P3) Base de autoestados de S_z de spin 1 : $\{|+\rangle, |0\rangle, |-\rangle\}$

a) $\frac{1}{2}$ de $+\hbar$ en $\hat{z}$ y $\frac{1}{2}$ $-\hbar$ en $\hat{z}$

$$\rho_1 = \frac{1}{2}\left(|+\rangle\langle+| + |-\rangle\langle-|\right) = \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$$

b) $\frac{1}{2}$ de $+\hbar$ en $\hat{z}$ y $\frac{1}{2}$ $-\hbar$ en $\hat{x}$

$$\begin{cases} |-\rangle^x = \frac{1}{2}\left(|+\rangle - \sqrt{2}|0\rangle + |-\rangle\right) \\ |0\rangle^x = \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right) \\ |+\rangle^x = \frac{1}{2}\left(|+\rangle + \sqrt{2}|0\rangle + |-\rangle\right) \end{cases}$$

$$\frac{1}{4}\left(|+\rangle\langle+| + \sqrt{2}\cancel{|+\rangle\langle 0|} + |+\rangle\langle-| + \sqrt{2}\cancel{|0\rangle\langle+|} + 2|0\rangle\langle 0| + \sqrt{2}\cancel{|0\rangle\langle-|} + |-\rangle\langle+| + \sqrt{2}\cancel{|-\rangle\langle 0|} + |-\rangle\langle-|\right)$$

$$\rho_2 = \frac{1}{2}\left(|+\rangle^x\langle+|^x\right) + \frac{1}{4}\left(|+\rangle - \sqrt{2}|0\rangle + |-\rangle\right)\left(\langle+|^z - \sqrt{2}\langle 0|^z + \langle-|\right)$$

$$\downarrow$$

$$\left(\frac{1}{4}\left(|+\rangle\langle+| - \sqrt{2}\cancel{|+\rangle\langle 0|} + |+\rangle\langle-| - \sqrt{2}\cancel{|0\rangle\langle+|}\right.\right.$$

$$+ 2|0\rangle\langle 0| - \sqrt{2}\cancel{|0\rangle\langle-|} + |-\rangle\langle+| - \sqrt{2}\cancel{|-\rangle\langle 0|} + |-\rangle\langle-|$$

$$\Rightarrow \rho_2 = \begin{pmatrix} 1/4 & 0 & 1/4 \\ 0 & 1/2 & 0 \\ 1/4 & 0 & 1/4 \end{pmatrix}$$

NOTA

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$$tr(S_z \rho_1) = tr\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}\right] =$$

$$= tr\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1/2 \end{bmatrix} = 0$$

$$tr(S_z \rho_2) = tr\begin{pmatrix} 1/4 & 0 & 1/4 \\ 0 & 0 & 0 \\ -1/4 & 0 & -1/4 \end{pmatrix} = 0$$

$$tr(S_x \rho_1) = tr\begin{pmatrix} 0 & 0 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 0 & 0 \end{pmatrix} = 0$$

$$tr(S_x \rho_2) = tr\begin{pmatrix} 0 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1/2 & 0 \end{pmatrix} = 0$$

$$tr(S_y \rho_1) = tr\begin{pmatrix} 0 & 0 & 0 \\ i1/2 & 0 & i1/2 \\ 0 & 0 & 0 \end{pmatrix} = 0$$

$$tr(S_y \rho_2) = tr\begin{pmatrix} 0 & -i/2 & 0 \\ \frac{i}{2} & 0 & \frac{i}{2} \\ 0 & -i/2 & 0 \end{pmatrix} = 0$$

$$\Rightarrow \langle \bar{S} \rangle_1 = \langle \bar{S} \rangle_2 = 0$$

NOTA

Escaneado con CamScanner

P10)

$$\rho_{12} = p\,|\psi^-\rangle\langle\psi^-| + \frac{1-p}{4}\,\mathbb{I}_{12}$$

$$\left\{\begin{array}{l} 0 < p < 1 \\ |\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) \\ \{|\uparrow\rangle, |\downarrow\rangle\} \text{ autoestados de } \sigma_z \end{array}\right\}$$

