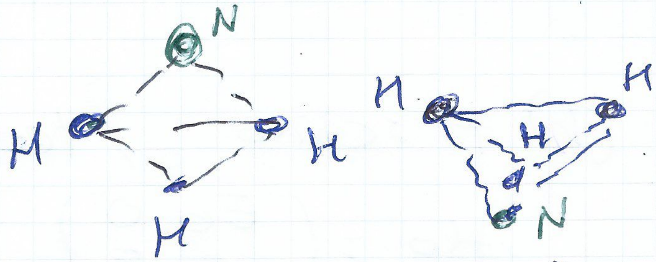
Guía ~~9~~: Perturbaciones

$\boxed{P2}$ NH_3 con campo eléctrico

$|0\rangle$ y $|1\rangle$

se acoplan por tuneleo



$$H = \begin{pmatrix} E_0 & F \\ F & E_1 \end{pmatrix}$$

$|0\rangle$ $|1\rangle$

$E_0 < E_1$

Si $F=0$ $H_0 = \begin{pmatrix} E_0 & \\ & E_1 \end{pmatrix}$

$|0\rangle$ y $|1\rangle$ son autoestados

(a) Solución exacta : Base $\{|0\rangle, |1\rangle\}$

$$H = \frac{(E_0+E_1)}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{(E_0-E_1)}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$+ F\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$H = \frac{(E_0+E_1)}{2}\mathbb{I} + \underbrace{\frac{(E_0-E_1)}{2}\sigma_z + F\sigma_x}_{\vec{b}\cdot\vec{\sigma}}$$

$$\vec{b} = \frac{(E_0-E_1)}{2}\hat{z} + F\hat{x}$$

Autoestados: $|\psi_+\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle$

$|\psi_-\rangle = -\sin\frac{\theta}{2}|0\rangle + \cos\frac{\theta}{2}|1\rangle$

$$E_\pm = \frac{E_0+E_1}{2} \pm \frac{1}{2}\sqrt{(E_0-E_1)^2 + 4F^2}$$

$$\operatorname{tg}\theta = \left(\frac{E_0 - E_1}{2F}\right)^{-1} = \frac{2F}{E_0 - E_1}$$

(b) F : chico , teoría de Perturbaciones

$$H_0 = \begin{pmatrix} E_0 & 0 \\ 0 & E_1 \end{pmatrix} \qquad V = F\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$E_0 \neq E_1$ no degenerado.

$$\begin{cases} E_g = E_0 + 0 + \overbrace{\dfrac{F^2}{E_0 - E_1}}^{<0} \\ E_e = E_1 + 0 - \underbrace{\dfrac{F^2}{E_0 - E_1}}_{<0} \end{cases}$$

$$|E_0 - E_1| \gg F^2$$

$$|g\rangle = |0\rangle + \frac{F}{E_0 - E_1}|1\rangle$$

$$|e\rangle = |1\rangle - \frac{F}{E_0 - E_1}|0\rangle$$

Si $F \ll (E_0 - E_1)$ $\quad \theta \simeq 0$, $\operatorname{tg}\theta \simeq \theta$ $\left\langle \begin{array}{l} \cos\theta/2 \simeq 1 \\ \sin\dfrac{\theta}{2} \simeq \dfrac{\operatorname{tg}\theta}{2} = \dfrac{F}{E_0 - E_1} \end{array} \right.$

$$\begin{cases} |\Psi_0\rangle \cong |0\rangle + \dfrac{F}{E_0 - E_1}|1\rangle \quad ✓ \\ |\Psi_1\rangle \cong -\dfrac{F}{E_0 - E_1}|0\rangle + |1\rangle \quad ✓ \end{cases}$$

$$\begin{cases} E_{ex0} \simeq \\ E_{ex1} \simeq \end{cases}$$

Guía 9: Perturbaciones.

