13) Matrices de Pauli

V: ev dim 2 base $\{|+\rangle, |-\rangle\}$

$|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\sigma_x = \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_y = \sigma_2 = \underline{\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}$ $\sigma_z = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

a) $\sigma_x \stackrel{?}{=} \sigma_x^\dagger = (\sigma_x^*)^t$ $\left(\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\right)^{*t} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}^t = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^*$ $\sigma_x = \sigma_x^\dagger$

Las matrices de Pauli son hermíticas.

Autovalores y Autovectores:

$\sigma_z = \begin{pmatrix} \underline{1} & 0 \\ 0 & \underline{-1} \end{pmatrix}$ $\lambda_\pm = \pm 1$ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |+\rangle$

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -|-\rangle$

$\left.\begin{aligned} \sigma_z|+\rangle &= |+\rangle \\ \sigma_z|-\rangle &= -|-\rangle \end{aligned}\right\}$

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\det\begin{pmatrix} -\lambda & 1 \\ 1 & -\lambda \end{pmatrix} = 0$

$\lambda^2 - 1 = 0$ $\boxed{\lambda_\pm = \pm 1}$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad y = x \qquad \boxed{\bar{v} = \begin{pmatrix} x \\ x \end{pmatrix}}$$

$$|\bar{v}|^2 = 1 \Rightarrow x^2 + x^2 = 1 \rightarrow x = \frac{1}{\sqrt{2}} \qquad \bar{v}_+ = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -\begin{pmatrix} x \\ y \end{pmatrix} \qquad x = -y \qquad \bar{v}_- = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

$$\boxed{|+_x\rangle = \frac{1}{\sqrt{2}}(\underbrace{|+\rangle}_{|+_z\rangle} + |-\rangle) \qquad |-_x\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)}$$

$$\boxed{|+_y\rangle = \frac{1}{\sqrt{2}}(|+\rangle + \underline{i}|-\rangle) \qquad |-_y\rangle = \frac{1}{\sqrt{2}}(|+\rangle - \underline{i}|-\rangle)}$$

b) $\det(\sigma_k) = -1$ $\quad \{ Tr[\sigma_k] = 0 \}$

la suma de los elementos de la diagonal

$$Tr[\sigma_z] = Tr\left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right] = 1 + (-1) = 0 \checkmark$$

$$\sigma_i^2 = \underline{\mathbb{1}} \qquad [\sigma_i, \sigma_j] = 2i\underbrace{\varepsilon_{ijk}}_{\text{símbolo de Levi-Civita}}\sigma_k \qquad (x, y, z)$$

$$[\sigma_x, \sigma_y] = 2i\,\varepsilon_{xyz}\,\sigma_z$$

$\varepsilon_{xyz} = 1$	$\varepsilon_{xzy} = -1$
$\varepsilon_{zxy} = 1$	$\varepsilon_{zyx} = -1$
$\varepsilon_{yzx} = 1$	$\varepsilon_{yxz} = -1$

$$\varepsilon_{iik} = \varepsilon_{kii} = \varepsilon_{iki} = 0$$

$$[\sigma_x, \sigma_y] = 2i \underbrace{\varepsilon_{xyz}}_{1} \sigma_z = \underline{\underline{2i\sigma_z}}$$

$$[\sigma_x, \sigma_y] = \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}_{\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}} - \underbrace{\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}} = \begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[\sigma_y, \sigma_x] = -[\sigma_x, \sigma_y] = -2i\sigma_z \qquad [\sigma_i, \sigma_i] = 0$$

$$\varepsilon_{iik} = 0$$

$$[\underline{\sigma_y}, \underline{\sigma_x}] = 2i \underbrace{\varepsilon_{yxz}}_{-1} \sigma_z = -2i\sigma_z$$

$$\{\sigma_i, \sigma_j\} = 2 \underline{\delta_{ij}} \mathbb{1} \qquad \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$\{\sigma_i, \sigma_i\} = \sigma_i\sigma_i + \sigma_i\sigma_i = 2\sigma_i^2 \qquad \sigma_i^2 = \underline{\mathbb{1}}$$

$$\{\sigma_i, \sigma_i\} = 2\underline{\mathbb{1}}$$

$$\{\sigma_x, \sigma_y\} = \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}_{\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}} + \underbrace{\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}} = 0$$

