{ik_i x\sin\theta}e^{-k_i\kappa z}. \tag{5}$$

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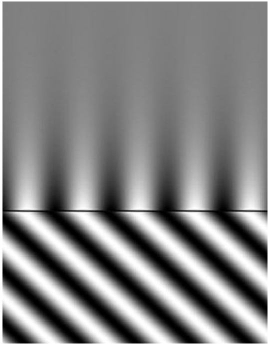
La onda se propaga oscilando en $\hat{x}$ y decae exponencialmente en $\hat{z}$ con una longitud típica de atenuación dada por la inversa de la parte imaginaria del vector de onda

$$e^{-1} \iff k_i\kappa\delta = 1 \iff \delta = \frac{1}{k_i\kappa}. \tag{6}$$

Problema 3



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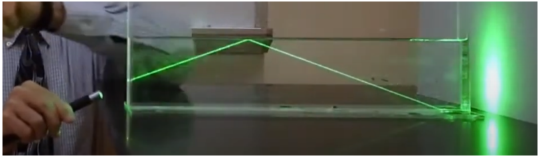
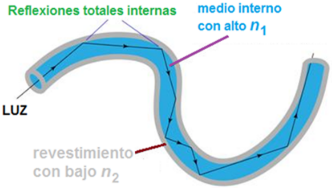
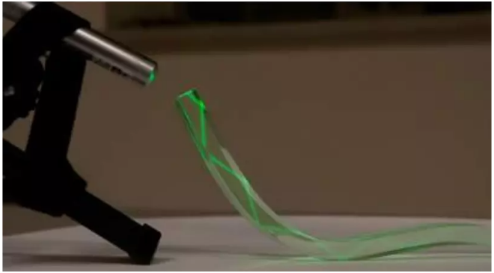



Reflexiones totales internas
medio interno
con alto n_1
LUZ
revestimiento
con bajo n_2



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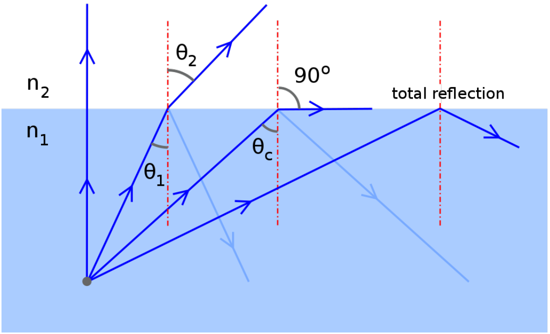
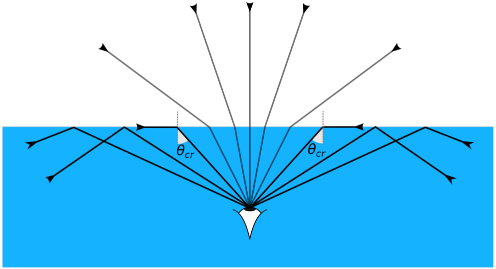
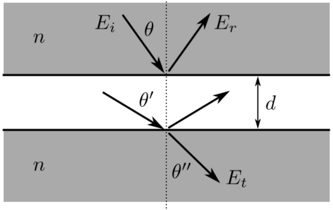




Problema 3

Problema 4



a)

$$\begin{matrix} n\sin\theta = \sin\theta' \\ \sin\theta' = n\sin\theta'' \end{matrix} \implies \theta'' = \theta \tag{7}$$

$$\mathbf{k}_2^{\pm} = k_i(\sin\theta\hat{x} \pm \cos\theta'\hat{z}) = k_i(\sin\theta\hat{x} \pm i\kappa\hat{z}) \tag{8}$$

Problema 4

Además tenemos que para calcular los coeficientes de reflexión y transmisión podemos usar los resultados del problema 2 tomando

$$R = \frac{R_{12}(1 - e^{2i\alpha_2})}{1 - R_{12}^2 e^{2i\alpha_2}} \tag{9}$$

$$T = \frac{T_{23}T_{12}e^{i\alpha_2}}{1 - R_{12}^2 e^{2i\alpha_2}} \tag{10}$$

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$$\bar{n}_1 = \bar{n}_3 = n\cos\theta \tag{11}$$

$$\bar{n}_2 = \cos\theta' \tag{12}$$

$$R_{12} = -R_{23} = \frac{\bar{n}_1 - \bar{n}_2}{\bar{n}_1 + \bar{n}_2} \tag{13}$$

