Representación de Coordenada de estados coherentes

$$\langle x | \alpha \rangle = \psi_\alpha(x)$$

Usaremos:
$$|\alpha\rangle = \underbrace{e^{\alpha a^\dagger - \alpha^* a}}_{\mathcal{D}(\alpha)} |0\rangle$$

$$\begin{cases} \hat{a} = \frac{1}{\sqrt{2}}\left(\frac{\hat{x}}{\sigma} + i\frac{\hat{p}\sigma}{\hbar}\right) \\ \hat{a}^\dagger = \frac{1}{\sqrt{2}}\left(\frac{\hat{x}}{\sigma} - i\frac{\hat{p}\sigma}{\hbar}\right) \end{cases} \qquad \sigma = \sqrt{\frac{\hbar}{m\omega}}$$

$$|\alpha\rangle = e^{\frac{1}{\sqrt{2}\sigma} 2i\,\mathrm{Im}(\alpha)\hat{x} - \frac{i\sigma}{\sqrt{2}\hbar} 2\,\mathrm{Re}(\alpha)\hat{p}} |0\rangle$$

$$|\alpha\rangle = e^{i\frac{\tilde{q}\hat{x}}{\hbar} - i\frac{\tilde{x}\hat{p}}{\hbar}} |0\rangle$$

con
$$\begin{cases} \tilde{q} = \frac{2\hbar}{\sqrt{2}\sigma}\,\mathrm{Im}(\alpha) \\ \tilde{x} = \frac{2\sigma}{\sqrt{2}}\,\mathrm{Re}(\alpha) \end{cases}$$

Usando $e^{A+B} = e^A e^B e^{-\frac{1}{2}[A,B]}$

$$e^{i\frac{\tilde{q}\hat{x}}{\hbar} - i\frac{\tilde{x}\hat{p}}{\hbar}} = e^{i\frac{\tilde{q}\hat{x}}{\hbar}} e^{-i\frac{\tilde{x}\hat{p}}{\hbar}} e^{-\frac{1}{2}\overbrace{\frac{\tilde{q}\tilde{x}}{\hbar^2}}^{\mathrm{Re}(\alpha)\,\mathrm{Im}(\alpha)}\underbrace{[\hat{x},\hat{p}]}_{i\hbar}}$$

$$= e^{i\frac{\tilde{q}\hat{x}}{\hbar}} e^{-i\frac{\tilde{x}\hat{p}}{\hbar}} e^{-i\,\mathrm{Re}(\alpha)\,\mathrm{Im}(\alpha)}$$

$$\langle x|\alpha\rangle = e^{-i\varphi_0}\langle x|e^{i\frac{\tilde{q}\hat{x}}{\hbar}}e^{-i\frac{\tilde{x}\hat{p}}{\hbar}}|0\rangle$$

Ojo! $\langle x|e^{i\frac{\tilde{q}\hat{x}}{\hbar}} = \left(e^{-i\frac{\tilde{q}\hat{x}}{\hbar}}|x\rangle\right)^\dagger$

$$= \left(e^{-i\frac{\tilde{q}x}{\hbar}}|x\rangle\right)^\dagger$$

$$= e^{i\frac{\tilde{q}x}{\hbar}}\langle x|$$

$$\langle x|\alpha\rangle = e^{-i\varphi_0}e^{i\frac{\tilde{q}x}{\hbar}}\langle x|e^{-i\frac{\tilde{x}\hat{p}}{\hbar}}|0\rangle$$

Ojo! $\langle x|e^{-i\frac{\tilde{x}\hat{p}}{\hbar}} = \left(e^{i\frac{\tilde{x}\hat{p}}{\hbar}}|x\rangle\right)^\dagger$

$$= \left(|x-\tilde{x}\rangle\right)^\dagger$$

$$= \langle x-\tilde{x}|$$

$$\therefore\ \langle x|\alpha\rangle = e^{-i\varphi_0}e^{i\frac{\tilde{q}x}{\hbar}}\langle x-\tilde{x}|0\rangle$$

$$\psi_\alpha(x) = e^{-i\varphi_0}\underbrace{e^{i\frac{\tilde{q}x}{\hbar}}\underbrace{\varphi_0(x-\tilde{x})}_{\text{Fundamental desplazado en }\tilde{x}.}}_{\text{Desplazado en }\tilde{q}\text{ en el espacio de momentos}}$$

$\varphi_0 = Re(\alpha)\,Im(\alpha)$ fase irrelevante.