$$\sigma_j\sigma_k = \underline{i\,\varepsilon_{jkl}\sigma_l} + \underline{\delta_{jk}\mathbb{1}}$$

$$\sigma_j\sigma_k = \frac{1}{2}\sigma_j\sigma_k + \frac{1}{2}\sigma_j\sigma_k + \underline{\frac{1}{2}\sigma_k\sigma_j} - \frac{1}{2}\sigma_k\sigma_j$$

$$\frac{1}{2}\{\sigma_j,\sigma_k\}$$

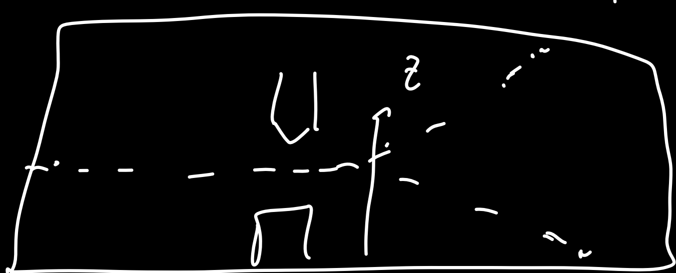
$$\frac{1}{2}[\sigma_j,\sigma_k]$$

$$\sigma_j\sigma_k = \underline{\frac{1}{2}[\sigma_j,\sigma_k]} + \underline{\frac{1}{2}\{\sigma_j,\sigma_k\}}$$

$$\sigma_j\sigma_k = i\,\varepsilon_{jkl}\,\sigma_l + \delta_{ij}\,\mathbb{1}$$

$$\underline{[\sigma_i,\sigma_j]}\neq 0 \qquad \Delta\sigma_i\,\Delta\sigma_j \geq \sqrt{\langle\sigma_k\rangle}$$

14) Spin 1/2 $\uparrow\downarrow$ cap 1 Sakurai



$$\underline{S = \frac{\hbar}{2}\sigma \quad - \frac{\hbar}{2}}$$

$$\underline{\vec{S} = (S_x, S_y, S_z)} \qquad \underline{S_i = \frac{\hbar}{2}\sigma_i} \qquad S_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

a) $[S_i, S_j] = i\hbar\,\varepsilon_{ijk}\,S_k$

$$[\underset{\uparrow \frac{\hbar}{2}}{\sigma_i}, \underset{\uparrow \hbar/2}{\sigma_j}] = 2i\,\varepsilon_{ijk}\,\sigma_k$$

$$\left[\frac{\hbar}{2}\sigma_i, \frac{\hbar}{2}\sigma_j\right] = \frac{\hbar^2}{\cancel{4}_2}\cancel{2}\,i\,\varepsilon_{ijk}\,\sigma_k$$

$$[aA, bB] = ab\,[A, B]$$

$$[S_i, S_j] = i\hbar\, \varepsilon_{ijk} \underbrace{\frac{\hbar}{2}\sigma_k}_{S_k} \Rightarrow [S_i, S_j] = i\hbar\, \varepsilon_{ijk} S_k$$

b) $\underline{\{|+\rangle, |-\rangle\}}$ autoestados de S_z

$$\begin{cases} S_x = \frac{\hbar}{2}\left[\,|+\rangle\langle -| + |-\rangle\langle +|\,\right] \\ S_y = \frac{\hbar}{2}\left[-i\,|+\rangle\langle -| + i\,|-\rangle\langle +|\,\right] \\ S_z = \frac{\hbar}{2}\left[\,|+\rangle\langle +| - |-\rangle\langle -|\,\right] \end{cases}$$

$$|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$S_x = \frac{\hbar}{2}\sigma_x \Rightarrow \underline{S_x} = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{cases} S_x|+\rangle = \frac{\hbar}{2}|-\rangle \\ S_x|-\rangle = \frac{\hbar}{2}|+\rangle \end{cases}$$

$\langle +|+\rangle \quad \langle +|+\rangle \nearrow 1 \qquad \langle +|-\rangle \nearrow 0$

$$\underline{S_x} = a\,\underline{|+\rangle\langle +|} + b\,|+\rangle\langle -| + c\,|-\rangle\langle +| + d\,|-\rangle\langle -|$$

$\underline{\langle +|S_x|+\rangle} = \underline{a} \qquad \underline{\langle -|S_x|+\rangle} = \underline{c}$

$\underline{\langle +|S_x|-\rangle} = \underline{b} \qquad \underline{\langle -|S_x|-\rangle} = d$

