$\boxed{P9}$ (c) Sea $|\psi\rangle$ autoestado de $\vec{\sigma}\cdot\hat{n}$ c/ autovalor 1:

$$\Pi_{|\psi\rangle} = \frac{1}{2}\left(\mathbb{1} + \vec{\sigma}\cdot\hat{n}\right).$$

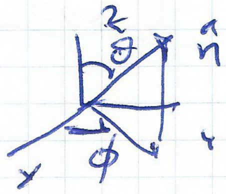
$$\langle \sigma_x \rangle = tr\left(\Pi_{|\psi\rangle}\cdot\sigma_x\right)$$

$$= tr\left[\frac{1}{2}\left(\mathbb{1} + \vec{\sigma}\cdot\hat{n}\right)\sigma_x\right]$$

$$= \frac{1}{2} tr\left[\cancel{\sigma_x} + n_x \overbrace{\sigma_x^2}^{\mathbb{1}} + \cancel{n_y \sigma_y \sigma_x} + \cancel{n_z \sigma_z \sigma_x}\right]$$

Se tachó lo que no tiene traza.

$\therefore$ $\boxed{\langle \sigma_x \rangle = n_x} = \sin\theta\cos\phi$



del mismo modo:

$\boxed{\langle \sigma_y \rangle = n_y.} = \sin\theta\sin\phi$

$$|\psi\rangle = \cos\frac{\theta}{2}|+\rangle + e^{i\phi}\sin\frac{\theta}{2}|-\rangle$$

- $(\sigma_x - \sin\theta\cos\phi)|\psi\rangle = \cos\frac{\theta}{2}|-\rangle + e^{i\phi}\sin\frac{\theta}{2}|+\rangle - \sin\theta\cos\phi\cos\frac{\theta}{2}|+\rangle - \sin\theta\cos\phi\sin\frac{\theta}{2}e^{i\phi}|-\rangle$

- $(\sigma_y - \sin\theta\sin\phi)|\psi\rangle = i\cos\frac{\theta}{2}|-\rangle - ie^{i\phi}\sin\frac{\theta}{2}|+\rangle - \sin\theta\sin\phi\cos\frac{\theta}{2}|+\rangle - \sin\theta\sin\phi\sin\frac{\theta}{2}e^{i\phi}|-\rangle$

igualando coeficientes de $|+\rangle$ y de $|-\rangle$:

en: $(\sigma_x - \langle\sigma_x\rangle)|\psi\rangle = \lambda(\sigma_y - \langle\sigma_y\rangle)$

$$e^{i\phi}\sin\frac{\theta}{2} - \sin\theta\cos\phi\cos\frac{\theta}{2} = \lambda\Big(-ie^{i\phi}\sin\frac{\theta}{2}$$
$$- \sin\theta\sin\phi\cos\frac{\theta}{2}\Big)$$

$$\cos\frac{\theta}{2} - \sin\theta\cos\phi\sin\frac{\theta}{2}e^{i\phi} = \lambda\Big(i\cos\frac{\theta}{2}$$
$$- \sin\theta\sin\phi\sin\frac{\theta}{2}e^{i\phi}\Big)$$

* Caso particular: $\phi = 0$:

Es posible ver: $\lambda = i(1 - 2\cos^2\frac{\theta}{2})$
$= -i(1 - 2\sin^2\frac{\theta}{2})$ ✓

imaginario puro.

Caso general. Dividiendo las igualdades por $\begin{cases} e^{i\phi}\sin\theta/2 \\ \cos\theta/2 \end{cases}$, y despejando λ:

$$\lambda = \frac{1 - 2\cos\phi\cos^2\theta/2\, e^{-i\phi}}{-i - 2\sin\phi\cos^2\theta/2\, e^{-i\phi}} = \frac{1 - 2\cos\phi\sin^2\theta/2\, e^{i\phi}}{i - 2\sin\phi\sin^2\frac{\theta}{2}e^{i\phi}}$$

Se puede verificar q' la identidad es correcta.

$\lambda \in \mathbb{C}$

(d) $A = \sigma_x$, $B = \sigma_y$ $[A, B] = 2i\sigma_z$

$Var\,\sigma_x = 1 - n_x^2$, $Var\,\sigma_y = 1 - n_y^2$ $\langle\sigma_z\rangle = n_z$

$K(\sigma_x, \sigma_y) = -n_x n_y$

$Var(\sigma_x)Var(\sigma_y) \geq \frac{1}{4}|K[\sigma_x,\sigma_y]|^2 + |K(\sigma_x,\sigma_y)|^2$

Como $\exists\, \lambda \in \mathbb{C}$ (satura la desigualdad)

$\Rightarrow$ $(1-n_x^2)(1-n_y^2) \equiv n_z^2 + n_x^2 n_y^2$

$\Rightarrow$ $1 = n_x^2 + n_y^2 + n_z^2$ (matamos un mosquito con un cañón)