$\boxed{P4}$

$$H_0 = \frac{P_x^2}{2m} + \frac{P_y^2}{2m} + \frac{m\omega^2}{2}(x^2+y^2)$$

Hay sistemas muy confinados que son casi planos (trampas electro-magnéticas).

$$\mathcal{H} = \mathcal{H}_x \otimes \mathcal{H}_y \qquad \{|n_x n_y\rangle\}$$

(a) $H_0 = \hbar\omega(a_x^\dagger a_x + \frac{1}{2}) + \hbar\omega(a_y^\dagger a_y + \frac{1}{2})$

$$H_0 |n_x, n_y\rangle = \hbar\omega(n_x + n_y + 1)|n_x, n_y\rangle$$

$\{|\varphi_0\rangle = |00\rangle \qquad E_0 = \hbar\omega$

$\begin{cases} |\varphi_1\rangle = |10\rangle \\ |\varphi_2\rangle = |01\rangle \end{cases} \qquad E_1 = 2\hbar\omega$

$\begin{cases} |\varphi_3\rangle = |20\rangle \\ |\varphi_4\rangle = |11\rangle \\ |\varphi_5\rangle = |02\rangle \end{cases} \qquad E_2 = 3\hbar\omega.$

$E_N = \hbar\omega(N+1)$ posee degeneración $N+1$

(b) Agregamos una perturbación $V = \delta m\omega^2 xy$

$$H = H_0 + V$$

i) $|\psi_0^{(0)}\rangle = |00\rangle$ No degenerado.

$$E_0 = \underbrace{E_0^{(0)}}_{\hbar\omega} + \underbrace{E_0^{(1)}}_{\langle 00|V|00\rangle} + \underbrace{E_0^{(2)}}_{\sum_{nm} \frac{|\langle 00|V|nm\rangle|^2}{E_0^{(0)} - E_{nm}^{(0)}}}$$

El único no nulo es:

$x = \sqrt{\frac{\hbar}{2m\omega}}(a_x + a_x^\dagger)$

$$\langle 00|V|11\rangle = \delta m\omega^2 \langle 0|x|1\rangle\langle 0|y|1\rangle$$
$$= \delta m\omega^2 \cdot \sqrt{\frac{\hbar}{2m\omega}} \cdot \sqrt{\frac{\hbar}{2m\omega}} = \frac{\delta\hbar\omega}{2}$$

$$\therefore \boxed{E_0 = \hbar\omega - \frac{\delta^2}{8}\hbar\omega.}$$

se usó: $E_{11}^{(0)} - E_{00}^{(0)} = 2\hbar\omega$

La corrección a primer orden se anula.

$$|\psi_0^{(1)}\rangle = \sum_{\substack{\{m,n\}\\ \neq\{0,0\}}} \frac{\langle m,n|V|00\rangle|mn\rangle}{(E_{00}^{(0)} - E_{m,n}^{(0)})} = \frac{\delta\hbar\omega}{\hbar\omega - 3\hbar\omega}|11\rangle = -\frac{\delta}{2}|11\rangle$$

$\therefore$ $|\psi_0\rangle \simeq |00\rangle - \frac{\delta}{2}|11\rangle$, normalizado a orden δ.

ii) Nivel doblemente degenerado $\{|\varphi_1\rangle, |\varphi_2\rangle\}$ $|10\rangle$ $|01\rangle$

Para obtener la corrección a primer orden de la energía y el orden cero de los autoestados de H, debemos diagonalizar V en el subespacio degenerado.

$$\langle 10|V|10\rangle = \delta m\omega^2 \langle 1|x|1\rangle\langle 0|y|0\rangle = 0$$
$$\langle 10|V|01\rangle = \delta m\omega^2 \langle 1|x|0\rangle\langle 0|y|1\rangle$$
$$= \delta\frac{\omega\hbar}{2} = \langle 01|V|10\rangle$$
$$\langle 01|V|01\rangle = 0$$

$$\therefore \quad V_1 = \frac{\hbar\omega\delta}{2}\underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\sigma_x}$$