$$T_{12} = \frac{2\bar{n}_1}{\bar{n}_1 + \bar{n}_2} \tag{14}$$

$$T_{23} = \frac{2\bar{n}_2}{\bar{n}_2 + \bar{n}_1} \tag{15}$$

$$\alpha_2 = dk_2\cos\theta' = ik_i\kappa \tag{16}$$

Problema 4

c) Así podemos tomar los límites en que la capa intermedia es muy delgada

$$\alpha_2 \to_{d\to 0} 0 \implies R \to_{d\to 0} \frac{R_{12}(1-1)}{1-R_{12}^2} = 0 \tag{17}$$

$$T \to_{d\to 0} \frac{T_{23}T_{12}e^0}{1-R_{12}^2 e^0} = \frac{T_{23}T_{12}}{1-R_{12}^2} = \frac{1}{1-\left(\frac{\bar{n}_1-\bar{n}_2}{\bar{n}_1+\bar{n}_2}\right)^2}\left(\frac{2\bar{n}_2}{\bar{n}_2+\bar{n}_1}\right)\left(\frac{2\bar{n}_1}{\bar{n}_1+\bar{n}_2}\right) \tag{18}$$

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o muy grande

$$i\alpha_2 = i(ik_i\kappa d) = -k_i\kappa d \to_{d\to\infty} -\infty \implies e^{2i\alpha_2} \to_{d\to\infty} 0 \tag{20}$$

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$$T = \frac{T_{23}T_{12}e^{-k_2\kappa d}}{1-R_{12}^2 e^{-2k_2\kappa d}} \propto T_{23}T_{12}e^{-k_2\kappa d} \to_{d\to\infty} 0 \tag{22}$$

Problema 4

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e) Recordemos que el vector de Poynting aparece en los balances de momento lineal y energía.

Recuerdo el flujo de energía a través de la superficie S de un volumen V es

$$\frac{d}{dt}U + \int_V d^3\mathbf{r}\, \mathbf{j} \cdot \mathbf{E} = -\int\int_S d^2r\, \mathbf{S} \cdot \mathbf{n}$$

$$U = \frac{1}{8\pi} \int_V d^3r \left(\mathbf{E} \cdot \mathbf{D} + \mathbf{H} \cdot \mathbf{B} \right), \quad \mathbf{S} = \frac{c}{4\pi} \mathbf{E} \times \mathbf{H}.$$

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Queremos calcular

$$\langle \mathbf{S}_t \cdot \hat{z} \rangle = \frac{c}{4\pi} \langle \mathbf{E}_t \times \mathbf{H_t} \rangle \cdot \hat{z}$$

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Como estamos trabajando con campos complejos que oscilan armónicamente

$$\tilde{A}(t) = \mathsf{Re}\left(Ae^{-i\omega t}\right), \quad \tilde{A}(t) = \mathsf{Re}\left(Be^{-i\omega t}\right) \tag{24}$$

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tenemos

$$\langle \tilde{A}\tilde{B} \rangle = \frac{1}{T} \int_0^T \tilde{A}(t)\tilde{B}(t)dt = \frac{1}{2}\mathsf{Re}\left(AB^*\right) \tag{25}$$

Problema 4

$$\langle \mathbf{S}_t \cdot \hat{z} \rangle = \frac{c}{8\pi} \mathsf{Re}\left\{(\mathbf{E}_t \times \mathbf{H_t}^*) \cdot \hat{z}\right\} \quad \leftarrow \mathbf{H} = \mathbf{B}/\mu = \mathbf{B} = \frac{\mathbf{k}_t \times \mathbf{E}_t}{k_t} \tag{26}$$

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sabemos que

$$0 = \mathbf{k}_t \cdot \mathbf{E}_t = (k_{tx} \hat{x} + k_{tx} \hat{z}) \cdot \mathbf{E}_t \implies k_{tx} \hat{x} \cdot \mathbf{E}_t = -k_{tz} \hat{z} \cdot \mathbf{E}_t \tag{30}$$