$\underline{S_x|+\rangle} = \frac{\hbar}{2}|-\rangle$ $\quad \langle+|S_x|+\rangle = \underbrace{\langle+|-\rangle}_{0}\frac{\hbar}{2} \Rightarrow a = 0$

$\underline{S_x|-\rangle} = \frac{\hbar}{2}|+\rangle$ $\quad \langle+|S_x|-\rangle = \langle+|+\rangle\frac{\hbar}{2} \Rightarrow b = \frac{\hbar}{2}$

$\langle-|S_x|+\rangle = \frac{\hbar}{2}\langle-|-\rangle \Rightarrow c = \frac{\hbar}{2}$

$\langle-|S_x|-\rangle = \frac{\hbar}{2}\langle-|+\rangle \Rightarrow d = 0$

$$\boxed{S_x = \frac{\hbar}{2}|+\rangle\langle-| + \frac{\hbar}{2}|-\rangle\langle+|}$$

$|i\rangle\langle i| \rightarrow$ diagonal de la matriz

$|i\rangle\langle j| \rightarrow$ fuera de la diagonal

$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\quad \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} = i\frac{\hbar}{2}|-\rangle$

$\frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} = -i\frac{\hbar}{2}|+\rangle$

$\underline{S_y}\,\underline{|+\rangle} = i\frac{\hbar}{2}|-\rangle$

$S_y|-\rangle = -i\frac{\hbar}{2}|+\rangle$

$$\boxed{S_y = \underline{i\frac{\hbar}{2}}|-\rangle\langle+| \underline{- i\frac{\hbar}{2}}|+\rangle\langle-|}$$

c) Autoestados de S_x $\{|+_x\rangle, |-_x\rangle\}$

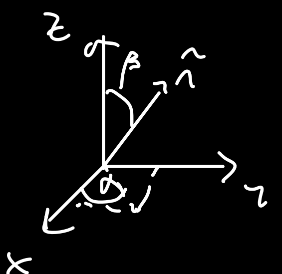
$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \begin{cases} S_x|+\rangle = \frac{\hbar}{2}|-\rangle \\ S_x|-\rangle = \frac{\hbar}{2}|+\rangle \end{cases}$$

$$\underline{S_x}\,\underline{(|+\rangle + |-\rangle)} = \frac{\hbar}{2}(|-\rangle + |+\rangle) = \frac{\hbar}{2}\,\underline{(|+\rangle + |-\rangle)}$$

$$S_x\,\underline{(|+\rangle - |-\rangle)} = \frac{\hbar}{2}(|-\rangle - |+\rangle) = -\frac{\hbar}{2}\,\underline{(|+\rangle - |-\rangle)}$$

$$\boxed{\begin{aligned} |+_x\rangle &= \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \\ |-_x\rangle &= \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) \end{aligned}}$$

$$\left[\bar{S}\cdot\hat{n}\,|\bar{S}\cdot\hat{n};+\rangle = \frac{\hbar}{2}|\bar{S}\cdot\hat{n};+\rangle\right]$$



$|\bar{S}\cdot\hat{n};+\rangle$ = Autostato di $\bar{S}\cdot\hat{n}$ con autovalore $+\hbar/2$

$$\boxed{\underline{|\bar{S}\cdot\hat{n};+\rangle} = \cos\beta/2\,\underline{|+\rangle} + \sin\beta/2\, e^{i\alpha}\,\underline{|-\rangle}}$$

$$\bar{S}\cdot\hat{n} = (S_x, S_y, S_z)\cdot\underbrace{(\sin\beta\cos\alpha, \sin\beta\sin\alpha, \cos\beta)}_{\hat{n}}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\left[\bar{S}\cdot\hat{n} = \sin\beta\cos\alpha\, \underline{S_x} + \sin\beta\sin\alpha\, \underline{S_y} + \underline{\cos\beta}\, S_z\right]$$

$$\bar{S}\cdot\hat{n} = \frac{\hbar}{2}\begin{pmatrix} \cos\beta & \sin\beta\cos\alpha - i\sin\beta\sin\alpha \\ \underline{\sin\beta\cos\alpha + i\sin\beta\sin\alpha} & -\cos\beta \end{pmatrix}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\boxed{\bar{S}\cdot\hat{n} = \frac{\hbar}{2}\begin{pmatrix} \cos\beta & \sin\beta\, e^{-i\alpha} \\ \sin\beta\, e^{i\alpha} & -\cos\beta \end{pmatrix}}$$

