Guía 4 Problema 9:

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\uparrow\rangle|\uparrow\rangle\right) = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\uparrow\uparrow\rangle\right)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right) = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

a) El estado compuesto es puro o mixto?

$$\rho_1 = |\psi_1\rangle\langle\psi_1| = \frac{1}{2}\left(|\uparrow\downarrow\rangle\langle\uparrow\downarrow| - |\uparrow\downarrow\rangle\langle\uparrow\uparrow| - |\uparrow\uparrow\rangle\langle\uparrow\downarrow| + |\uparrow\uparrow\rangle\langle\uparrow\uparrow|\right)$$

Estado puro $\Leftrightarrow$ $Tr(\rho_1^2) = 1$.

$$\rho_1^2 = \frac{1}{4}\Big(|\uparrow\downarrow\rangle\langle\uparrow\downarrow| - |\uparrow\downarrow\rangle\langle\uparrow\uparrow| + |\uparrow\downarrow\rangle\langle\uparrow\downarrow| - |\uparrow\downarrow\rangle\langle\uparrow\uparrow|$$
$$- |\uparrow\uparrow\rangle\langle\uparrow\downarrow| + |\uparrow\uparrow\rangle\langle\uparrow\uparrow| - |\uparrow\uparrow\rangle\langle\uparrow\downarrow| + |\uparrow\uparrow\rangle\langle\uparrow\uparrow|\Big)$$

$$\rho_1^2 = \rho_1 \Rightarrow Tr(\rho_1^2) = Tr(\rho_1) = 1 \Rightarrow \text{Puro.}$$

$$\rho_2 = |\psi_2\rangle\langle\psi_2| \Rightarrow \rho_2^2 = (|\psi_2\rangle\langle\psi_2|)^2 \underset{\substack{\uparrow \\ \text{Proyector}}}{=} |\psi_2\rangle\langle\psi_2| = \rho_2$$

$$\Rightarrow Tr(\rho_2^2) = Tr(\rho_2) = 1 \Rightarrow \text{Puro.}$$

b) Matriz densidad reducida.

$$\rho_1 = \frac{1}{2}\Big(|\uparrow\rangle|\downarrow\rangle\langle\uparrow|\langle\downarrow| - |\uparrow\rangle|\downarrow\rangle\langle\uparrow|\langle\uparrow| - |\uparrow\rangle|\uparrow\rangle\langle\uparrow|\langle\downarrow| + |\uparrow\rangle|\uparrow\rangle\langle\uparrow|\langle\uparrow| \Big)$$

$$\rho_1 = \frac{1}{2}\Big(|\uparrow\rangle\langle\uparrow| \otimes |\downarrow\rangle\langle\downarrow| - |\uparrow\rangle\langle\uparrow| \otimes |\downarrow\rangle\langle\uparrow| - |\uparrow\rangle\langle\uparrow| \otimes |\uparrow\rangle\langle\downarrow| + |\uparrow\rangle\langle\uparrow| \otimes |\uparrow\rangle\langle\uparrow| \Big)$$

$$\rho_1^A = Tr_B(\rho_1) = \frac{1}{2}\Big(|\uparrow\rangle\langle\uparrow| + |\uparrow\rangle\langle\uparrow| \Big) = |\uparrow\rangle\langle\uparrow|$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\uparrow\rangle - |\uparrow\downarrow\rangle \Big) = |\uparrow\rangle \otimes \frac{1}{\sqrt{2}}\big(|\uparrow\rangle - |\downarrow\rangle \big)$$

$$\rho_1^B = Tr_A(\rho_1) = \frac{1}{2}\Big(|\downarrow\rangle\langle\downarrow| - |\downarrow\rangle\langle\uparrow| - |\uparrow\rangle\langle\downarrow| + |\uparrow\rangle\langle\uparrow| \Big)$$

$$\rho_1^B = \frac{1}{\sqrt{2}}\big(|\uparrow\rangle - |\downarrow\rangle \big)$$

ρ_1^A y ρ_1^B son puros

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle \big) \longrightarrow$$ Separable

$$\rho_2 = \frac{1}{2}\Big(|\uparrow\rangle\langle\uparrow| \otimes |\downarrow\rangle\langle\downarrow| - |\uparrow\rangle\langle\downarrow| \otimes |\downarrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\uparrow| \otimes |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\downarrow| \otimes |\uparrow\rangle\langle\uparrow| \Big)$$

$$\rho_2^A = Tr_B(\rho_2) = \frac{1}{2}\left(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| \right) = \frac{1}{2}\mathbb{1}$$

$$\rho_2^B = Tr_A(\rho_2) = \frac{1}{2}\left(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| \right) = \frac{1}{2}\mathbb{1}$$

ρ_2^A y ρ_2^B son mixtos $\Rightarrow$ ρ_2 es entrelazado

d) Observable cuya medición me de 100% certeza?

$|\psi_1\rangle \rightarrow$ Mido en la base $\{|\psi_1\rangle, \text{ortogonales}\}$

$$|\psi_1\rangle = |\uparrow\rangle \otimes \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$$

Mido localmente en $A = \{|\uparrow\rangle, |\downarrow\rangle\}$

Mido localmente en $B = \{|+x\rangle, |-x\rangle\}$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Mido en la base $\{|\psi_2\rangle, \text{ortogonales}\}$

$\rho_2^A = \rho_2^B = \frac{\mathbb{1}}{2}$, en cualquier base local que mida mis resultados son 50% y 50% $\Rightarrow$ max incerteza.

Cuando hay entrelazamiento la información del conjunto es más que la suma de la información de las partes.

$$\mathrm{Tr}\left(|\psi\rangle\langle\phi|\right) = \langle\phi|\psi\rangle\langle\phi|\phi\rangle + \langle\phi_\perp|\psi\rangle\cancelto{0}{\langle\phi|\phi_\perp\rangle}$$

$$= \langle\phi|\psi\rangle$$