Problema 4

$$\langle \mathbf{S}_t \cdot \hat{z} \rangle = \frac{c}{8\pi} \mathsf{Re} \left\{ (\mathbf{E}_t \times \mathbf{H_t}^*) \cdot \hat{z} \right\} \quad \leftarrow \mathbf{H} = \mathbf{B}/\mu = \mathbf{B} = \frac{\mathbf{k}_t \times \mathbf{E}_t}{k_t} \tag{26}$$

$$= \frac{c}{8\pi} \mathsf{Re} \left\{ \frac{1}{k_t} \mathbf{E}_t \times (\mathbf{k}_t \times \mathbf{E}_t)^* \cdot \hat{z} \right\} \tag{27}$$

Usando $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$ tenemos

$$\mathbf{E}_t \times (\mathbf{k}_t^* \times \mathbf{E}_t^*) = \mathbf{k}_t^* \left(\mathbf{E}_t \cdot \mathbf{E}_t^* \right) - \mathbf{E}_t^* \left(\mathbf{E}_t \cdot \mathbf{k}_t^* \right) \tag{28}$$

$$= \mathbf{k}_t^* |\mathbf{E}_t|^2 - \mathbf{E}_t^* \left(\mathbf{E}_t \cdot \mathbf{k}_t^* \right) \tag{29}$$

sabemos que

$$0 = \mathbf{k}_t \cdot \mathbf{E}_t = (k_{tx} \hat{x} + k_{tx} \hat{z}) \cdot \mathbf{E}_t \implies k_{tx} \hat{x} \cdot \mathbf{E}_t = -k_{tz} \hat{z} \cdot \mathbf{E}_t \tag{30}$$

reemplazando en

$$\mathbf{E}_t \cdot \mathbf{k}_t^* = \mathbf{E}_t \cdot (k_{tx}^* \hat{x} + k_{tz}^* \hat{z}) = \mathbf{E}_t \cdot (k_{tx} \hat{x} + k_{tz}^* \hat{z}) = \mathbf{E}_t \cdot (-k_{tz} \hat{z} + k_{tz}^* \hat{z}) \tag{31}$$

Problema 4

$$\langle \mathbf{S}_t \cdot \hat{z} \rangle = \frac{c}{8\pi} \mathrm{Re}\left\{ (\mathbf{E}_t \times \mathbf{H_t}^*) \cdot \hat{z} \right\} \quad \leftarrow \mathbf{H} = \mathbf{B}/\mu = \mathbf{B} = \frac{\mathbf{k}_t \times \mathbf{E}_t}{k_t} \tag{26}$$

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Usando $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$ tenemos

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sabemos que

$$0 = \mathbf{k}_t \cdot \mathbf{E}_t = (k_{tx}\hat{x} + k_{tx}\hat{z}) \cdot \mathbf{E}_t \implies k_{tx}\hat{x} \cdot \mathbf{E}_t = -k_{tz}\hat{z} \cdot \mathbf{E}_t \tag{30}$$

reemplazando en

$$\mathbf{E}_t \cdot \mathbf{k}_t^* = \mathbf{E}_t \cdot (k_{tx}^*\hat{x} + k_{tz}^*\hat{z}) = \mathbf{E}_t \cdot (k_{tx}\hat{x} + k_{tz}^*\hat{z}) = \mathbf{E}_t \cdot (-k_{tz}\hat{z} + k_{tz}^*\hat{z}) \tag{31}$$

donde usamos que $k_{tx} \in \mathrm{Re}$

Problema 4

y mediante la ecuación de ortogonalidad anterior tenemos

$$\mathbf{E}_t \cdot \mathbf{k}_t^* = -\mathbf{E}_t \cdot \hat{z}\left(k_{tz} - k_{tz}^*\right) = -E_{tz} 2i \mathrm{Im} k_{tz} \tag{32}$$

Problema 4

y mediante la ecuación de ortogonalidad anterior tenemos

$$\mathbf{E}_t \cdot \mathbf{k}_t^* = -\mathbf{E}_t \cdot \hat{z}\left(k_{tz} - k_{tz}^*\right) = -E_{tz} 2i \mathsf{Im} k_{tz} \tag{32}$$

Volviendo al vector de Poynting

$$\langle \mathbf{S}_t \cdot \hat{z} \rangle = \frac{c}{8\pi} \frac{1}{k_t} \mathsf{Re} \left\{ |\mathbf{E}_t|^2 \mathbf{k}_t^* \cdot \hat{z} + E_{tz} 2i \mathsf{Im} k_{tz} \mathbf{E}_t^* \cdot \hat{z} \right\} \tag{33}$$

