Diego Wallace

Ensayo final 8

$|W\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$

a) 3 cavidades $C_1\, C_2\, C_3$ en resonancia con átomo A de 2 niveles

inicialmente $|e\rangle, |0\rangle_1, |0\rangle_2, |0\rangle_3$ ($\omega_{C_1} = \omega_{C_2} = \omega_{C_3}$ porque todos están en resonancia)

cavidad 1 $\Rightarrow |\psi(t)\rangle = e^{-\frac{iH_{JC}t}{\hbar}}|e,0\rangle = e^{-\frac{iH_{JC}t}{\hbar}}(\cos\theta_1 |1+\rangle + \sin\theta_1 |1-\rangle) = e^{-\frac{iH_{JC}t}{\hbar}} \frac{|1+\rangle + |1-\rangle}{\sqrt{2}}$

$$|\psi(t)\rangle = \frac{e^{-\frac{iE_1^+ t}{\hbar}}|1+\rangle + e^{-\frac{iE_1^- t}{\hbar}}|1-\rangle}{\sqrt{2}} \quad \text{con } E_1^{\pm} = \hbar\omega_{C_1} \pm \frac{\Omega}{2}$$

$$|\psi(t)\rangle = \frac{e^{-i\omega_C t}}{\sqrt{2}}\left(e^{-\frac{i\Omega t}{2}}|1+\rangle + e^{\frac{i\Omega t}{2}}|1-\rangle\right)$$

evaluo en $T_1 = \operatorname{arctg}\left(\frac{1}{\sqrt{2}}\right)\frac{2}{\Omega} \Rightarrow \; = \frac{e^{-i\omega_{C_1}T_1}}{\sqrt{2}}\left(e^{-i\operatorname{arctg}(\frac{1}{\sqrt{2}})}|1+\rangle + e^{i\operatorname{arctg}(\frac{1}{\sqrt{2}})}|1-\rangle\right)$

$$\left.\begin{array}{l}\cos(\operatorname{arctg}(\frac{1}{\sqrt{2}})) = \frac{1}{\sqrt{1+\frac{1}{2}}} = \sqrt{\frac{2}{3}} \\ \sin(\operatorname{arctg}(\frac{1}{\sqrt{2}})) = \frac{1/\sqrt{2}}{\sqrt{1+\frac{1}{2}}} = \frac{1}{\sqrt{3}}\end{array}\right\} = \frac{e^{-i\omega_{C_1}T_1}}{\sqrt{2}}\left(\sqrt{\frac{2}{3}}(|1+\rangle + |1-\rangle) + \frac{1}{\sqrt{3}}(-i|1+\rangle + i|1-\rangle)\right)$$

$$= \frac{e^{-i\omega_{C_1}T_1}}{\sqrt{2}}\left(\frac{2}{\sqrt{3}}|e,0\rangle + \sqrt{\frac{2}{3}}|g,1\rangle\right)$$

cavidad 2 $\Rightarrow |\psi(T_1)\rangle = \frac{e^{-i\omega_{C_1}T_1}}{\sqrt{2}}\left(\frac{2}{\sqrt{3}}|e,0\rangle_1 + \sqrt{\frac{2}{3}}|g,1\rangle_1\right)|0\rangle_2$ } uso T_1 tiempo 0 en cavidad 2

$$|\psi(t)\rangle = \frac{e^{-i\omega_{C_1}T_1}}{\sqrt{2}}\left(\frac{2}{\sqrt{3}}e^{-\frac{iH_{JC}t}{\hbar}}|e0\rangle_2|0\rangle_1 + \sqrt{\frac{2}{3}}e^{-\frac{iH_{JC}t}{\hbar}}|g,0\rangle_2|1\rangle_1\right)$$

por facilidad los ordeno en realidad son $|e,0,0\rangle$ y $|g,1,0\rangle$

$e^{-\frac{iH_{JC}t}{\hbar}}|g,0\rangle_2 = |g,0\rangle_2$ porque están en resonancia

