GUÍA 2

PALACIOS

P1

a) S_j observables

$$S_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix} \qquad S_x^\dagger = \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}$$

$S_x = S_x^\dagger \Rightarrow$ hermítico

$$S_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0&-i&0\\i&0&-i\\0&i&0\end{pmatrix} \qquad S_y^\dagger = \begin{pmatrix}0&-i&0\\-(-i)&0&-i\\0&-(-i)&0\end{pmatrix} = \begin{pmatrix}0&-i&0\\i&0&-i\\0&i&0\end{pmatrix}$$

$S_y = S_y^\dagger \Rightarrow$ hermítico

$$S_z = \frac{\hbar}{\sqrt{2}}\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix} \qquad S_z^\dagger = \frac{\hbar}{2}\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}$$

$S_z = S_z^\dagger \Rightarrow$ hermítico

b) $$P_j = \prod_{i\neq j}\frac{(A-a_i)}{(a_j-a_i)} \qquad a_i = \{-\hbar, 0, \hbar\}$$

$|-,x\rangle \; [-\hbar]$

$$P_{-\hbar} = \frac{(S_x-0)}{(-\hbar-0)}\cdot\frac{(S_x-\hbar\mathbb{1})}{(-\hbar-\hbar)} = \frac{S_x(S_x-\hbar\mathbb{1})}{-\hbar(-2\hbar)} = \frac{S_x^2-\hbar S_x}{2\hbar^2}$$

$$P_{-\hbar}|+\rangle = \left(\frac{S_x^2-\hbar S_x}{2\hbar^2}\right)|+\rangle = \frac{1}{2\hbar^2}\left(\frac{\hbar^2}{2}(|+\rangle+|-\rangle)-\frac{\hbar^2}{\sqrt{2}}|0\rangle\right) = \frac{1}{2}\left[\frac{|+\rangle}{2}+\frac{|-\rangle}{2}-\frac{|0\rangle}{\sqrt{2}}\right]$$

Normalizado $$\left[P_{-\hbar}|+\rangle = \frac{|+\rangle}{2}+\frac{|0\rangle}{\sqrt{2}}-\frac{|-\rangle}{2}\right]$$

PALACIOS

$$S_x = \frac{\hbar}{\sqrt{2}}\left[|+\rangle\langle 0| + |0\rangle\langle +| + |0\rangle\langle -| + |-\rangle\langle 0| \right]$$

$$S_x|+\rangle = \frac{\hbar}{\sqrt{2}}|0\rangle$$

$$S_x^2 = \frac{\hbar^2}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \frac{\hbar^2}{2}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$S_x^2 = \frac{\hbar^2}{2}\left(|+\rangle\langle +| + |+\rangle\langle -| + 2|0\rangle\langle 0| + |-\rangle\langle +| + |-\rangle\langle -| \right)$$

$$S_x^2|+\rangle = \frac{\hbar^2}{2}(|+\rangle + |-\rangle) \quad S_x^2|0\rangle = \hbar^2|0\rangle \quad S_x^2|-\rangle = \frac{\hbar^2}{2}(|+\rangle + |-\rangle)$$

$$P_\hbar = \frac{(S_x - 0)}{(\hbar - 0)}\frac{(S_x + \hbar \mathbb{1})}{(\hbar + \hbar)} = \frac{S_x(S_x + \hbar\mathbb{1})}{2\hbar^2} = \frac{(S_x^2 + \hbar S_x)}{2\hbar^2}$$

Normalizado

$$\left[P_\hbar|+\rangle = \frac{|+\rangle}{2} + \frac{|0\rangle}{\sqrt{2}} + \frac{|-\rangle}{2} \right]$$

$$P_0 = \frac{(S_x + \hbar\mathbb{1})}{(0+\hbar)}\frac{(S_x - \hbar\mathbb{1})}{(0-\hbar)} = \frac{S_x^2 - \cancel{S_x\hbar\mathbb{1} + \hbar\mathbb{1}S_x} - \hbar^2\mathbb{1}}{-\hbar^2} = \frac{\hbar^2\mathbb{1} - S_x^2}{\hbar^2}$$

$$P_0|+\rangle = |+\rangle - \frac{1}{2}\left[|+\rangle + |-\rangle \right] = \frac{1}{2}(|+\rangle - |-\rangle)$$

Normalizado

$$\left[P_0|+\rangle = \frac{\sqrt{2}}{2}\left[|+\rangle - |-\rangle \right] \right]$$

c)

$$S_y = \frac{\hbar i}{\sqrt{2}}\left[-|+\rangle\langle 0| + |0\rangle\langle +| - |0\rangle\langle -| + |-\rangle\langle 0| \right]$$

