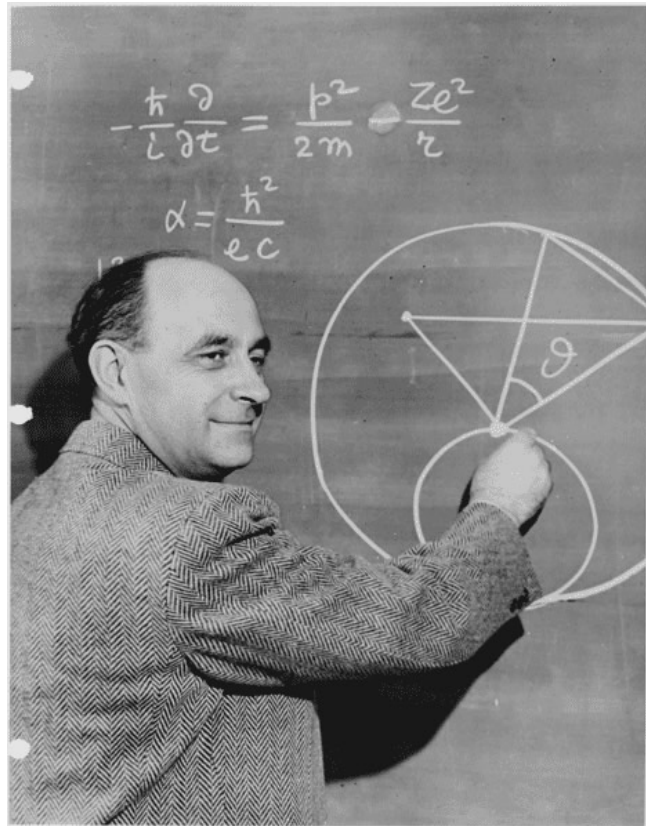


# Quantal\_3



## Sistema cuántico

Debemos tener en cuenta que las partículas no pueden ser localizadas en el espacio de fases mejor que en un volumen  $h^3$  de una celda en el espacio de fases

Esto esta asociado a la relación  $\Delta x \Delta p \geq h$

Solo sabemos la probabilidad de encontrar partículas en la celda pero no cual partícula

Sea el operador de permutación

$$P_{ij} \Psi(r_1, \dots, r_i, \dots, r_j, \dots) = \lambda \Psi(r_1, \dots, r_j, \dots, r_i, \dots)$$

Sea un Hamiltoniano que satisface

$$[\hat{P}_{ij}, \hat{H}] = 0 \quad \forall i, j \text{ con } i \neq j$$

Tendran entonces autofunciones comunes

Entonces

$$P_{ij}\Psi(r_1, \dots, r_i, \dots, r_j, \dots) = \lambda\Psi(r_1, \dots, r_j, \dots, r_i, \dots)$$

$$P_{ij}^2\Psi(r_1, \dots, r_i, \dots, r_j, \dots) = \lambda^2\Psi(r_1, \dots, r_i, \dots, r_j, \dots)$$

$$\Psi(r_1, \dots, r_i, \dots, r_j, \dots) = \lambda^2\Psi(r_1, \dots, r_i, \dots, r_j, \dots)$$

Por lo tanto se cumple que

$$\lambda^2 = 1 \Rightarrow \lambda = \pm 1$$

Las funciones de onda con  $\lambda=1$  son simétricas

Las funciones de onda con  $\lambda=-1$  son antisimétricas

$$\Psi(r_1, \dots, r_i, \dots, r_j, \dots) = +\Psi(r_1, \dots, r_i, \dots, r_j, \dots)$$

$$\text{si } P_{ij}\Psi(r_1, \dots, r_i, \dots, r_j, \dots) = -\Psi(r_1, \dots, r_j, \dots, r_i, \dots)$$

y  $r_i=r_j, \dots$  imposible

Si introducimos ahora el operador  $P$

$$P\Psi(r_1, \dots, r_i, \dots, r_j, \dots) = \lambda^P \Psi(r_{P1}, \dots, r_{Pi}, \dots, r_{Pj}, \dots)$$

Donde  $P_i$  denota las permutaciones de los índices

Si el  $H$  conmuta con todas las  $P_{ik}$  o sea con  $P$  se puede construir un conjunto de autofunciones que son totalmente simétricas o antisimétricas  
Que satisfacen

$$\Psi^S(\vec{r}_1, \dots, \vec{r}_N) = A \sum_P \hat{P} \Psi(\vec{r}_1, \dots, \vec{r}_N)$$

$$\Psi^A(\vec{r}_1, \dots, \vec{r}_N) = B \sum_P (-1)^P \hat{P} \Psi(\vec{r}_1, \dots, \vec{r}_N)$$

$$(-1)^P = \begin{cases} +1 & \text{Permutación par} \\ -1 & \text{Permutación impar} \end{cases}$$

Para Bose

$$\langle n_p \rangle = \frac{ze^{-\beta\varepsilon_p}}{1 - ze^{-\beta\varepsilon_p}}$$

$$0 \leq \frac{ze^{-\beta\varepsilon_p}}{1 - ze^{-\beta\varepsilon_p}} = \frac{1}{e^{\beta(\varepsilon_p - \mu)} - 1}$$

$$0 \leq e^{\beta(\varepsilon_p - \mu)} - 1 \Rightarrow 1 \leq e^{\beta(\varepsilon_p - \mu)} \Rightarrow 0 \leq (\varepsilon_p - \mu) \Rightarrow \varepsilon_p \geq \mu \Rightarrow$$

para el fundamental  $\varepsilon_p = 0 \Rightarrow 0 \geq \mu$  de otra forma el fundamental tendria poblacion negativa

ademas si  $\mu = 0$  para  $\varepsilon_p = 0 \Rightarrow \langle n_p \rangle = \infty$

Para Fermi

$$\langle n_{\mathbf{p}} \rangle = \frac{ze^{-\beta\varepsilon_{\mathbf{p}}}}{1 + ze^{-\beta\varepsilon_{\mathbf{p}}}}$$

Vemos que en este caso

$$0 \leq \frac{ze^{-\beta\varepsilon_{\mathbf{p}}}}{1 + ze^{-\beta\varepsilon_{\mathbf{p}}}} = \frac{1}{1 + e^{\beta(\varepsilon_{\mathbf{p}} - \mu)}} \leq 1$$