Problema 4

y mediante la ecuación de ortogonalidad anterior tenemos

$$\mathbf{E}_t \cdot \mathbf{k}_t^* = -\mathbf{E}_t \cdot \hat{z}\left(k_{tz} - k_{tz}^*\right) = -E_{tz} 2i \mathrm{Im} k_{tz} \tag{32}$$

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$$= \frac{c}{8\pi} \frac{1}{k_t} \mathrm{Re}\left\{ |\mathbf{E}_t|^2 k_{tz}^* + E_{tz} 2i \mathrm{Im} k_{tz} E_{tz}^* \right\} \tag{34}$$

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Volviendo al vector de Poynting

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$$= \frac{c}{8\pi}\frac{1}{k_t} \mathsf{Re}\left\{ |\mathbf{E}_t|^2 k_{tz}^* + E_{tz} 2i \mathsf{Im} k_{tz} E_{tz}^* \right\} \tag{34}$$

$$= \frac{c}{8\pi}\frac{1}{k_t} \mathsf{Re}\left\{ |\mathbf{E}_t|^2 k_{tz}^* + |E_{tz}|^2 2i \mathsf{Im} k_{tz} \right\} = \frac{c}{8\pi}\frac{1}{k_t} \mathsf{Re}\left\{ |\mathbf{E}_t|^2 k_{tz}^* \right\} = 0 \tag{35}$$

Problema 4

y mediante la ecuación de ortogonalidad anterior tenemos

$$\mathbf{E}_t \cdot \mathbf{k}_t^* = -\mathbf{E}_t \cdot \hat{z}\left(k_{tz} - k_{tz}^*\right) = -E_{tz} 2i \mathsf{Im} k_{tz} \tag{32}$$

Volviendo al vector de Poynting

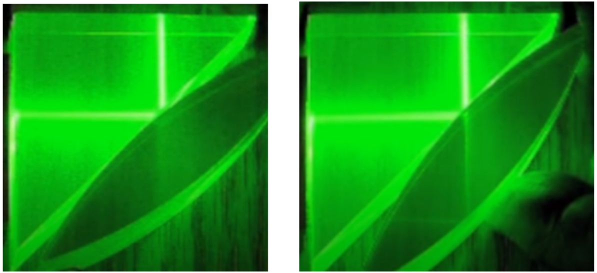
$$\langle \mathbf{S}_t \cdot \hat{z} \rangle = \frac{c}{8\pi}\frac{1}{k_t}\mathsf{Re}\left\{|\mathbf{E}_t|^2 \mathbf{k}_t^* \cdot \hat{z} + E_{tz} 2i \mathsf{Im} k_{tz} \mathbf{E}_t^* \cdot \hat{z}\right\} \tag{33}$$

$$= \frac{c}{8\pi}\frac{1}{k_t}\mathsf{Re}\left\{|\mathbf{E}_t|^2 k_{tz}^* + E_{tz} 2i \mathsf{Im} k_{tz} E_{tz}^*\right\} \tag{34}$$

$$= \frac{c}{8\pi}\frac{1}{k_t}\mathsf{Re}\left\{|\mathbf{E}_t|^2 k_{tz}^* + |E_{tz}|^2 2i \mathsf{Im} k_{tz}\right\} = \frac{c}{8\pi}\frac{1}{k_t}\mathsf{Re}\left\{|\mathbf{E}_t|^2 k_{tz}^*\right\} = 0 \tag{35}$$

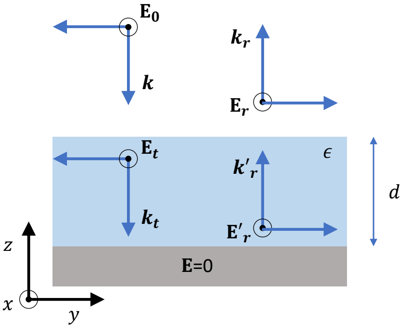
El flujo del vector de Poynting se anula dependiendo de si el vector de onda es complejo o real y hay transmisión de energía o no.