$$|\psi_1^{(0)\pm}\rangle = \frac{1}{\sqrt{2}}(|10\rangle \pm |01\rangle)$$

$$E_1^{(1)\pm} = 2\hbar\omega \pm \delta\frac{\hbar\omega}{2}$$

Se rompe la degeneración

Guía 9: Perturbaciones

Cont. $\boxed{P4}$

iii) Nivel triplemente degenerado $\{|\varphi_3\rangle, |\varphi_4\rangle, |\varphi_5\rangle\}$
$|20\rangle, |11\rangle, |02\rangle$

$$\langle 20|V|20\rangle = \langle 11|V|11\rangle = \langle 02|V|02\rangle = 0$$

$$\langle 20|V|02\rangle = \delta m\omega^2 \langle 2|x|0\rangle\langle 0|y|2\rangle$$
$$= 0 = \langle 02|V|20\rangle$$

$$\langle 20|V|11\rangle = \langle 11|V|02\rangle = \langle 11|V|20\rangle = \langle 02|V|11\rangle$$
$$= \delta m\omega^2 \langle 2|x|1\rangle\langle 0|y|1\rangle$$
$$= \delta m\omega^2 \frac{\hbar}{2m\omega}\sqrt{2}\cdot 1 = \delta\frac{\sqrt{2}}{2}\hbar\omega$$

$$V_2 = \frac{\delta\sqrt{2}}{2}\hbar\omega \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}}_{\sqrt{2}\frac{S_x}{\hbar}} \quad \text{spin 1.}$$

$$E_2^{(1)+} = 3\hbar\omega + \delta\hbar\omega \qquad |\psi_2^{(0)+}\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$$

$$E_2^{(1)0} = 3\hbar\omega \qquad |\psi_2^{(0)0}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$E_2^{(1)-} = 3\hbar\omega - \delta\hbar\omega \qquad |\psi_2^{(0)-}\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$$

Se rompe la degeneración.

(c) Resolver exactamente

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{m\omega^2}{2}(x^2+y^2) + \delta m\omega^2 xy$$

Como en Mecánica Clásica (Transformación canónica) rotación en $\pi/4$

$$\begin{cases} x = \dfrac{Q_1+Q_2}{\sqrt{2}} & p_x = \dfrac{P_1+P_2}{\sqrt{2}} \\ y = \dfrac{Q_1-Q_2}{\sqrt{2}} & p_y = \dfrac{P_1-P_2}{\sqrt{2}} \end{cases}$$

$$\begin{cases} Q_1 = \dfrac{x+y}{\sqrt{2}} & P_1 = \dfrac{p_x+p_y}{\sqrt{2}} \\ Q_2 = \dfrac{x-y}{\sqrt{2}} & P_2 = \dfrac{p_x-p_y}{\sqrt{2}} \end{cases}$$

$$[Q_i, Q_j] = [P_i, P_j] = 0$$
$$[Q_i, P_j] = i\hbar\delta_{ij}.$$

variables canónicamente conjugadas.

$$H = \frac{P_1^2}{2m} + \frac{P_2^2}{2m} + \frac{m\omega^2}{2}(Q_1^2+Q_2^2) + \frac{\delta m\omega^2}{2}(Q_1^2 - Q_2^2)$$

$$= \frac{P_1^2}{2m} + \frac{P_2^2}{2m} + \frac{m}{2}\omega^2(1+\delta)Q_1^2 + \frac{m\omega^2}{2}(1-\delta)Q_2^2$$

Son dos osciladores con frecuencias distintas

$$\omega_1^2 = \omega^2(1+\delta) \qquad \omega_2^2 = \omega^2(1-\delta)$$

$$|\Psi_0\rangle = |0_1 0_2\rangle \qquad E_0 = \frac{\hbar}{2}(\omega_1+\omega_2) = \frac{\hbar\omega}{2}\left[\sqrt{1+\delta}+\sqrt{1-\delta}\right]$$

$$E_0 \simeq \hbar\omega - \hbar\omega\frac{(\delta)^2}{8} \checkmark$$

$\Psi_0(x,y)$.

Ayuda: $\sqrt{1+x} \sim 1 + \frac{x}{2} - \frac{x^2}{8}$