$$0 \leq 1 \leq 1 + e^{\beta(\varepsilon_{\mathbf{p}} - \mu)}$$

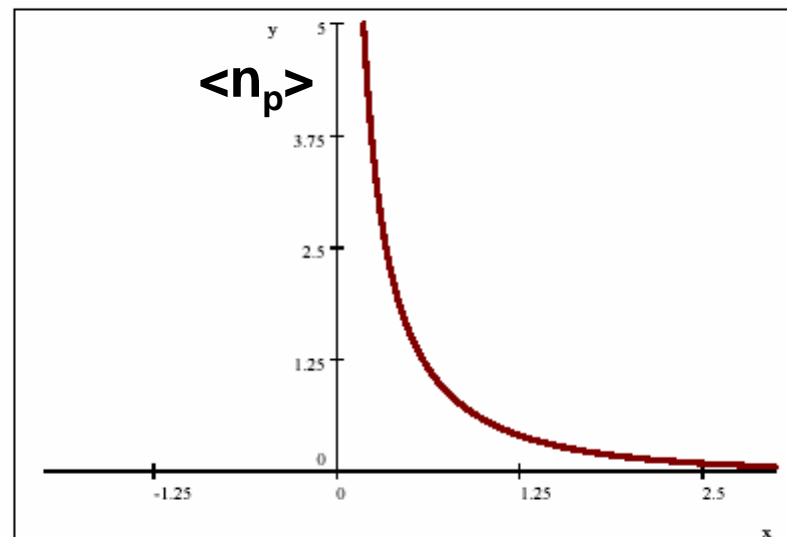
se cumple para todo  $\mu$

# Acerca de los numeros de ocupacion

Estudiamos la ocupacion media de una dado nivel

Para Bose

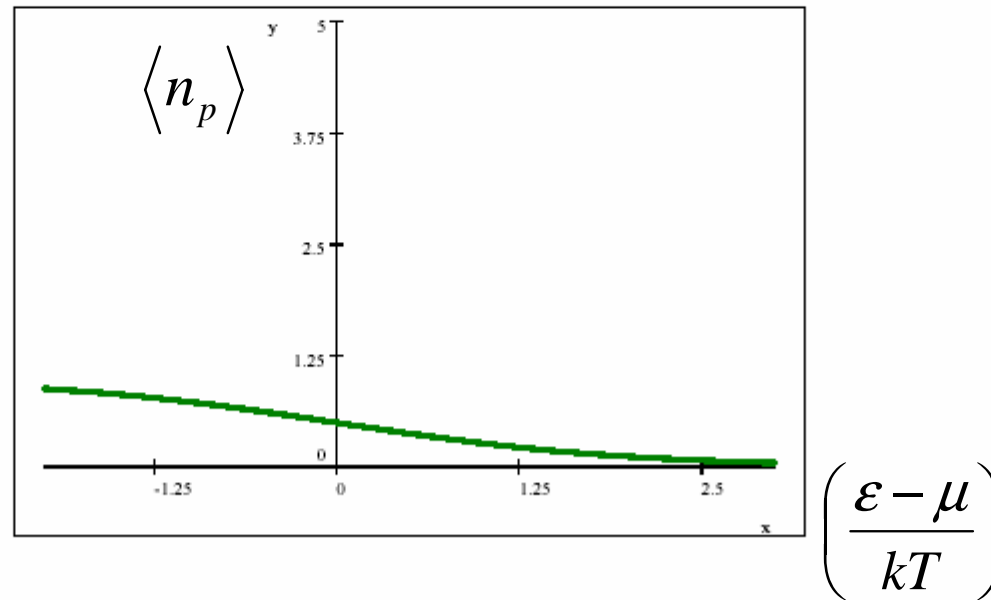
$$\langle n_p \rangle = \frac{ze^{-\beta\epsilon_p}}{1 - ze^{-\beta\epsilon_p}} = \frac{1}{e^{\beta(\epsilon_p - \mu)} - 1}$$



$$\left( \frac{\epsilon - \mu}{kT} \right)$$

## Para Fermi

$$\langle n_{\mathbf{p}} \rangle = \frac{1}{e^{\beta(\varepsilon_{\mathbf{p}} - \mu)} + 1}$$





Recordemos que para Boltzmann

$$Q_N = \frac{1}{N!} [Q_1(V, T)]^N$$

Ademas

$$4\pi p^2 dp = 2\pi p^2 p dp = 2\pi \sqrt{2m\epsilon} d\epsilon$$

$$Q_1(V, T) = \sum e^{-\beta\epsilon} \approx \frac{2\pi V}{h^3} (2m)^{3/2} \int_0^\infty e^{-\beta\epsilon} \epsilon^{1/2} d\epsilon$$

$$\int_0^\infty e^{-\beta x} x^{1/2} dx \text{ es del tipo } \int_0^\infty e^{-ax^2} x^n dx$$

$$\text{sea } y = x^{1/2}$$

$$dy = \frac{1}{2x^{1/2}} dx$$

$$2y dy = dx$$

$$\Rightarrow 2 \int_0^\infty e^{-\beta y^2} y^2 dy = \frac{1}{2} \sqrt{\pi} \beta^{-3/2} = \frac{\sqrt{\pi}}{2(kT)^{-3/2}}$$

$$\langle n_p \rangle = \frac{1}{\Xi} \sum_{N=0}^{\infty} z^N \sum_{n_p; \sum n_p = N} n_p \exp(-\beta \sum_p \epsilon_p n_p) = \frac{-1}{\beta} \frac{\partial}{\partial \epsilon_p} \log \Xi$$

$$Q_1(V, T) = \frac{2\pi V}{h^3} (2m)^{3/2} \frac{\sqrt{\pi}}{2(kT)^{-3/2}} = \frac{V(2\pi mkT)^{3/2}}{h^3}$$

Como  $\lambda^3 = h^3/(2\pi mkT)^{3/2}$

Resulta

$$Q_1(V, T) = V/\lambda^3$$

$$\Xi(z, V, T) = \sum_0^{\infty} z^N Q_N = \sum_0^{\infty} z^N \frac{V^N}{\lambda^{3N} N!} = \exp\left(z \frac{V}{\lambda^3}\right)$$

Ademas

$$\langle n_\epsilon \rangle = \frac{1}{\Xi(z, V, T)} \left[ -\frac{1}{\beta} \left( \frac{\partial \Xi}{\partial \epsilon} \right) \right] = -\frac{1}{\beta} \left( \frac{\partial \log \Xi}{\partial \epsilon} \right)$$

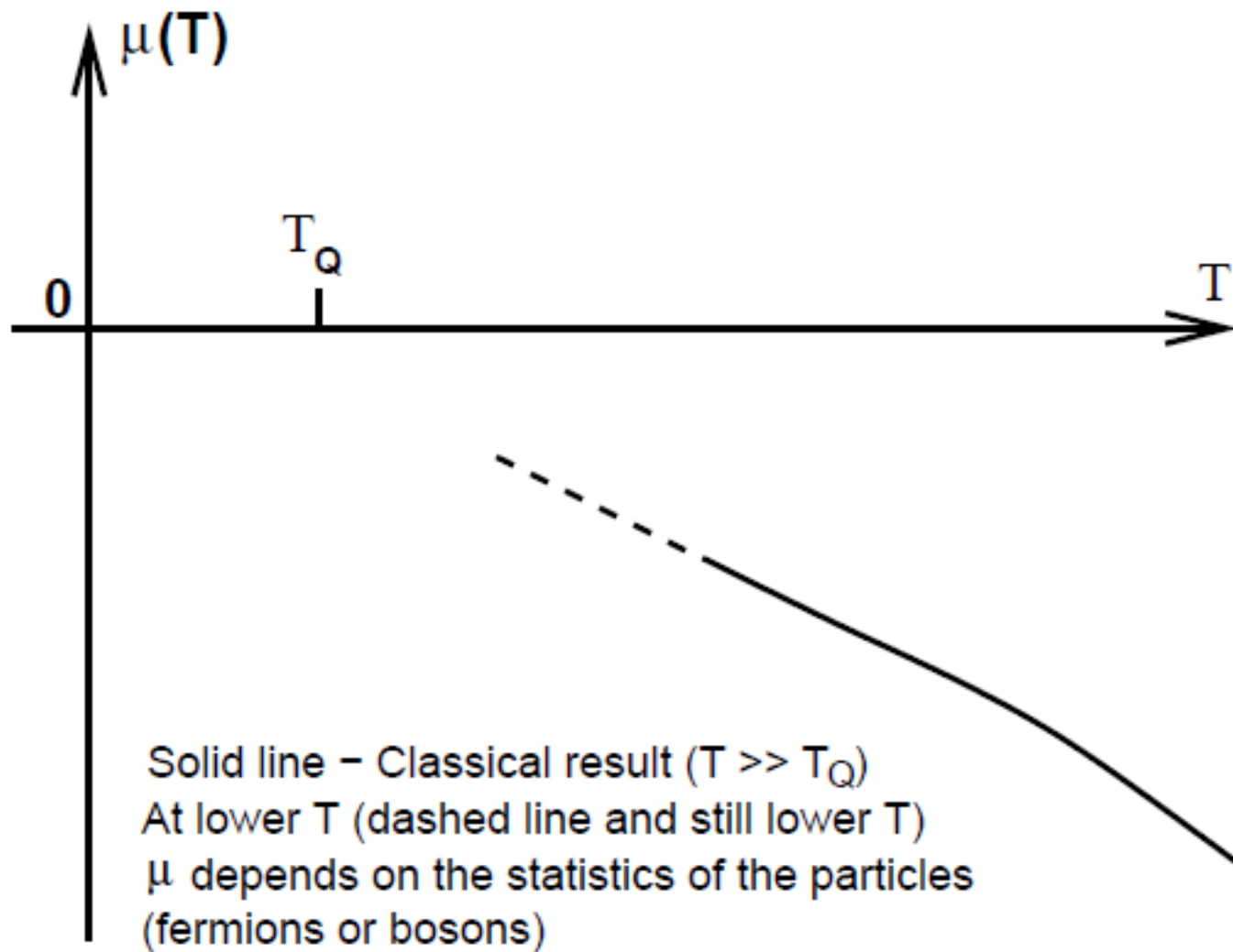
$$q_{MB} = zQ_1 \Rightarrow \bar{N} = z \left( \frac{\partial q}{\partial z} \right) = zQ_1 \Rightarrow \langle n_\epsilon \rangle = ze^{-\beta\epsilon}$$