$$\sin\beta(\cos\alpha + i\sin\alpha) = \sin\beta\, e^{i\alpha}$$

$$\det\begin{pmatrix} \cos\beta - \lambda & \sin\beta\, e^{-i\alpha} \\ \sin\beta\, e^{i\alpha} & -\cos\beta - \lambda \end{pmatrix} = 0$$

$$-\underline{(\cos\beta + \lambda)(\cos\beta - \lambda)} - \sin^2\beta = 0$$

$$-\cos^2\beta + \lambda^2 - \sin^2\beta = 0 \qquad \lambda^2 - 1 = 0$$

$$\lambda_\pm = \pm 1$$

$$\left[\begin{pmatrix} \cos\beta & \sin\beta\, e^{-i\alpha} \\ \sin\beta\, e^{i\alpha} & -\cos\beta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}\cdot(+1)\right]$$

$$\rightarrow \cos\beta\, x + \sin\beta\, e^{-i\alpha} y = x \qquad \beta = 0 \Rightarrow \vec{S}\cdot\hat{n} = S_z$$

$$y \sin\beta\, e^{-i\alpha} = x(1-\cos\beta)$$

$$\boxed{y = x\,\frac{(1-\cos\beta)\,e^{i\alpha}}{\sin\beta}}$$

$$\bar{v} = \underline{\begin{pmatrix} \underline{x} \\ x\,\frac{(1-\cos\beta)\,e^{i\alpha}}{\sin\beta} \end{pmatrix}} \qquad |\bar{v}|^2 = 1$$

$$x^2 + x^2\,\frac{(1-\cos\beta)^2}{\sin^2\beta} = 1$$

$$\frac{(1-\cos\beta)^2}{\sin^2\beta} = \frac{(1-\cos\beta)\,(1-\cos\beta)}{\sin^2\beta}\,\frac{(1+\cos\beta)}{(1+\cos\beta)}$$

$$= \cancel{\frac{\overset{\sin^2\beta}{(1-\cos^2\beta)}}{\sin^2\beta}}\,\frac{(1-\cos\beta)}{(1+\cos\beta)} = \frac{(1-\cos\beta)}{(1+\cos\beta)}$$

$$\boxed{|\vec{S}\cdot\hat{n};+\rangle = \underline{\cos\beta_{/2}}\,|+\rangle + \underline{e^{i\alpha}\sin\beta_{/2}}\,|-\rangle}$$

$$\frac{(1-\cos\beta)}{(1+\cos\beta)} \qquad \cos\beta = \cos\left(\frac{\beta}{2}+\frac{\beta}{2}\right) = \cos^2\beta/2 - \sin^2\beta/2$$

$$\cos\beta = 1 - 2\sin^2\beta/2 \rightarrow 2\sin^2\beta/2 = 1-\cos\beta$$

$$\cos\beta = 2\cos^2\beta/2 - 1 \rightarrow 2\cos^2\beta/2 = 1+\cos\beta$$

$$= \frac{\cancel{2}\sin^2(\beta/2)}{\cancel{2}\cos^2(\beta/2)} \qquad x^2 + x^2\frac{(1-\cos\beta)^2}{\sin^2\beta} = 1$$

$$x^2 + x^2\frac{\sin^2(\beta/2)}{\cos^2(\beta/2)} = 1$$

$$x^2\left(1 + \frac{\sin^2(\beta/2)}{\cos^2(\beta/2)}\right) = 1$$

$$x^2\left(\frac{\cos^2(\beta/2)+\sin^2(\beta/2)}{\cos^2(\beta/2)}\right) = 1$$

$$x^2 = \cos^2(\beta/2) \rightarrow x = \cos(\beta/2)$$

$$\bar{v} = \begin{pmatrix} x \\ x\frac{(1-\cos\beta)}{\sin\beta}e^{i\alpha} \end{pmatrix}$$

$$\bar{v} = \begin{pmatrix} \cos\beta/2 \\ \cos\beta/2\,\dfrac{(1-\cos\beta)}{\sin\beta}\, e^{i\alpha} \end{pmatrix}$$