Problema 3



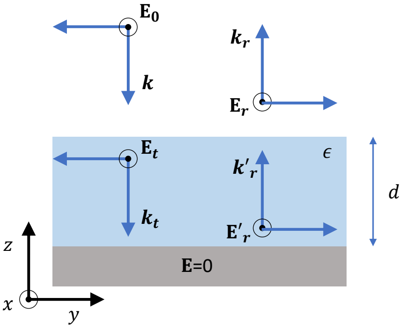
Problema 5

Enunciado



Problema 5

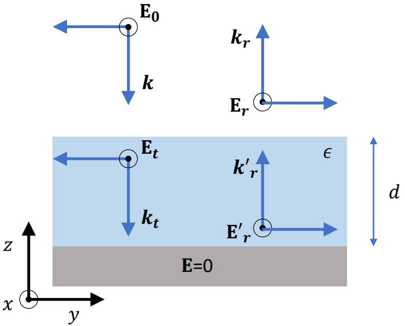
Enunciado



$$\mathbf{E}_{\text{vac}} = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t} + \mathbf{E}_r e^{i\mathbf{k}_r\cdot\mathbf{x} - i\omega t}$$

Problema 5

Enunciado

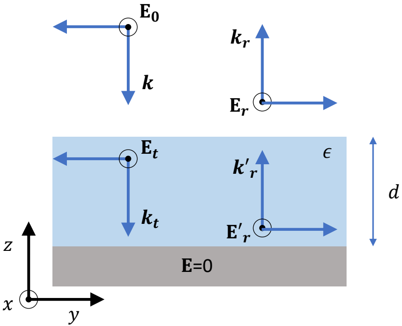


$$\mathbf{E}_{\text{vac}} = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t} + \mathbf{E}_r e^{i\mathbf{k}_r\cdot\mathbf{x}-i\omega t}$$

$$\mathbf{E}_\epsilon = \mathbf{E}_t e^{i\mathbf{k}_t\cdot\mathbf{x}-i\omega t} + \mathbf{E}'_r e^{i\mathbf{k}'_r\cdot\mathbf{x}-i\omega t}$$

Problema 5

Enunciado



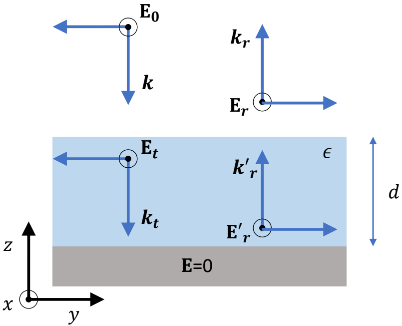
$$\mathbf{E}_{\text{vac}} = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t} + \mathbf{E}_r e^{i\mathbf{k}_r\cdot\mathbf{x}-i\omega t}$$

$$\mathbf{E}_\epsilon = \mathbf{E}_t e^{i\mathbf{k}_t\cdot\mathbf{x}-i\omega t} + \mathbf{E}'_r e^{i\mathbf{k}'_r\cdot\mathbf{x}-i\omega t}$$

Condiciones de contorno en $z = 0$:

$$0 = \mathbf{E}_\epsilon \,|_{z=0} = \mathbf{E}_t + \mathbf{E}'_r \implies \mathbf{E}'_r = -\mathbf{E}_t$$

Enunciado



$$\mathbf{E}_{\text{vac}} = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t} + \mathbf{E}_r e^{i\mathbf{k}_r\cdot\mathbf{x}-i\omega t}$$

$$\mathbf{E}_{\epsilon} = \mathbf{E}_t e^{i\mathbf{k}_t\cdot\mathbf{x}-i\omega t} + \mathbf{E}'_r e^{i\mathbf{k}'_r\cdot\mathbf{x}-i\omega t}$$

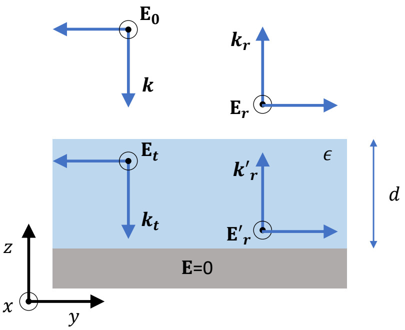
Condiciones de contorno en $z = 0$:

$$0 = \mathbf{E}_{\epsilon}\ \big|_{z=0} = \mathbf{E}_t + \mathbf{E}'_r \implies \mathbf{E}'_r = -\mathbf{E}_t$$

$$\mathbf{E}_{\text{vac}} = [E_0 e^{-ikz-i\omega t} + E_r e^{ikz-i\omega t}]\hat{x}$$