$$q_{MB} = zQ_1 \quad \Rightarrow \quad \bar{N} = z \left( \frac{\partial q}{\partial z} \right) = zQ_1 \quad \Rightarrow \quad \langle n_\varepsilon \rangle = z \frac{V}{\lambda^3}$$

$$V_Q = \lambda^3$$

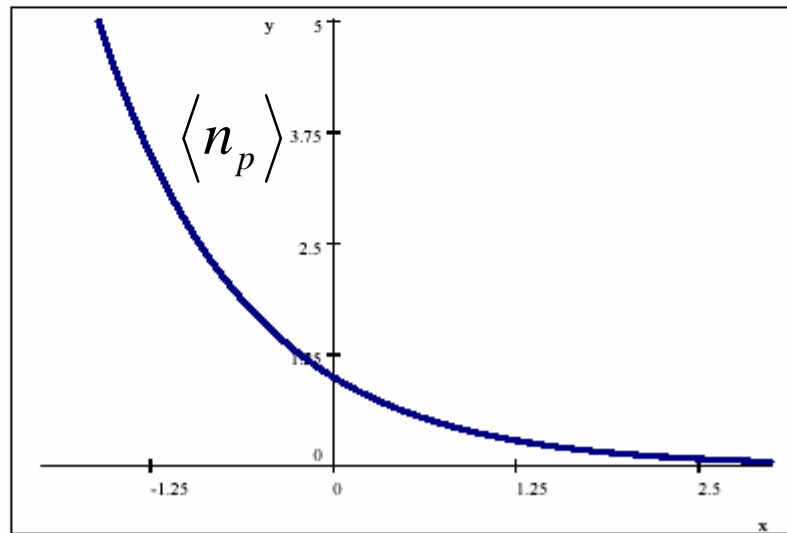
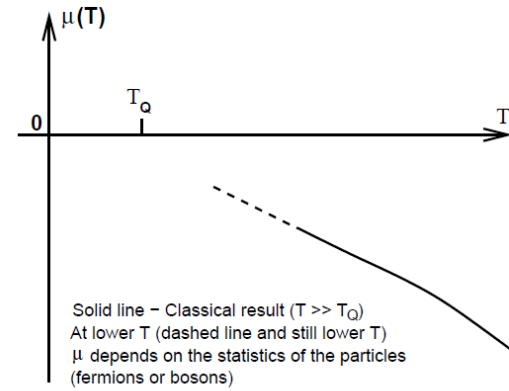
$$\langle N \rangle = e^{\beta\mu} \frac{V}{V_Q},$$

$$\mu(T) = k_B T \ln(nV_Q) = -k_B T \ln \left[ \frac{1}{n} \left( \frac{mk_B T}{2\pi\hbar^2} \right)^{3/2} \right] = -\frac{3}{2} k_B T \ln \left( \frac{T}{T_Q} \right)$$



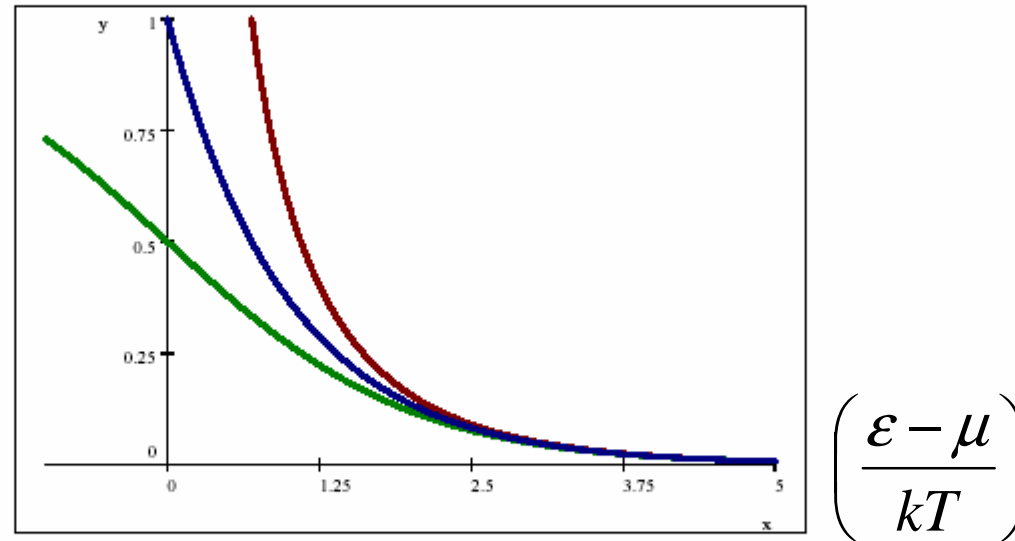
## Para Boltzmann

$$\langle n_p \rangle = e^{\beta(-\varepsilon_p + \mu)} \propto e^{\beta(-\varepsilon_p)}$$



$$\left( \frac{\varepsilon - \mu}{kT} \right)$$

Todo junto  $\Rightarrow$



la diferencia entre las estadísticas cuánticas y la clásica se hace imperceptible cuando  $e^{\beta(\epsilon_p - \mu)} \gg 1$

$$e^{\beta(\epsilon_p)}/z \gg 1 \Rightarrow z \ll 1 \Rightarrow e^{\frac{\mu}{kT}} \ll 1 \Rightarrow \mu < 0 \text{ y } |\mu| \gg 1$$

como  $0 \geq \mu$  para Bose

y para Fermi **no** había restricción  $\Rightarrow$  es consistente.

