

Guía 9 - 23/06

Bryson No partida

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Sistema sin perturbar:

$$H_0 \text{ Hamiltoniano} \longrightarrow \{|n\rangle\} \text{ tq } H_0|n\rangle = E_n|n\rangle$$

$$U_0(t) \text{ evo. temp} \text{ tq } \frac{dU_0(t)}{dt} = \frac{1}{i\hbar} H_0 U_0(t)$$

Sistema perturbado: (dependiente del tiempo)

$$H = H_0 + \lambda V(t) \longrightarrow U(t) \text{ tq } \frac{dU(t)}{dt} = \frac{1}{i\hbar} H(t) U(t)$$

Pictore de interacción:

$$|\Psi_I(t)\rangle = U_0^+(t)U(t)|\Psi_0\rangle \rightarrow \frac{d|\Psi_I(t)\rangle}{dt} = \frac{\lambda}{i\hbar} V_I(t) |\Psi_I(t)\rangle$$

$$A_I(t) = U_0^+(t) A(t) U_0(t) \rightarrow \frac{dA_I(t)}{dt} = \frac{1}{i\hbar} [A_I(t), H_0]$$

$$\Rightarrow U(t)|\Psi_0\rangle = U_0(t)|\Psi_I(t)\rangle \quad \text{Esto es lo que
busco calcular}$$

$$\text{Defino } T(t) = U_0^+(t)U(t) \Rightarrow |\Psi_I(t)\rangle = T(t)|\Psi_0\rangle$$

$$\frac{dT(t)}{dt} = \frac{\lambda}{i\hbar} V_I(t) T(t) \quad T(0) = 1I$$

$$\Rightarrow T(t) = 1I + \frac{\lambda}{i\hbar} \int_0^t V_I(t_s) T(t_s) dt_s$$

$$= 1I + \frac{\lambda}{i\hbar} \int_0^t dt_s V_I(t_s) + \left(\frac{\lambda}{i\hbar}\right)^2 \int_0^t \int_0^{t_s} dt_2 V_I(t_s) V(t_2) T(t_2)$$

$$T(t) = \sum_{n=0}^{+\infty} \left(\frac{\lambda}{i\hbar}\right)^n \int_0^t dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{n-1}} dt_n V_I(t_1) V_I(t_2) \cdots V_I(t_n)$$

Serie de Dyson

A primer orden: $T(t) = 1I + \frac{\lambda}{i\hbar} \int_0^t V_I(t_s) dt_s$

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Notemos que:

$$T(t) = U_0(t)U(t) \rightarrow U(t) = U_0(t)T(t)$$

$$U_0(t)T(t) = U_0(t)T(t)U_0^+(t)U_0(t)$$

$$= \left[\sum_{m=0}^{+\infty} \left(\frac{\lambda}{i\hbar} \right)^m \int dt_1 \cdots \int dt_m U_0(t) V_I(t_1) \cdots \overset{U_0^+(t)}{\cancel{V_I(t_m)}} U_0^+(t) \right] U_0(t)$$

$$= \left[\sum_{m=0}^{+\infty} \left(\frac{\lambda}{i\hbar} \right)^m \int \cdots \int dt_m U_0(t) U_0^+(t_1) V(t_1) U_0(t_2) U_0^+(t) \cdots \right] U_0(t)$$

$$= \underbrace{\left[\sum_{m=0}^{+\infty} \left(\frac{\lambda}{i\hbar} \right)^m \int_0^t \int_0^{t_m} U_0^+(t_1-t) V(t_1) U_0(t_2-t) \cdots \right]}_{U_0(t)} U_0(t)$$

$$U(t) = U_0(t)T(t) = \tilde{T}(t)U_0(t) \quad \text{regla de commutación}$$

$$\text{Si } |\psi\rangle = |m\rangle \Rightarrow U(t)|m\rangle = e^{-iE_m t \frac{\lambda}{i\hbar}} \overset{\dagger}{\tilde{T}(t)} |m\rangle$$

fase global

El estado físico está dado por $\tilde{T}(t)|m\rangle$

A orden L:

$$\tilde{T}(t)|m\rangle = \left[1 + \frac{\lambda}{i\hbar} \int_0^t dt_+ U_o^+(t_+-t) V(t_+) U_o(t_+-t) \right] |m\rangle$$

[P12] $V(t) = -e(a \times -b(3z^2 - r^2)) \sin(\omega t)$

$|100\rangle$ fundamental del átomo de hidrógeno

Función de onda: $\Psi_{nlm}(r, \Omega) = R_{nl}(r) Y_l^m(\Omega) \equiv \langle r, \Omega | nl m \rangle$

$$H_0(nlm) = \frac{E_1}{mr^2} |nlm\rangle \quad 0 \leq l \leq n-1 \\ -l \leq m \leq l$$

$$|\Psi(t)\rangle = \left[1 + \frac{i}{\hbar} \int_0^t U_o^+(t_+-t) V(t_+) U_o(t_+-t) dt_+ \right] |100\rangle = \sum C_{nlm}(t) |nlm\rangle$$