$$\mathbf{E}_{\epsilon} = E_t[e^{-ik'z-i\omega t} - e^{ik'z-i\omega t}]\hat{x}$$

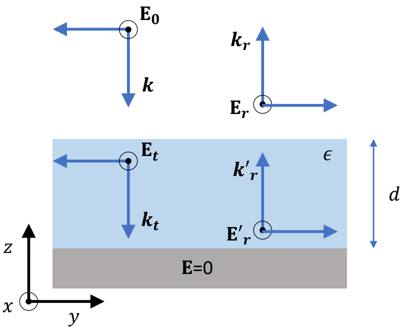
Problema 5



Condiciones de contorno en $z = d$

- $(\mathbf{D}_{\text{vac}} - \mathbf{D}_{\epsilon}) \cdot \hat{z} \mid_{z=d} = 0$
- $(\mathbf{B}_{\text{vac}} - \mathbf{B}_{\epsilon}) \cdot \hat{z} \mid_{z=d} = 0$
- $(\mathbf{E}_{\text{vac}} - \mathbf{E}_{\epsilon}) \times \hat{z} \mid_{z=d} = 0$
- $(\mathbf{H}_{\text{vac}} - \mathbf{H}_{\epsilon}) \times \hat{z} \mid_{z=d} = 0$

Problema 5



Condiciones de contorno en $z = d$

- $(\mathbf{D}_{\text{vac}} - \mathbf{D}_\epsilon) \cdot \hat{z} \,|_{z=d} = 0$ ✓
- $(\mathbf{B}_{\text{vac}} - \mathbf{B}_\epsilon) \cdot \hat{z} \,|_{z=d} = 0$ ✓
- $(\mathbf{E}_{\text{vac}} - \mathbf{E}_\epsilon) \times \hat{z} \,|_{z=d} = 0$
- $(\mathbf{H}_{\text{vac}} - \mathbf{H}_\epsilon) \times \hat{z} \,|_{z=d} = 0$

Las primeras dos se satisfacen inmediatamente porque los campos son paralelos a la superficie.

Problema 5

$$(E_{\mathsf{vac}} - E_\epsilon)\hat{x} \times \hat{z}|_{z=d} = 0 \quad \implies [E_0 e^{-ikd} + E_r e^{ikd}] - E_t[e^{-ik'd} - e^{ik'd}] = 0 \quad (1)$$

Problema 5

$$(E_{\textsf{vac}} - E_\epsilon)\hat{x} \times \hat{z}|_{z=d} = 0 \quad \implies [E_0 e^{-ikd} + E_r e^{ikd}] - E_t[e^{-ik'd} - e^{ik'd}] = 0 \quad (1)$$

Usando $\mathbf{H} = \sqrt{\frac{\epsilon}{\mu}}\hat{k} \times \mathbf{E}$

$$(H_{\textsf{vac}} - H_\epsilon)(-\hat{y}) \times \hat{z}|_{z=d} = 0 \quad \implies [E_0 e^{-ikd} - E_r e^{ikd}] - E_t\sqrt{\epsilon}[e^{-ik'd} + e^{ik'd}] = 0 \quad (2)$$

Problema 5

$$(E_{\textsf{vac}} - E_\epsilon)\hat{x} \times \hat{z}|_{z=d} = 0 \quad \implies [E_0 e^{-ikd} + E_r e^{ikd}] - E_t[e^{-ik'd} - e^{ik'd}] = 0 \quad (1)$$

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$$(H_{\textsf{vac}} - H_\epsilon)(-\hat{y}) \times \hat{z}|_{z=d} = 0 \quad \implies [E_0 e^{-ikd} - E_r e^{ikd}] - E_t\sqrt{\epsilon}[e^{-ik'd} + e^{ik'd}] = 0 \quad (2)$$

Sumando (1) y (2):

$$[E_0 e^{-ikd} + \cancel{E_r e^{ikd}}] - E_t(\frac{-2i}{-2i})[e^{-ik'd} - e^{ik'd}] + [E_0 e^{-ikd} \cancel{- E_r e^{ikd}}] - E_t\sqrt{\epsilon}(\frac{2}{2})[e^{-ik'd} + e^{ik'd}] = 0$$

Problema 5

$$(E_{\text{vac}} - E_\epsilon)\hat{x} \times \hat{z}|_{z=d} = 0 \quad \Longrightarrow [E_0 e^{-ikd} + E_r e^{ikd}] - E_t[e^{-ik'd} - e^{ik'd}] = 0 \quad (1)$$