O sea que cuando

$$\langle n_{\bar{p}} \rangle = \frac{1}{\exp \beta(\varepsilon_p - \mu) \pm 1} \Big|_B^F$$

$$\left( \frac{\varepsilon - \mu}{kT} \right) \gg 1 \Rightarrow \exp\left( \frac{\varepsilon - \mu}{kT} \right) \gg 1 \Rightarrow \langle n_{\varepsilon} \rangle \ll 1$$

Tomando en cuenta que

$$g_{Bose} \{n_e\} = 1$$

$$g_{Fermi} \{n_e\} = 1 \Leftrightarrow n_e = 0 \text{ o } 1$$

$$g_{Fermi} \{n_e\} = 0 \quad \text{En otro caso}$$

$$g_{Boltzmann} \{n_e\} = \prod \frac{1}{n_e!}$$

Sabemos que a  
T grande la aprox. clasica  
es apropiada.  
Entonces  $\mu$  debe hacerse  
grande y negativo

Como es el caso para Fermi  
Bose y Boltzmann

$$g_{Boltzmann} \{n_e\} = 1$$

# Fermi Gas

Como habiamos visto, las dos ecuaciones basicas son:

$$\frac{P}{kT} = \frac{4\pi}{h^3} \int_0^\infty dp p^2 \log\left(1 + ze^{-\beta \frac{p^2}{2m}}\right)$$

$$\frac{1}{v} = \frac{4\pi}{h^3} \int_0^\infty dp p^2 \frac{1}{1 + z^{-1} e^{\beta \frac{p^2}{2m}}}$$

A partir de la primera habiamos escrito

Sea  $\left(\frac{\beta}{2m}\right)^{\frac{1}{2}} p = x$ ;  $dx = dp \left(\frac{\beta}{2m}\right)^{\frac{1}{2}}$ , entonces

$$\frac{P}{kT} = \frac{4}{\lambda^3 \sqrt{\pi}} \int_0^\infty dx x^2 \log\left(1 + ze^{-x^2}\right) = \frac{1}{\lambda^3} f_{\frac{5}{2}}(z)$$



y del mismo modo

$$\frac{1}{v} = \frac{1}{\lambda^3} f_{\frac{3}{2}}(z)$$

con  $\lambda = \sqrt{2\pi\hbar^2/mkT}$

luego hicimos

$$f_{\frac{5}{2}}(z) = \frac{4}{\sqrt{\pi}} \int_0^{\infty} dx x^2 \log(1 + ze^{-x^2})$$

$$= \int_0^{\infty} dx x^2 \left[ 0 + ze^{-x^2} - \frac{1}{2!} z^2 e^{-2x^2} + \frac{2}{3!} z^3 e^{-3x^2} - \dots \right]$$

como  $\int_0^{\infty} dx x^2 e^{-ax^2} = \frac{1}{4} \frac{1}{a^{\frac{3}{2}}} \sqrt{\pi}$

$$\frac{P}{kT} = \frac{1}{\lambda^3} \sum_{l=1}^{\infty} \frac{(-1)^{l+1} z^l}{l^{\frac{5}{2}}}$$

Entonces tenemos 2 ecuaciones



$$\frac{P}{kT} = \frac{1}{\lambda^3} f_{\frac{5}{2}}(z)$$

$$\frac{1}{v} = \frac{1}{\lambda^3} f_{\frac{3}{2}}(z)$$

y podemos despejar  $z$  de una de ellas y reemplazar en la otra.

Observar que a partir del desarrollo de  $f_i(z)$  en potencias de  $z$ , es inmediato que

$$f_{v-1}(z) = z \frac{\partial}{\partial z} f_v(z)$$

# Como se comporta $f_{\frac{3}{2}}(z)$ ? $z$ grande

$$f_{\frac{3}{2}}(z) = \sum_{l=1}^{\infty} \frac{(-1)^{l+1} z^l}{l^{\frac{3}{2}}} = z - \frac{z^2}{2^{\frac{3}{2}}} + \frac{z^3}{3^{\frac{3}{2}}} - \frac{z^4}{4^{\frac{3}{2}}} \dots$$

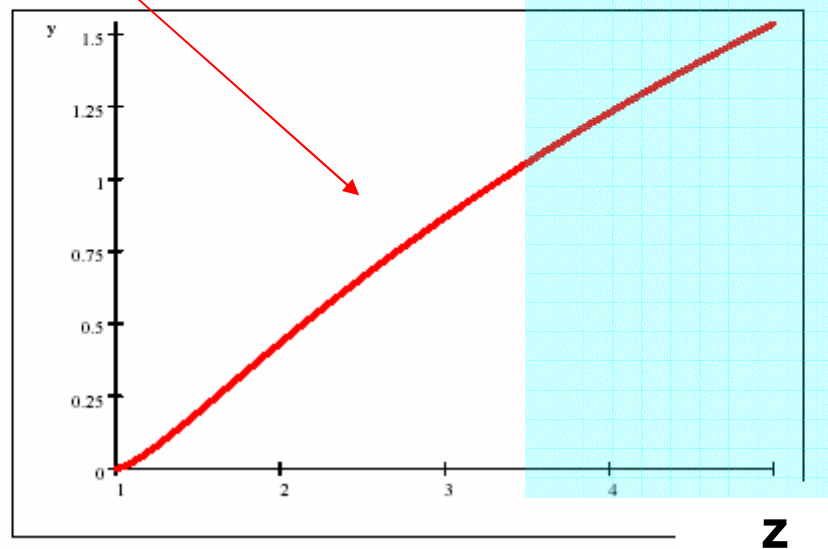
si  $z$  es grande  $\rightarrow$  "lemma de Sommerfeld"

$$f_{\frac{3}{2}}(z) = \frac{4}{3\sqrt{\pi}} [(\log z)^{\frac{3}{2}} + \frac{\pi^2}{8} (\log z)^{\frac{-1}{2}} + \dots] \quad [*]$$

por ejemplo

$$y = \frac{4}{3\sqrt{\pi}} ((\log x)^{\frac{3}{2}})$$

$$\lambda^3/\nu = f_{3/2}(z)$$



\* Huang, Pathria, etc.

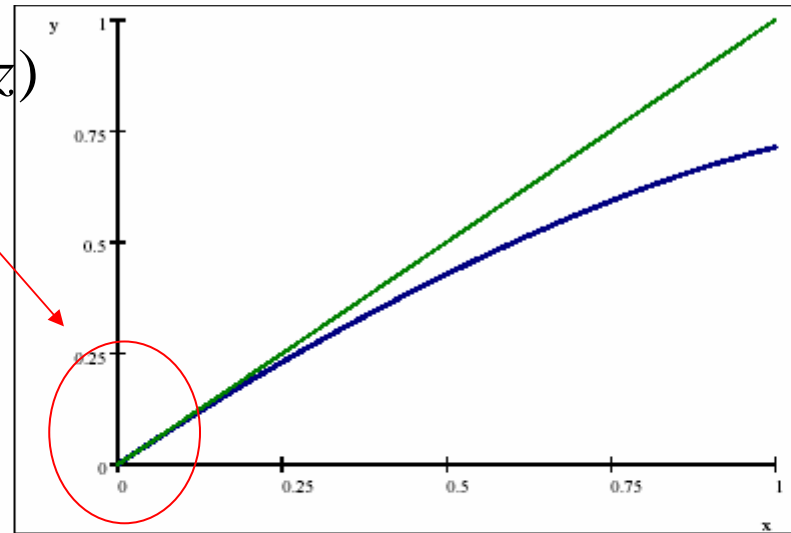
## Como se comporta $f_{\frac{3}{2}}(z)$ ?