$e^{-\frac{iH_{JC}t}{\hbar}}|e0\rangle_2 = \frac{e^{-i\omega_{C_2}t}}{\sqrt{2}}\left(e^{-\frac{i\Omega t}{2}}|1+\rangle + e^{\frac{i\Omega t}{2}}|1-\rangle\right)$ y en $T_2 = \frac{\pi}{4}\frac{2}{\Omega}$ me queda

$$\frac{e^{-i\omega_C T_2}}{\sqrt{2}}\left(e^{-i\frac{\pi}{4}}|1+\rangle + e^{i\frac{\pi}{4}}|1-\rangle\right) = \frac{e^{-i\omega_C T_2}}{\sqrt{2}}(|e,0\rangle + |g,1\rangle)$$

$$|\psi(T_2)\rangle = \frac{e^{-i\omega_{C_1}T_1}}{\sqrt{2}}\left(\sqrt{\frac{2}{3}}e^{-i\omega_{C_2}T_2}(|e,0,0\rangle + |g,0,1\rangle) + \sqrt{\frac{2}{3}}|g,1,0\rangle\right)$$

cavidad 3 $\Rightarrow$ análogamente

$$|\psi(t)\rangle = \frac{e^{-i\omega_{C_1}T_1}}{\sqrt{2}}\left(\sqrt{\frac{2}{3}}e^{-i\omega_{C_2}T_2}\left(e^{-\frac{iH_{JC}t}{\hbar}}|e,0,0,0\rangle + e^{-\frac{iH_{JC}t}{\hbar}}|g,0,1,0\rangle\right) + \sqrt{\frac{2}{3}}e^{-\frac{H_{JC}t_4}{\hbar}}|g,1,0,0\rangle\right)$$

$e^{-\frac{iH_{JC}t}{\hbar}}|g,0\rangle_3 = |g,0\rangle_3$

$e^{-\frac{iH_{JC}t}{\hbar}}|e,0\rangle_3 = \frac{e^{-i\omega_{C_3}t}}{\sqrt{2}}\left(e^{-\frac{i\Omega t}{2}}|1+\rangle_3 + e^{\frac{i\Omega t}{2}}|1-\rangle_3\right)$ } $T_3 = \frac{\pi}{\Omega}$

$$\frac{e^{-i\omega_{C_3}T_3}}{\sqrt{2}}\left(e^{-\frac{i\pi}{2}}|1+\rangle_3 + e^{\frac{i\pi}{2}}|1-\rangle_3\right) = \frac{e^{-i\omega_{C_3}T_3}}{\sqrt{2}}\sqrt{2}\,|g,1\rangle$$

$$\Rightarrow |\psi(T_3)\rangle = \frac{e^{-i\omega_{C_1}T_1}}{\sqrt{2}}\left(\sqrt{\frac{1}{3}}e^{-i\omega_{C_2}T_2}\left(e^{-i\omega_3 T_3}|g,0,0,1\rangle + |g,0,1,0\rangle\right) + \sqrt{\frac{2}{3}}|g,1,0,0\rangle\right)$$

Ahora los estados fundamentales de cada cavidad $|g,0\rangle$ los puedo multiplicar por cualquier fase y quedan invariantes (siguen en el mismo subespacio de dim 1)

$$\Rightarrow |\psi(T_3)\rangle = \frac{1}{\sqrt{3}}(|g,0,0,1\rangle + |g,0,1,0\rangle + |g,1,0,0\rangle) = |W\rangle \otimes |g\rangle$$

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(b) El tiempo de interacción lo puedo controlar variando la velocidad con lo que entra el átomo a la cavidad

1^er átomo $|e\rangle_1$ y $|0\rangle_c$

$$|\Psi(T_1)\rangle = e^{-\frac{iH_{JC}T_1}{\hbar}}|e,0\rangle = \frac{e^{-\frac{iE_+T_1}{\hbar}}|1+\rangle + e^{-\frac{iE_-T_1}{\hbar}}|1-\rangle}{\sqrt{2}} = \frac{e^{-i\omega_c T_1}}{\sqrt{2}}\left(e^{-\frac{i\Omega T_1}{2}}|1+\rangle + e^{\frac{i\Omega T_1}{2}}|1-\rangle\right.$$

$$\left.\begin{array}{l}\cos\left(\frac{\Omega}{2}T_1\right) = \cos(\arctan(\sqrt{2})) = \frac{1}{\sqrt{3}} \\ \sin\left(\frac{\Omega}{2}T_1\right) = \sin(\arctan(\sqrt{2})) = \sqrt{\frac{2}{3}}\end{array}\right\} |\Psi(T_1)\rangle = \left(\frac{1}{\sqrt{3}}|e,0\rangle + \sqrt{\frac{2}{3}}|g,1\rangle\right)$$