Usando $\mathbf{H} = \sqrt{\frac{\epsilon}{\mu}}\hat{k} \times \mathbf{E}$

$$(H_{\text{vac}} - H_\epsilon)(-\hat{y}) \times \hat{z}|_{z=d} = 0 \quad \Longrightarrow [E_0 e^{-ikd} - E_r e^{ikd}] - E_t\sqrt{\epsilon}[e^{-ik'd} + e^{ik'd}] = 0 \quad (2)$$

Sumando (1) y (2):

$$[E_0 e^{-ikd} + \cancel{E_r e^{ikd}}] - E_t(\frac{-2i}{-2i})[e^{-ik'd} - e^{ik'd}] + [E_0 e^{-ikd} \cancel{- E_r e^{ikd}}] - E_t\sqrt{\epsilon}(\frac{2}{2})[e^{-ik'd} + e^{ik'd}] = 0$$

$$\cancel{2}E_0 e^{-ikd} - E_t(-\cancel{2}i)\sin k'd - E_t\sqrt{\epsilon}\cancel{2}\cos k'd = 0$$

Problema 5

$$(E_{\mathsf{vac}} - E_\epsilon)\hat{x} \times \hat{z}|_{z=d} = 0 \quad \Longrightarrow [E_0 e^{-ikd} + E_r e^{ikd}] - E_t[e^{-ik'd} - e^{ik'd}] = 0 \quad (1)$$

Usando $\mathbf{H} = \sqrt{\frac{\epsilon}{\mu}}\hat{k} \times \mathbf{E}$

$$(H_{\mathsf{vac}} - H_\epsilon)(-\hat{y}) \times \hat{z}|_{z=d} = 0 \quad \Longrightarrow [E_0 e^{-ikd} - E_r e^{ikd}] - E_t\sqrt{\epsilon}[e^{-ik'd} + e^{ik'd}] = 0 \quad (2)$$

Sumando (1) y (2):

$$[E_0 e^{-ikd} + \cancel{E_r e^{ikd}}] - E_t(\frac{-2i}{-2i})[e^{-ik'd} - e^{ik'd}] + [E_0 e^{-ikd} \cancel{- E_r e^{ikd}}] - E_t\sqrt{\epsilon}(\frac{2}{2})[e^{-ik'd} + e^{ik'd}] = 0$$

$$\cancel{2}E_0 e^{-ikd} - E_t(-\cancel{2}i)\sin k'd - E_t\sqrt{\epsilon}\cancel{2}\cos k'd = 0$$

$$\boxed{\frac{E_t}{E_0} = \frac{e^{-ikd}}{\sqrt{\epsilon}\cos(k'd) - i\sin(k'd)}}$$

Problema 5

Partiendo de (2) tenemos

$$[E_0 e^{-ikd} - E_r e^{ikd}] - E_t\sqrt{\epsilon}[e^{-ik'd} + e^{ik'd}] = 0$$

Reemplazando el resultado anterior

$$[E_0 e^{-ikd} - E_r e^{ikd}] - \frac{E_0 e^{-ikd}}{\sqrt{\epsilon}\cos k'd - i\sin k'd}\sqrt{\epsilon} 2\cos k'd = 0$$

Despejamos

$$\boxed{\frac{E_r}{E_0} = -\frac{\sqrt{\epsilon}\cos(k'd) + i\sin(k'd)}{\sqrt{\epsilon}\cos(k'd) - i\sin(k'd)} e^{-2ikd}}$$

Problema 5

Partiendo de (2) tenemos

$$[E_0 e^{-ikd} - E_r e^{ikd}] - E_t\sqrt{\epsilon}[e^{-ik'd} + e^{ik'd}] = 0$$

Reemplazando el resultado anterior

$$[E_0 e^{-ikd} - E_r e^{ikd}] - \frac{E_0 e^{-ikd}}{\sqrt{\epsilon}\cos k'd - i\sin k'd}\sqrt{\epsilon}2\cos k'd = 0$$

Despejamos

$$\boxed{\frac{E_r}{E_0} = -\frac{\sqrt{\epsilon}\cos(k'd) + i\sin(k'd)}{\sqrt{\epsilon}\cos(k'd) - i\sin(k'd)}e^{-2ikd}}$$

Queda tomar $\epsilon_1 = 1$ y $\epsilon_3 \to \infty$ (límite conductor) en el problema 5 y ver que da lo mismo.