$z$  pequeño

$$f_{\frac{3}{2}}(z) = \sum_{l=1}^{\infty} \frac{(-1)^{l+1} z^l}{l^{\frac{3}{2}}} = z - \frac{z^2}{2^{\frac{3}{2}}} + \frac{z^3}{3^{\frac{3}{2}}} - \frac{z^4}{4^{\frac{3}{2}}} \dots$$

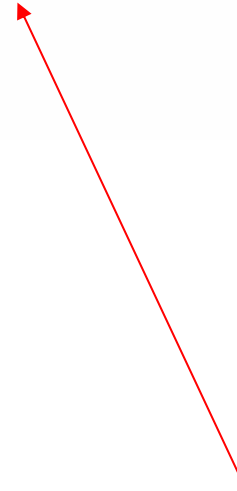
si  $z$  es pequeño...

$$\lambda^3/\nu = f_{3/2}(z)$$



Vemos que hay solución de  $\lambda^3/\nu \nabla z$

Tomar en cuenta que fijar  $\frac{\lambda^s}{v}$  es determinar  $T$ ,  $N$  y  $V$ , que son los parametros del sistema que determino  $\Rightarrow$  *para toda condicion macroscopica tendremos solucion.*  $\Rightarrow$  para mantener fijo  $v$  al variar  $T$  varia  $z$



# Condiciones varias

$$f_{3/2}(z) = \frac{\lambda^3}{v} \quad \text{con} \quad \lambda = \sqrt{\frac{2\pi\hbar^2}{mkT}}$$

Es decir,  $f_{3/2}(z)$  expresa la relación entre la longitud de onda térmica y el Volumen que le "corresponde" a cada partícula.

$$\mathbf{1) \frac{\lambda^3}{v} \ll 1}$$

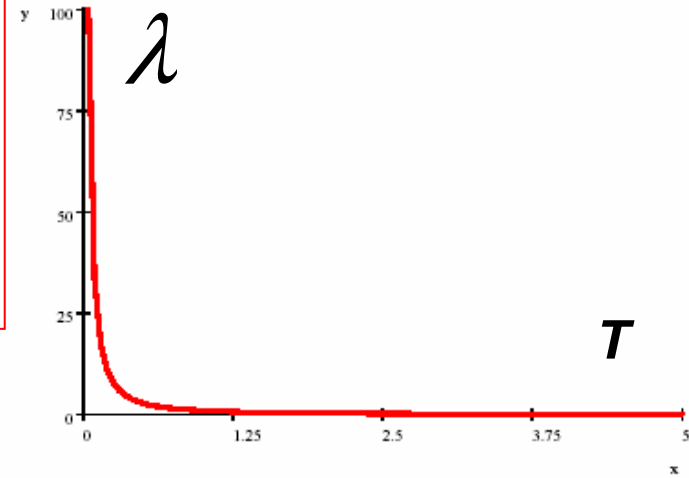
$$\Rightarrow \lambda^3 \ll v$$

la región en la que el paquete es relevante es mucho menor que el volumen específico

**Altas temperaturas y bajas densidades**

- altas temperaturas

$$\lambda = \sqrt{2\pi\hbar^2 / mkT} \Rightarrow \text{decrece al aumentar } T$$



- bajas densidades  $v = \frac{V}{N} \Rightarrow$  crece al aumentar  $V$ ,  $\rho \propto \frac{1}{V}$

la distancia interparticular es mucho mayor que la  $\approx$  longitud de la onda de De Broglie media a la temperatura correspondiente.

Estas condiciones corresponden a  $z$  pequeño (ver figura),  
luego



$$f_{\frac{3}{2}}(z) = \sum_{l=1}^{\infty} \frac{(-1)^{l+1} z^l}{l^{\frac{3}{2}}} = z - \frac{z^2}{2^{\frac{3}{2}}} + \frac{z^3}{3^{\frac{3}{2}}} - \frac{z^4}{4^{\frac{3}{2}}} \dots$$

si  $z$  es pequeño...

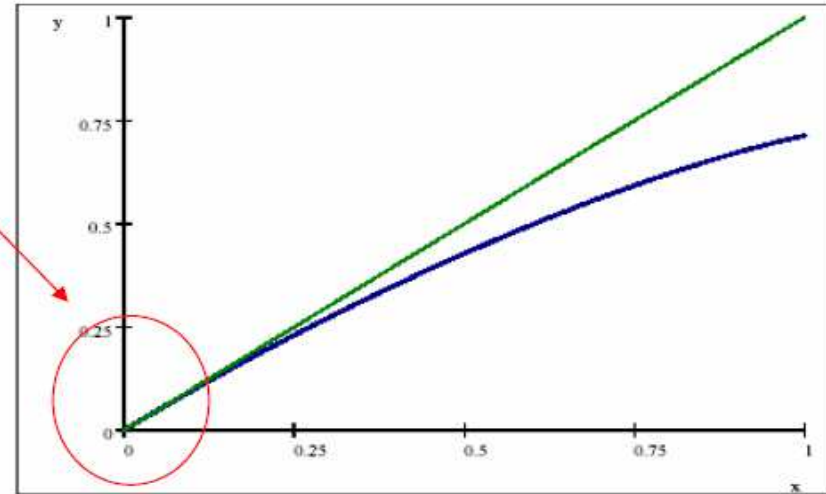
$$\frac{\lambda^3}{v} \approx z - \frac{z^2}{2^{\frac{3}{2}}}$$

de donde

$$z \approx \frac{\lambda^3}{v} + \frac{1}{2^{\frac{3}{2}}} \left( \frac{\lambda^3}{v} \right)^2$$

cuando  $\frac{\lambda^3}{v} \rightarrow 0 \Rightarrow z \rightarrow \frac{\lambda^3}{v}$  que es la solución para el gas

de Boltzmann





Recordemos que para Boltzmann

$$\bar{n}_i = \begin{cases} \frac{g_i}{z^{-1} e^{\beta\epsilon_i} \mp 1} & \text{(Bose and Fermi)} \\ g_i z e^{-\beta\epsilon_i} & \text{(Boltzmann)} \end{cases}$$

$$N = z \sum_i g_i e^{-\beta\epsilon_i} = z \sum_{\mathbf{p}} e^{-\beta\epsilon_{\mathbf{p}}} = \frac{zV}{h^3} \int_0^\infty dp 4\pi p^2 e^{-\beta p^2 / 2m} = \frac{zV}{\lambda^3}$$

Entonces :

$$\frac{N}{V} = \frac{z}{\lambda^3} \Rightarrow z = \frac{\lambda^3}{v}$$

entonces

$$\langle n_p \rangle = \frac{ze^{-\beta \epsilon_p}}{1+ze^{-\beta \epsilon_p}} \rightarrow \frac{\lambda^3}{v} e^{-\beta \epsilon_p} \quad \left[ \frac{\lambda^3}{v} \ll 1 \Rightarrow z \ll 1 \Rightarrow 1 + ze^{-\beta \epsilon_p} \cong 1 \right]$$