La probabilidad de transición es $P(|100\rangle \rightarrow |nlm\rangle) = |\langle nl m | \Psi(t) \rangle|^2$

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$$\begin{aligned}
 \langle m l m | V(t) \rangle &\simeq \langle m l m | \mathbb{1} | 100 \rangle + \frac{1}{i\hbar} \int_0^t \langle m l m | V(t_1-t) V(t_1) V(t_1-t) | 100 \rangle dt_1 \\
 &\simeq \delta_{m,l} \delta_{l00} \delta_{m0} + \frac{1}{i\hbar} \int_0^t e^{iE_m \frac{(t-t_1)}{\hbar}} \langle m l m | V(t_1) e^{-iE_l \frac{(t_1-t)}{\hbar}} | 100 \rangle dt_1 \\
 &= \delta_{m,l} \delta_{l00} \delta_{m0} + \frac{1}{i\hbar} \int_0^t e^{-i(E_l - E_m) \frac{(t_1-t)}{\hbar}} \langle m l m | V(t_1) | 100 \rangle dt_1
 \end{aligned}$$

(*)

④ $\langle m l m | V(t_1) | 100 \rangle = -e \sin(\omega t) \langle m l m | (a x - b(3z^2 - r^2)) | 100 \rangle$

$$3z^2 - r^2 = T_0^{(2)} \quad \text{tensor esférico de orden 2}$$

$$X = \frac{1}{2} (T_{-1}^{(+)} - T_1^{(+)}) \quad \text{...} \quad \text{...} \quad \text{...} \quad \text{...} \quad \text{...} \quad 2$$

$$= -e \sin(\omega t) \left[\frac{a}{2} \left(\langle m l m | T_{-1}^{(+)} | 100 \rangle - \langle m l m | T_1^{(+)} | 100 \rangle \right) - b \langle m l m | T_0^{(2)} | 100 \rangle \right]$$

Regla de ↓

segun m = ±

l = 1

m = 1

l = 1

m = 0

l = 2

Por Teorema de Wigner-Eckart

integral radial

$$\langle m l m | V(t_1) | 100 \rangle = -e \sin(\omega t) \left[\frac{a}{2} (\delta_{l1}) (\delta_{m,-1} - \delta_{m,1}) T_m^1 - b \delta_{l2} \delta_{m0} T_m^2 \right]$$

Es claro que si $a \neq 0$ y $b = 0$

Solo se transiciones a $|m, l, \pm 1\rangle$ con $m \geq 2$

Si $b \neq 0$ y $a = 0$, se transiciones a $|m, 2, 0\rangle$ con $m \geq 3$

La probabilidad de transición es ($\langle m|lm\rangle \neq 0$)

$$P(|100\rangle \rightarrow |mlm\rangle) = \left| \frac{1}{\hbar} \int_0^t e^{-i(E_l - E_m)(t_f - t)} \frac{\hbar}{\hbar} \sin(\omega t_f) \left[\frac{a}{2} (\delta_{l2} \delta_{m-1} - \delta_{l1} \delta_{m+1}) T_m^L \right. \right.$$
$$\left. \left. - b \delta_{l2} \delta_{m0} T_m^z \right] dt_f \right|^2$$

$$= \frac{e^2 a^2}{4\hbar^2} |T_m^L|^2 \delta_{l2} \delta_{m+1} \left| \int_0^t \sin(\omega t_f) e^{-i(E_l - E_m)(t_f - t)} \frac{\hbar}{\hbar} dt_f \right|^2$$

$$+ \frac{e^2 b^2}{\hbar} |T_m^z|^2 \delta_{l2} \delta_{m0} \left| \int_0^t \sin(\omega t_f) e^{-i(E_l - E_m)(t_f - t)} \frac{\hbar}{\hbar} dt_f \right|^2$$

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$$\int_0^t \sin(\omega t) e^{-i \frac{(E_L - E_m)(t_f - t)}{\hbar}} dt = \frac{1}{\Omega_m^2 - \omega^2} \left[\omega (\cos \omega t - \cos \Omega_m t) + i (\Omega_m \sin \omega t - \omega \sin \Omega_m t) \right]$$

⇒

$$P(|100\rangle \rightarrow |m\bar{m}\rangle) = \left\{ \frac{e^2 a^2}{4\hbar^2} |T_m^1| \delta_{\bar{m},\bar{m}+1} + \frac{e^2 b^2}{\hbar^2} |T_m^2| \delta_{\bar{m},\bar{m}-1} \right\} \left| \int_0^t \sin \frac{i(\Omega_m t + \phi)}{\hbar} dt \right|^2$$

$$P(|100\rangle \rightarrow |100\rangle) = 1 - P(|100\rangle \rightarrow |m\bar{m}\rangle)$$