PALACIOS

$$S_y|+\rangle = \frac{\hbar i}{\sqrt{2}}|0\rangle$$

$$S_y|0\rangle = \frac{\hbar i}{\sqrt{2}}\left[-|+\rangle + |-\rangle\right]$$

$$S_y|-\rangle = -\frac{\hbar i}{\sqrt{2}}|0\rangle$$

d) $$S_x = \frac{\hbar}{\sqrt{2}}\left[|+\rangle\langle 0| + |0\rangle\langle +| + |0\rangle\langle -| + |-\rangle\langle 0|\right]$$

e) $$\left(\frac{S_y}{\hbar}\right)^3 = \left(\frac{S_y}{\hbar}\right)$$

$$\frac{S_y}{\hbar} = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\left(\frac{S_y}{\hbar}\right)^3 = \frac{i^3}{(\sqrt{2})^3}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \frac{-i}{(\sqrt{2})^3}\begin{pmatrix} -1 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \frac{-i}{(\sqrt{2})^3}\begin{pmatrix} 0 & 2 & 0 \\ -2 & 0 & 2 \\ 0 & -2 & 0 \end{pmatrix} = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} = \frac{S_y}{\hbar} \checkmark$$

PALACIOS

$$R(\theta,\hat{y}) = e^{-iS_y/\hbar}$$

$$R(\theta,\hat{y}) = \mathbb{1} + (\cos(\theta) - 1)\left(\frac{S_y}{\hbar}\right)^2 - i\sin(\theta)\frac{S_y}{\hbar}$$

$$R(\tfrac{\pi}{2},\hat{y}) = \mathbb{1} - \frac{S_y^2}{\hbar^2} - i\frac{S_y}{\hbar}$$

$$S_y^2 = \frac{\hbar^2}{2}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix} = \frac{\hbar^2}{2}\Big[|+\rangle\langle+| - |+\rangle\langle-| + 2|0\rangle\langle 0| - |-\rangle\langle+| + |-\rangle\langle-|\Big]$$

$$R(\tfrac{\pi}{2},\hat{y})|+\rangle = |+\rangle - \frac{1}{\cancel{\hbar^2}}\frac{\cancel{\hbar^2}}{2}\Big[|+\rangle - |-\rangle\Big] - \frac{i}{\cancel{\hbar}}\frac{i\cancel{\hbar}}{\sqrt{2}}|0\rangle = \frac{|+\rangle}{2} + \frac{|0\rangle}{\sqrt{2}} + \frac{|-\rangle}{2}$$

$$R(\tfrac{\pi}{2},\hat{y})|0\rangle = \cancel{|0\rangle} - \frac{1\cancel{\hbar^2}}{\cancel{\hbar^2}}\frac{\cancel{2}}{\cancel{2}}\cancel{|0\rangle} - i\frac{1}{\cancel{\hbar}}\frac{\cancel{\hbar}\,i}{\sqrt{2}}\Big[-|+\rangle + |-\rangle\Big] = \frac{1}{\sqrt{2}}\Big[-|+\rangle + |-\rangle\Big]$$

$$R(\tfrac{\pi}{2},\hat{y})|-\rangle = |-\rangle - \frac{1\cancel{\hbar^2}}{\cancel{\hbar^2}\,2}\Big[-|+\rangle + |-\rangle\Big] + \frac{i\cancel{\hbar}\,i}{\cancel{\hbar}\sqrt{2}}|0\rangle = \frac{|+\rangle}{2} - \frac{|0\rangle}{\sqrt{2}} + \frac{|-\rangle}{2}$$

Los vectores son los mismos

P2 m, q $\quad E_x = E_0$

a) En representación de coordenadas

$$H = \hbar^2\frac{\partial^2\psi(x)}{\partial x^2} - qE_0X$$

En representación de momentos

$$H = \frac{P^2}{2m} - qE_0 i\hbar\frac{\partial\psi(p)}{\partial p}$$

b)

$$H|\psi_E\rangle = H\psi_E(P)$$

$$H\psi_E(P) = \left(\frac{P^2}{2m} - i\hbar qE_0\frac{\partial}{\partial p}\right)\psi_E(P) = E\,\psi_E(P)$$

PALACIOS

$$\psi'(p) = \left(\frac{p^2}{2m} - E\right)\frac{-i}{\hbar q E_0}\,\psi(p)$$

$$\frac{d\psi}{dp} = \frac{-i}{\hbar q E_0}\left[\frac{p^2}{2m} - E\right]\psi$$

$$\frac{d\psi}{\psi} = \frac{-i}{\hbar q E_0}\left[\frac{p^2}{2m} - E\right]dp$$

$$\int \frac{d\psi}{\psi} = \frac{-i}{\hbar q E_0}\int\left(\frac{p^2}{2m} - E\right)dp$$

$$\ln(\psi) = \frac{-i}{\hbar q E_0}\left[\frac{p^3}{6m} - pE\right]$$

$$\boxed{\psi(p) = e^{\frac{-ip}{\hbar q E_0}\left[\frac{p^2}{6m} - E\right]}}$$

c) $$\langle x|\psi\rangle = \int_{-\infty}^{+\infty}\langle x|p\rangle\langle p|\psi\rangle = \int_{-\infty}^{+\infty}dp\,\frac{1}{\sqrt{2\pi\hbar}}\,e^{\frac{ipx}{\hbar}}\,e^{\frac{-ip}{\hbar q E_0}\left[\frac{p^2}{6m} - E\right]}$$

d) $$\psi(x) = \int_{-\infty}^{+\infty}dp\,\frac{1}{\sqrt{2\pi\hbar}}\,e^{\frac{ipx}{\hbar}}\,e^{\frac{-ip}{\hbar q E_0}\left[\frac{p^2}{6m} - E\right]}$$

potencial $\phi = -E_0 x$