La ecuacion de estado es entonces

$$\frac{Pv}{kT} = \frac{v}{\lambda^3} \sum_{l=1}^{\infty} \frac{(-1)^{l+1} z^l}{l^{\frac{5}{2}}} = \frac{v}{\lambda^3} \left( z - \frac{z^2}{2^{\frac{5}{2}}} \dots \right)$$

reemplazando  $z$

$$\frac{Pv}{kT} = \left( 1 + \frac{\lambda^3}{v} \frac{1}{2^{\frac{3}{2}}} \dots \right)$$

$$\left[ z \approx \frac{\lambda^3}{v} + \left[ \frac{\lambda^3}{v} \right]^2 \frac{1}{2^{3/2}} \right]$$

que es el gas ideal mas una correccion de origen cuantico

$$P_f > P_c$$

$$2) \frac{\lambda^3}{\nu} \gg 1$$

Bajas temperaturas

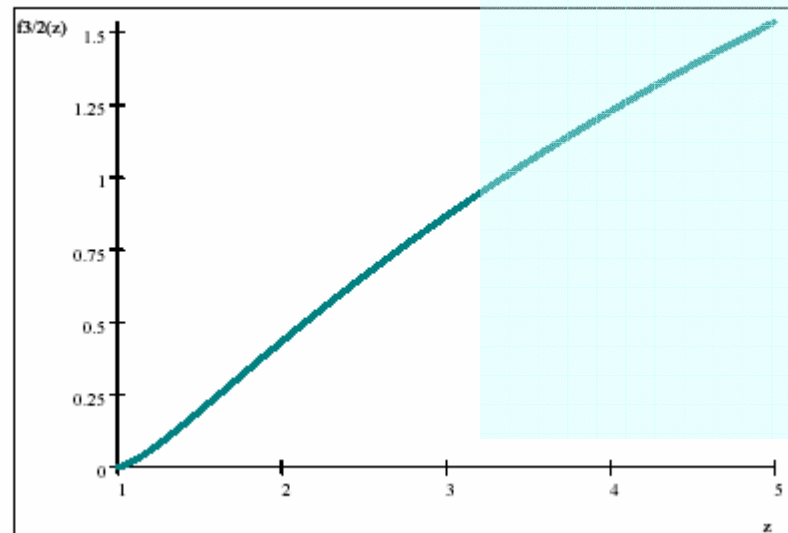
$$(\lambda^3 \gg 1)$$

Altas densidades

$$(\nu \ll 1)$$

Como  $\lambda$  es mucho mayor que  $\sqrt[3]{\nu}$  hay "fuerte overlap"

Para valores altos de  $\frac{\lambda^3}{\nu} \Rightarrow$  valores altos de  $z$



del lemma de sommerfeld

$$\frac{\lambda^3}{v} \approx \frac{4}{3\sqrt{\pi}} (\log z)^{\frac{3}{2}} \approx \frac{1}{v} \left[ \sqrt{2\pi\hbar^2/mkT} \right]^3$$

$$z \approx \exp \left[ \frac{1}{kT} \frac{\hbar^2}{2m} 4\pi \left( \frac{1}{v} \frac{3\sqrt{\pi}}{4} \right)^{\frac{2}{3}} \right] = \exp \beta \left[ \frac{\hbar^2}{2m} \left( \frac{6\pi^2}{v} \right)^{\frac{2}{3}} \right]$$

definimos

$\epsilon_f$

$$\epsilon_f = \frac{\hbar^2}{2m} \left( \frac{6\pi^2}{v} \right)^{\frac{2}{3}}$$

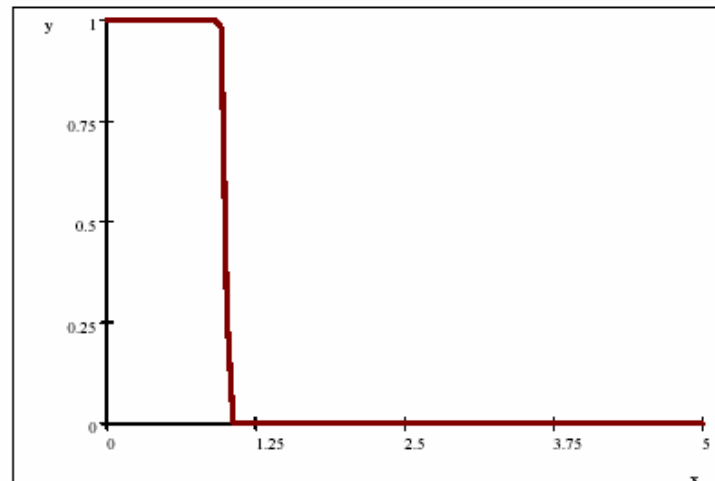
luego

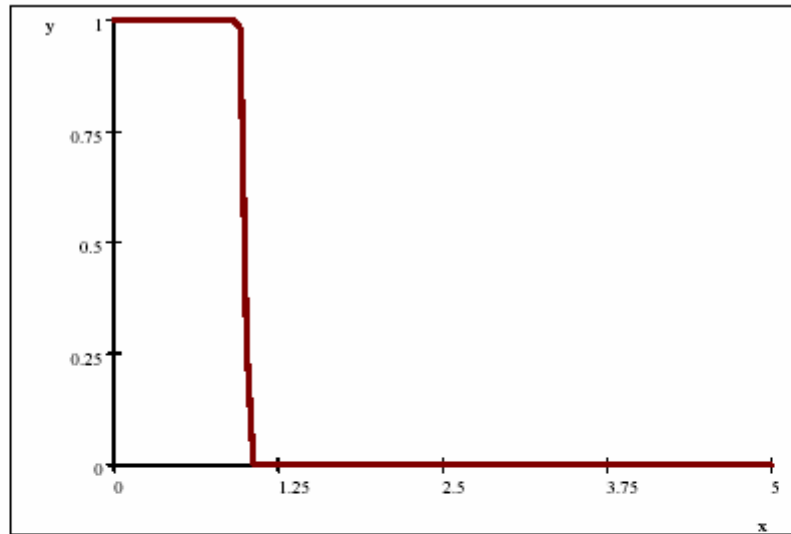
$$z = \exp \beta \epsilon_f$$

Entonces podemos reescribir la poblacion media por estado

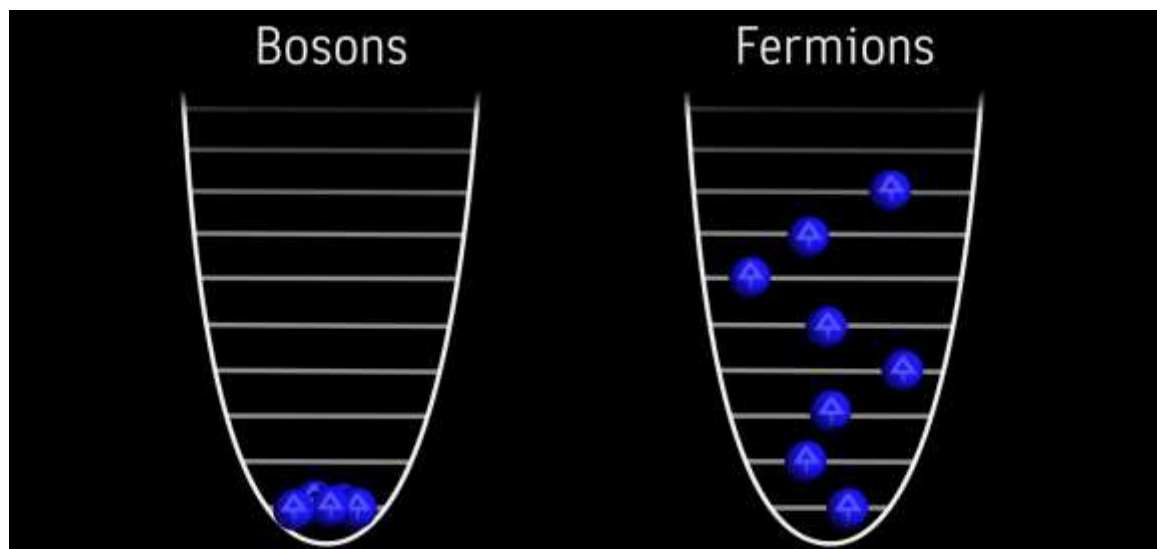
$$\langle n_p \rangle = \frac{ze^{-\beta \epsilon_p}}{1 + ze^{-\beta \epsilon_p}} = \frac{1}{1 + \exp \beta(\epsilon_p - \epsilon_f)}$$