a) $\gamma = tr\left(\rho_{12}^2\right)$

$$\rho_{12}^2 = \left[p\,|\psi^-\rangle\langle\psi^-| + \frac{1-p}{4}\mathbb{I}_{12}\right]\left[p\,|\psi^-\rangle\langle\psi^-| + \frac{1-p}{4}\mathbb{I}_{12}\right]$$

$$= p^2|\psi^-\rangle\langle\psi^-| + \frac{p-p^2}{2}|\psi^-\rangle\langle\psi^-| + \frac{(1-p)^2}{16}\mathbb{I}_{12}$$

$$\Rightarrow Tr\left(\rho_{12}^2\right) = p^2\,Tr\left(|\psi^-\rangle\langle\psi^-|\right) + \frac{p-p^2}{2}\,Tr\left(|\psi^-\rangle\langle\psi^-|\right) + \frac{(1-p)^2}{16}\underbrace{Tr\left(\mathbb{I}_{12}\right)}_{4}$$

$$* \; Tr\left(|\psi^-\rangle\langle\psi^-|\right) = Tr\left[\frac{1}{2}\Big(|\uparrow\downarrow\rangle\langle\uparrow\downarrow| - |\uparrow\downarrow\rangle\langle\downarrow\uparrow| - |\downarrow\uparrow\rangle\langle\uparrow\downarrow| + |\downarrow\uparrow\rangle\langle\downarrow\uparrow|\Big)\right] = 1$$

$$\Rightarrow Tr\left(\rho_{12}^2\right) = \frac{1}{2}\left(p^2 + p\right) + \frac{1}{4}\left(1 + p^2 - 2p\right)$$

$$\boxed{\gamma = \frac{3}{4}p^2 + \frac{1}{4}}$$

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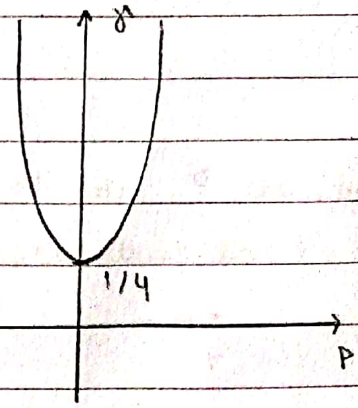
Grafico γ:



Observamos que para $p = 1$ tenemos un estado puro y para $P = 0$ un estado máximamente mixto.

b)

- $\rho_A = Tr_B(\rho_{12})$

$$= p\, Tr_B\left(|\psi^-\rangle\langle\psi^-|\right) + \frac{1-p}{4}\, Tr_B\left(\mathbb{I}_{12}\right)$$

$$\begin{cases} Tr_B\left(|\psi^-\rangle\langle\psi^-|\right) = \frac{1}{2}\left[\,|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|\,\right] = Tr_A\left(|\psi^-\rangle\langle\psi^-|\right) \\ Tr_B\left(\mathbb{I}_{12}\right) = 2\left[\,|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|\,\right] = Tr_A\left(\mathbb{I}_{12}\right) \end{cases}$$

$$\Rightarrow \rho_A = \frac{1}{2}\left(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|\right)$$

- $\rho_B = Tr_A(\rho_{12})$

$$= p\, Tr_A\left(|\psi^-\rangle\langle\psi^-|\right) + \frac{1-p}{4}\, Tr_A\left(\mathbb{I}_{12}\right)$$

$$= \frac{1}{2}\left(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|\right)$$

Las matrices ρ_A y ρ_B no dependen de p.

NOTA

• Sabemos que un estado es entrelazado $\Leftrightarrow$ los estados ρ_A y ρ_B son mixtos:

$$Tr(\rho_A^2) = \frac{1}{2} < 1 = Tr(\rho_B^2)$$

y como ρ_A y ρ_B no dependen de P $\Rightarrow$ tendremos un estado global entrelazado independientemente de su pureza.

c)