$$(1-\cos\beta) = 2\sin^2(\beta/2)$$

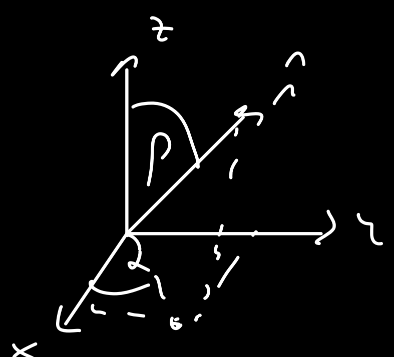
$$\sin\beta = \sin\left(\tfrac{\beta}{2}+\tfrac{\beta}{2}\right) = 2\sin\beta/2\,\cos\beta/2$$

$$\cos\beta/2\,\frac{(1-\cos\beta)}{\sin\beta}\, e^{i\alpha} = \frac{\cancel{\cos\beta/2}\cdot 2\sin^2(\beta/2) e^{i\alpha}}{\cancel{2\sin\beta/2}\ \cancel{\cos\beta/2}} = \sin(\beta/2) e^{i\alpha}$$

$$\bar{v} = \begin{pmatrix} \cos\beta/2 \\ \sin\beta/2\, e^{i\alpha} \end{pmatrix} \qquad |+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\boxed{|\bar{S}\cdot\hat{n};+\rangle = \cos\beta/2\,|+\rangle + \sin\beta/2\, e^{i\alpha}|-\rangle}$$

$\hat{n} = \hat{x}$



$\hat{x} \Rightarrow \beta = \frac{\pi}{2} \quad \alpha = 0$

$$\boxed{|\bar{S}\cdot\hat{x}\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle}$$

16) dim 3 $\quad \{|1\rangle, |2\rangle, |3\rangle\}$

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix} \qquad B = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix}$$

a) A degenerato

$$A|1\rangle = a|1\rangle \qquad A|2\rangle = -a|2\rangle \qquad A|3\rangle = -a|3\rangle$$

$$\boxed{B = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix}}$$

$b\sigma_y$

$$|2'\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle + i|3\rangle\right) \quad (b)$$

$$|3'\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle - i|3\rangle\right) \quad (-b)$$

$$B' = \begin{pmatrix} b & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & -b \end{pmatrix} \Rightarrow \text{degenerato}$$

$$B = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix} \qquad \{|2\rangle, |3\rangle\}$$

$$B|1\rangle = b|1\rangle$$

b) $[A, B] \stackrel{?}{=} 0$

$$[A, B] = \underbrace{\begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix}\begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix}} - \underbrace{\begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix}\begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix}}$$

$$\begin{pmatrix} [illegible] & & \\ 0 & 0 & +iba \\ 0 & -iba & 0 \end{pmatrix} - \begin{pmatrix} ab & 0 & 0 \\ 0 & 0 & iba \\ 0 & -iba & 0 \end{pmatrix}$$

$$[A, B] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix} \qquad B = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix}$$

$$\{|2\rangle, |3\rangle\} \qquad \{|2\rangle, |3\rangle\}$$

$$A|2\rangle = -a|2\rangle$$

$$A|3\rangle = -a|3\rangle$$

$$A(\alpha|2\rangle + \beta|3\rangle) = -a(\alpha|2\rangle + \beta|3\rangle)$$

$$|2'\rangle = \frac{1}{\sqrt{2}}(|2\rangle + i|3\rangle) \qquad |3'\rangle = \frac{1}{\sqrt{2}}(|2\rangle - i|3\rangle)$$

Base $\{|1\rangle, |2'\rangle, |3'\rangle\}$

$$A = \begin{pmatrix} a & & \\ & -a & \\ & & -a \end{pmatrix} \qquad B = \begin{pmatrix} b & & \\ & b & \\ & & -b \end{pmatrix}$$

$A|1\rangle = a|1\rangle$ $\quad B|1\rangle = b|1\rangle$

$A|2'\rangle = -a|2'\rangle$ $\quad B|2'\rangle = b|2'\rangle$

$A|3'\rangle = -a|3'\rangle$ $\quad B|3'\rangle = -b|3'\rangle$

$$\boxed{\begin{aligned} |1\rangle &\to a, b \\ |2'\rangle &\to -a, b \\ |3'\rangle &\to -a, -b \end{aligned}}$$

CCOC

$\{|a,b\rangle, |-a,b\rangle, |-a,-b\rangle\}$