2^do átomo $|\Psi(T_2)\rangle = e^{-\frac{iH_{JC}T_2}{\hbar}}\left(\frac{|e,g,0\rangle}{\sqrt{3}} + \sqrt{\frac{2}{3}}|g,g,1\rangle\right)$

$$e^{-\frac{iH_{JC}T_2}{\hbar}}|g,0\rangle_2 = |g,0\rangle_2 = e^{-i\omega_c T_2}|g,0\rangle_2$$

$$e^{-\frac{iH_{JC}T_2}{\hbar}}|g,1\rangle_2 = \frac{e^{-i\omega_c T_2}}{\sqrt{2}}\left(-i\, e^{-\frac{i\Omega T_2}{2}}|1+\rangle_2 + i\, e^{\frac{i\Omega T_2}{2}}|1-\rangle\right) \quad y \quad T_2 = \frac{5}{2}\frac{\pi}{\Omega}$$

$$= \frac{e^{-i\omega_c T_2}}{\sqrt{2}}\left(-i\left(-\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)|1+\rangle_2 + i\left(-\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)|1-\rangle\right) = \frac{e^{-i\omega_c T_2}}{\sqrt{2}}\left(\underbrace{\frac{i|1+\rangle - i|1-\rangle}{\sqrt{2}}}_{-|g,1\rangle_2} + \underbrace{\frac{|1+\rangle + |1-\rangle}{\sqrt{2}}}_{|e,0\rangle_2}\right.$$

$$|\Psi(T_2)\rangle = \frac{1}{\sqrt{3}}|e,g,0\rangle + \frac{1}{\sqrt{3}}|g,e,0\rangle - \frac{1}{\sqrt{3}}|g,g,1\rangle$$

3^er átomo: $|\Psi(T_3)\rangle = \frac{e^{-\frac{iH_{JC}T_3}{\hbar}}}{\sqrt{3}}\left(|e,g,g,0\rangle + |g,e,g,0\rangle - |g,g,g,1\rangle\right)$

$$e^{-\frac{iH_{JC}T_3}{\hbar}}|g,1\rangle_3 = \frac{e^{-i\omega_c T_3}}{\sqrt{2}}\left(-i\, e^{-\frac{i\Omega T_3}{2}}|1+\rangle + i\, e^{\frac{i\Omega T_3}{2}}|1-\rangle\right) \quad y \quad T_3 = \frac{\pi}{\Omega}$$

$$= \frac{e^{-i\omega_c T_3}}{\sqrt{2}}\left(-|1+\rangle - |1-\rangle\right) = -e^{-i\omega_c T_3}|e\,0\rangle_3$$

$$e^{-\frac{iH_{JC}T_3}{\hbar}}|g,0\rangle_3 = |g,0\rangle_3 = e^{-i\omega_c T_3}|g,0\rangle$$

$$\Rightarrow |\Psi(T_3)\rangle = \frac{1}{\sqrt{3}}\left(|e,g,g,0\rangle + |g,e,g,0\rangle + |g,g,e,0\rangle\right) = |W\rangle \otimes |0\rangle$$

(c) $|W\rangle = \frac{1}{\sqrt{3}}\left(|0\rangle_1 \otimes |0\rangle_2 \otimes |1\rangle_3 + |0\rangle_1 \otimes |1\rangle_2 \otimes |0\rangle_3 + |1\rangle_1 \otimes |0\rangle_2 \otimes |0\rangle_3\right)$

$$= \frac{1}{\sqrt{3}}\left(|0\rangle_2 \otimes |0\rangle_1 \otimes |1\rangle_3 + |0\rangle_2 \otimes |1\rangle_1 \otimes |0\rangle_3 + |1\rangle_2 \otimes |0\rangle_1 \otimes |0\rangle_3\right)$$

y etc. ya que son todos vectores normales de dimensión 2 que forman base en sus respectivos espacios vectoriales y son todos iguales en forma

$$\rho_{123} = |W\rangle\langle W| = \frac{1}{3}\left(|100\rangle + |010\rangle + |001\rangle\right)\left(\langle 100| + \langle 010| + \langle 001|\right)$$

$$= \frac{1}{3}\left(|100\rangle\langle 100| + |100\rangle\langle 010| + |100\rangle\langle 001| + |010\rangle\langle 100| + |010\rangle\langle 010| + |010\rangle\langle 001|\right.$$

$$\left. + |001\rangle\langle 100| + |001\rangle\langle 010| + |001\rangle\langle 001|\right)$$

$$\rho_{12} = \mathrm{tr}_3(\rho_{123}) = \frac{1}{3}\left(|10\rangle\langle 10| + |10\rangle\langle 01| + |01\rangle\langle 10| + |01\rangle\langle 01| + |00\rangle\langle 00|\right) \neq |\Psi\rangle\langle\Psi|$$

con $|\Psi\rangle$ genérico

$$\rho_1 = \mathrm{tr}_{23}(\rho_{123}) = \frac{1}{3}\left(|1\rangle\langle 1| + |0\rangle\langle 0| + |0\rangle\langle 0|\right) \neq |\Psi\rangle\langle\Psi|$$

$\Rightarrow$ el estado está entrelazado

NOTA