$\frac{1}{1 + \exp(100(x-1))}$  ; temperatura baja,  $\beta$  alto  $\Rightarrow$

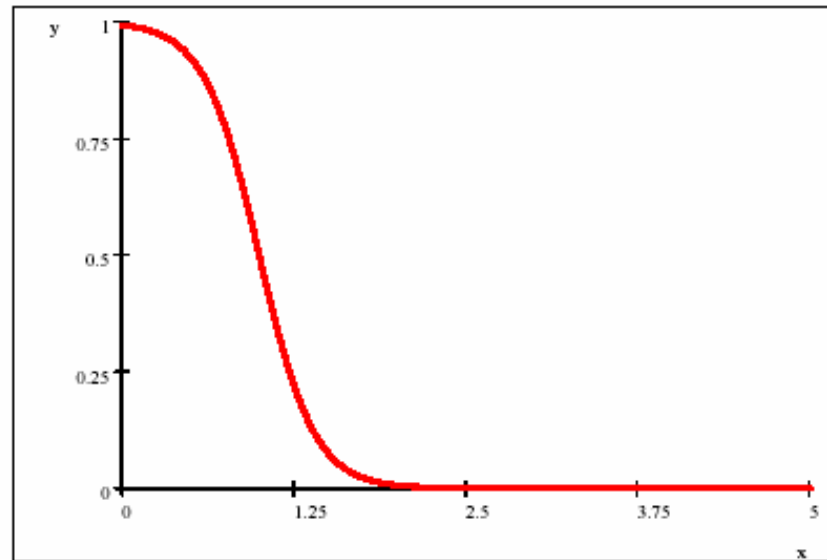




Esto es así porque a temperaturas bajas al "apilar partículas" en los niveles más bajos...



Si la temperatura es "un poco mas alta"



Entonces para el caso de  $T \rightarrow 0 \Rightarrow \beta \rightarrow \infty \Rightarrow$

$$\langle n_p \rangle_{T \rightarrow 0, \epsilon_p < \epsilon_f} = 1 \quad \langle n_p \rangle_{T \rightarrow 0, \epsilon_p > \epsilon_f} = 0$$

Entonces cuando  $T \rightarrow 0$ ,  $\epsilon_f$  es el nivel tal que debajo de el hay  $N$  estados. En el espacio de momentos las particulas ocupan una esfera de radio  $p_f$  y determina la superficie de

Fermi. (para poder acomodar  $N$  fermiones)

Sea ahora el caso en que tomamos en cuenta otro grado de libertad por lo que los niveles  $\vec{p}$  estan  $g$  veces degenerados.

Por ejemplo sea el spin  $s \Rightarrow g = 2s + 1$

La condicion que determina el valor de  $p_f$  es

$$g \sum_{\mathbf{p}} \langle n_{\mathbf{p}} \rangle_{T=0} = N$$

Volumen en p

Integrando

$$\frac{g}{(2\pi\hbar)^3} \frac{4\pi}{3} p_f^3 = \frac{N}{V}$$

$$\Rightarrow N = \frac{4\pi g V}{3h^3} p_F^3$$



si llamamos  $\epsilon_f = \frac{p_f^2}{2m} \Rightarrow$

$$p_F = h \left[ \frac{3N}{4\pi g V} \right]^{1/3}$$

$$\epsilon_f = \frac{\hbar^2}{2m} \left( \frac{6\pi^2}{gV} \right)^{2/3}$$

De donde se ve que un sistema fermionico con  $N$  particulas y una degeneracion  $g$  es similar a tener  $g$  sistemas fermionicos independientes con  $\frac{N}{g}$  particulas C/u

$$n = \frac{N}{V} = \frac{1}{v}$$

$$E_0 = \frac{4\pi g V}{h^3} \int_0^{p_F} \left( \frac{p^2}{2m} \right) p^2 dp = g \frac{2\pi V}{5m h^3} p_F^5$$

$$N = g \frac{4\pi V}{3h^3} p_F^3 = gV \left[ \frac{4\pi}{3} \frac{p_F^3}{h^3} \right]$$

$$\frac{E_0}{N} = \frac{3p_F^2}{10m} = \frac{3}{5} \epsilon_F$$

$$P_0 = \frac{2}{3} \frac{E_0}{V} = \frac{2}{3} \frac{3}{5} n \epsilon_F \Rightarrow P_0 = \left[ \left( \frac{6\pi^2}{g} \right)^{2/3} \frac{\hbar^2}{5m} \right] n^{5/3} \Rightarrow P_0 \propto n^{5/3}$$

# Funciones termodinamicas a temperaturas bajas y altas densidades

$$1) \frac{\lambda^3}{v} = f_{\frac{3}{2}}(z)$$

y en este limite

$$2) f_{\frac{3}{2}}(z) = \frac{4}{3\sqrt{\pi}} [(\log z)^{\frac{3}{2}} + \frac{\pi^2}{8} (\log z)^{-\frac{1}{2}} + \dots] \Rightarrow$$

$$f_{\frac{3}{2}}(z) = \frac{4}{3\sqrt{\pi}} (\log z)^{\frac{3}{2}} [1 + \frac{\pi^2}{8} (\log z)^{-2} + \dots]$$

$$3) f_{\frac{5}{2}}(z) = \frac{8}{15\sqrt{\pi}} (\log z)^{\frac{5}{2}} [1 + \frac{5\pi^2}{8} (\log z)^{-2} + \dots]$$

$$4) f_{\frac{1}{2}}(z) = \frac{2}{\sqrt{\pi}} (\log z)^{\frac{1}{2}} [1 - \frac{\pi^2}{24} (\log z)^{-2} + \dots]$$

ademas

$$\frac{P}{kT} = \frac{1}{\lambda^3} f_{\frac{5}{2}}(z)$$

$$\left[ f_{\nu-1}(z) = z \frac{\partial}{\partial z} f_{\nu}(z) \right]$$

$$\frac{P}{kT} = \frac{1}{\lambda^3} f_{5/2}(z)$$

$$\frac{1}{v} = \frac{1}{\lambda^3} f_{3/2}(z)$$

Entonces, con  $\lambda = \sqrt{2\pi\hbar^2/mkT} = h\sqrt{1/2\pi mkT}$

$$\frac{1}{v} = \frac{1}{\lambda^3} f_{3/2}(z) = \frac{4\pi}{3} (2m/h^2)^{3/2} (kT \log z)^{3/2} \left(1 + \frac{\pi^2}{8} (\log z)^{-2} + \dots\right)$$

de donde

a) primer orden  $kT \log z \approx \left(\frac{3N}{4\pi V}\right)^{2/3} \frac{h^2}{2m} = \frac{\hbar^2}{2m} \left(\frac{6\pi^2}{gV}\right)^{2/3}$

