Fórmulas útiles:

- Definiciones y relaciones constitutivas para M.L.I.H.:

$$\mathbf{D} = \mathbf{E} + 4\pi\,\mathbf{P} \qquad \mathbf{D} = (1 + 4\pi\,\chi_e)\,\mathbf{E} = \epsilon\,\mathbf{E}$$
$$\mathbf{B} = \mathbf{H} + 4\pi\,\mathbf{M} \qquad \mathbf{B} = (1 + 4\pi\,\chi_m)\,\mathbf{H} = \mu\,\mathbf{H}$$

- Ecuaciones de Maxwell y las condiciones que se obtienen sobre una interfase:
[$\hat{\mathbf{n}}$ normal unitaria apuntando desde el medio (1) hacia el (2)]

$$\nabla\cdot\mathbf{D} = 4\pi\rho_\ell \quad \Longrightarrow \quad (\mathbf{D}_2 - \mathbf{D}_1)\cdot\hat{\mathbf{n}} = 4\pi\sigma_\ell \tag{1}$$
$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} \quad \Longrightarrow \quad \hat{\mathbf{n}}\times(\mathbf{E}_2 - \mathbf{E}_1) = 0 \tag{2}$$
$$\nabla\cdot\mathbf{B} = 0 \quad \Longrightarrow \quad (\mathbf{B}_2 - \mathbf{B}_1)\cdot\hat{\mathbf{n}} = 0 \tag{3}$$
$$\nabla\times\mathbf{H} = \frac{4\pi}{c}\mathbf{j}_\ell + \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t} \quad \Longrightarrow \quad \hat{\mathbf{n}}\times(\mathbf{H}_2 - \mathbf{H}_1) = \frac{4\pi}{c}\,\mathbf{g}_\ell \tag{4}$$

- Balance de energía promediado:

$$\langle \frac{d}{dt}\left(\int_{\mathrm{V}} d^3r\; u\right)\rangle + \langle \int_{\mathrm{V}} d^3r\; \mathbf{j}\cdot\mathbf{E}\,\rangle = -\,\langle \oint_{\mathrm{A}=\partial\mathrm{V}} d^2r\; \mathbf{n}\cdot\mathbf{S}\,\rangle \tag{5}$$

dónde la densidad de energía es $u = \frac{1}{8\pi}(\mathbf{E}\cdot\mathbf{D} + \mathbf{H}\cdot\mathbf{B})$.

- Dados $A(t) = Re[\mathbb{A}e^{-i\omega t}]$ y $B(t) = Re[\mathbb{B}e^{-i\omega t}]$, el promedio temporal en un número entero de períodos $\tau = 2\pi/\omega$ es

$$\langle AB\rangle \equiv \langle AB\rangle_\tau = \frac{1}{2}\,Re[\mathbb{A}\,\mathbb{B}^*] \tag{6}$$

- Algunos cuadrivectores y cuadritensores conocidos:

$$x^\mu = (ct, \mathbf{x}) \quad \partial_\mu = (\partial_t/c, \nabla) \quad k^\mu = (\omega/c, \mathbf{k}) \tag{7}$$
$$J^\mu = (c\rho, \mathbf{j}) \quad A^\mu = (\Phi, \mathbf{A}) \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{8}$$
$$f^\mu \equiv \frac{1}{c}F^\mu_{\ \nu}\,J^\nu = \left(\frac{\mathbf{j}\cdot\mathbf{E}}{c}, \mathbf{f}_{\text{Lorentz}}\right) \quad \mathbf{f}_{\text{Lorentz}} = \rho\mathbf{E} + \frac{\mathbf{j}}{c}\times\mathbf{B} \tag{9}$$

- Boost de un cuadrivector: $C'^\mu = \Lambda^\mu{}_\nu\, C^\nu$, con $C^\mu = (C^0, \vec{C})$:

$$C'^0 = \gamma(C^0 - \vec{\beta}\cdot\vec{C}_\parallel) \tag{10}$$
$$\vec{C}'_\parallel = \gamma(\vec{C}_\parallel - \vec{\beta}C^0) \tag{11}$$
$$\vec{C}'_\perp = \vec{C}_\perp \tag{12}$$

donde $\parallel$ es la componente paralela a la velocidad $\vec{\beta}$ del boost y $\gamma = 1/\sqrt{1-\beta^2}$.

- Transformación de los campos:

$$\mathbf{E}'_\parallel = \mathbf{E}_\parallel \qquad \mathbf{E}'_\perp = \gamma\,(\mathbf{E}_\perp + \boldsymbol{\beta}\times\mathbf{B}_\perp) \tag{13}$$
$$\mathbf{B}'_\parallel = \mathbf{B}_\parallel \qquad \mathbf{B}'_\perp = \gamma\,(\mathbf{B}_\perp - \boldsymbol{\beta}\times\mathbf{E}_\perp) \tag{14}$$

- Algunos invariantes:

$$F_{\mu\nu}F^{\mu\nu} = -2(E^2 - B^2) \tag{15}$$
$$\mathcal{F}_{\mu\nu}F^{\mu\nu} = -4(\mathbf{E}\cdot\mathbf{B}) \tag{16}$$
$$F_\mu{}^\mu = 0 \tag{17}$$

- La potencia por unidad de ángulo sólido en un punto $\mathbf{r}$ a tiempo t radiada por una carga en movimiento arbitrario:

$$\frac{dP}{d\Omega} \equiv I(\mathbf{r},t) = [\mathbf{S}(\mathbf{r},t')\cdot R^2\mathbf{n}]_{t'=t_{ret}}, \qquad \mathbf{S}(\mathbf{r},t) = \frac{c}{4\pi}\,\mathbf{E}\times\mathbf{H}. \tag{18}$$

- Una primitiva innecesaria:

$$\int dx\ \sin^2 x = \tfrac{1}{2}(x - \sin x\cos x) \tag{19}$$

- Expresiones del desarrollo de un campo bajo ciertas hipótesis:

$$\mathbf{E}(\mathbf{r},t) = \frac{\hat{r}}{c^2 r}\times[\hat{r}\times\ddot{\mathbf{p}} + \ddot{\mathbf{m}} + \frac{1}{6c}\hat{r}\times\dddot{\mathbf{Q}}]|_{t_{ret}} + \mathcal{O}[(1/\lambda)^4] \tag{20}$$

$$\mathbf{B}(\mathbf{r},t) = -\frac{1}{c^2 r}[\hat{r}\times\ddot{\mathbf{p}} + (\hat{r}\times\ddot{\mathbf{m}})\times\hat{r} + \frac{1}{6c}\hat{r}\times\dddot{\mathbf{Q}}]|_{t_{ret}} + \mathcal{O}[(1/\lambda)^4] \tag{21}$$

$$\mathbf{p} = \int d^3r'\rho(\boldsymbol{r'},t)\boldsymbol{r'} \ \ \rightarrow\ \ \mathbf{p} = \sum_\alpha q_\alpha \boldsymbol{r}_\alpha(t) \tag{22}$$

$$\mathbf{m} = \frac{1}{2c}\int d^3r'\boldsymbol{r'}\times\mathbf{J}(\boldsymbol{r'},t) \ \ \rightarrow\ \ \mathbf{m} = \frac{1}{2c}\sum_\alpha \boldsymbol{r}_\alpha(t)\times q_\alpha\dot{\boldsymbol{r}}_\alpha(t) \tag{23}$$

$$\mathbf{Q} = \bar{\bar{Q}}\cdot\hat{r}$$

$$[\bar{\bar{Q}}]_{ij} = \int d^3r'\,(3r'_i r'_j - \delta_{ij}r'^2)\rho(\mathbf{r}',t) \ \ \rightarrow\ \ [\bar{\bar{Q}}]_{ij} = \sum_\alpha q_\alpha\,(3r_{\alpha i}(t)r_{\alpha j}(t) - \delta_{ij}r'^2_\alpha) \tag{24}$$

- algunos desarrollos a primer orden no nulo:

$$\sin x = x + \mathcal{O}[x^3] \qquad \cos x = 1 + \mathcal{O}[x^2] \tag{25}$$

- Conservación de la carga (ec. de continuidad):

$$\nabla\cdot\mathbf{j} + \frac{\partial\rho}{\partial t} = 0 \tag{26}$$

- propiedad Delta de Dirac:

$$\delta(f(x)) = \sum_i \frac{\delta(x-x_i)}{\left|\frac{df}{dx}|_{x_i}\right|} \tag{27}$$

- versores:

$$\begin{aligned}\hat{x} &= \sin\theta\,\cos\varphi\,\hat{r} + \cos\theta\,\cos\varphi\,\hat{\theta} - \sin\varphi\,\hat{\varphi}\\ \hat{y} &= \sin\theta\sin\varphi\,\hat{r} + \cos\theta\sin\varphi\,\hat{\theta} + \cos\,\varphi\,\hat{\varphi}\\ \hat{z} &= \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}\end{aligned} \tag{28}$$