$\mathcal{E}_F$

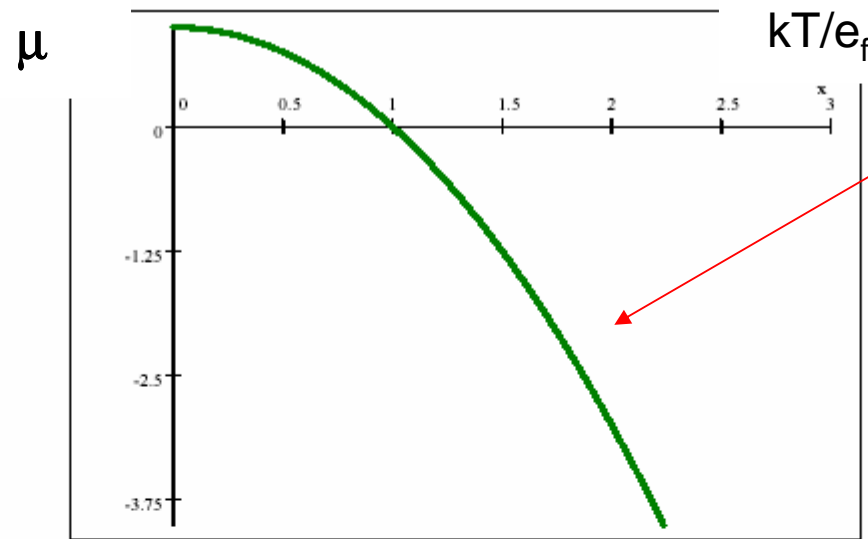
b) al siguiente orden

$$kT \log z \equiv \mu \approx \epsilon_f \left(1 - \frac{\pi^2}{12} (kT/\epsilon_f)^2\right)$$

$$\epsilon_f = \frac{\hbar^2}{2m} \left(\frac{6\pi^2}{gV}\right)^{2/3}$$

De donde  $\mu$  se comporta...

# Entonces



$$\frac{P}{kT} = \frac{1}{\lambda^3} f_{5/2}(z)$$

$$\frac{N}{V} = \frac{1}{\lambda^3} f_{3/2}(z)$$

$$U = -\left(\frac{\partial}{\partial \beta} \log \Xi\right)_{z,V} = kT^2 \left(\frac{\partial}{\partial T} \log \Xi\right) = kT^2 \left(\frac{\partial}{\partial T} \frac{PV}{kT}\right)$$

$$= \frac{3}{2} kT \frac{V}{\lambda^3} f_{5/2}(z) = \frac{3}{2} NkT \frac{f_{5/2}(z)}{f_{3/2}(z)} \Rightarrow$$

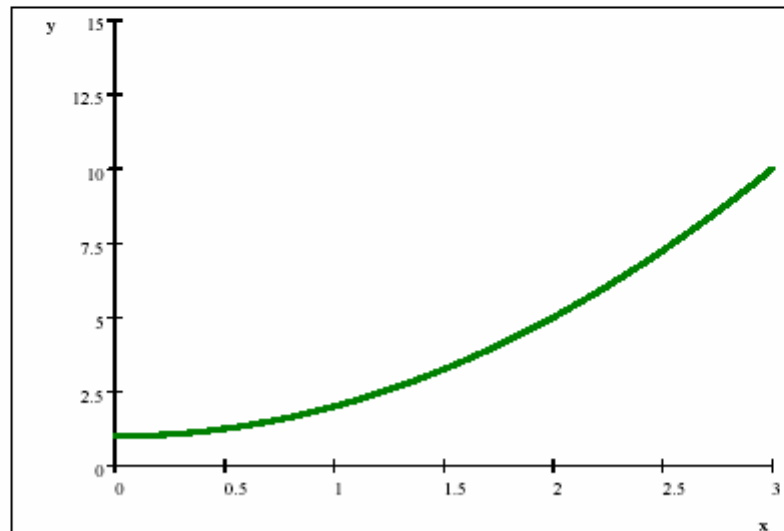
$$\frac{U}{N} = \frac{3}{2} kT \frac{f_{5/2}(z)}{f_{3/2}(z)} = \frac{3}{2} kT \frac{\frac{8}{15\sqrt{\pi}} (\log z)^{\frac{5}{2}} [1 + \frac{5\pi^2}{8} (\log z)^{-2} + \dots]}{\frac{4}{3\sqrt{\pi}} (\log z)^{\frac{3}{2}} [1 + \frac{\pi^2}{8} (\log z)^{-2} + \dots]} \Rightarrow$$

Dividiendo...  
36

$$\frac{U}{N} = \frac{3}{5}kT \log z \left( 1 + \frac{\pi^2}{2} (\log z)^{-2} + \dots \right)$$

Que puede reescribirse como (usando la aprox. para  $kT \ln z$ )

$$\frac{U}{N} = \frac{3}{5} \epsilon_f \left( 1 + \frac{5\pi^2}{12} \left( \frac{kT}{\epsilon_f} \right)^2 + \dots \right)$$



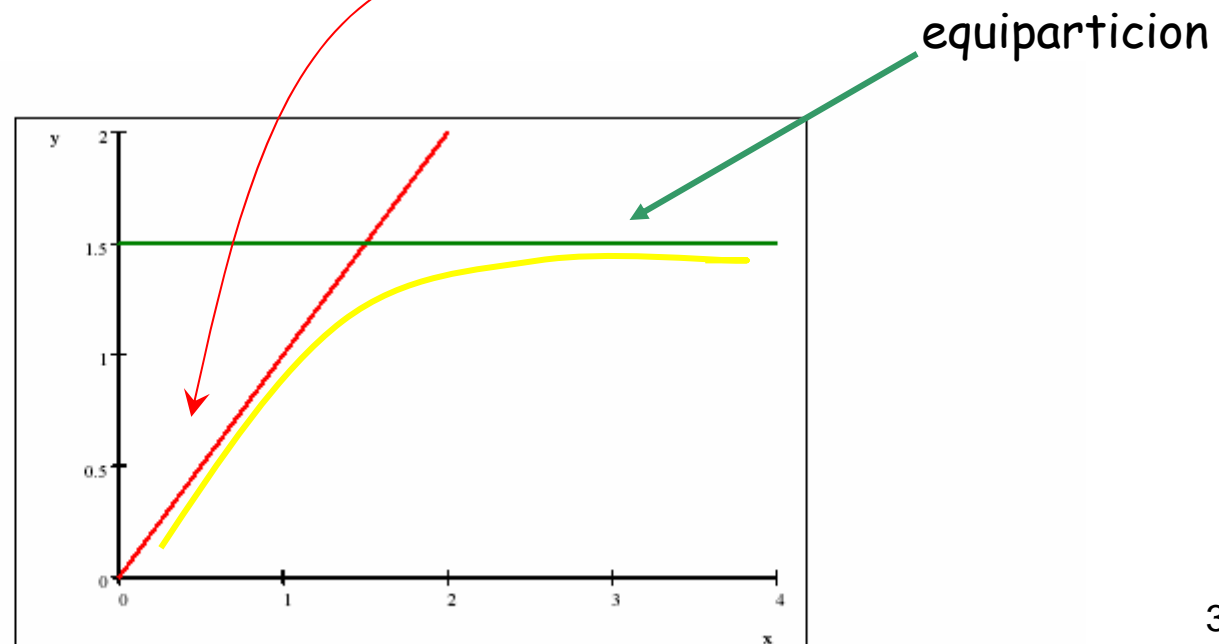
Del mismo modo

$$P = \frac{2}{3} \frac{U}{V}$$

Y para  $C_V$

$$C_V = \frac{\pi^2}{2} \frac{kT}{\epsilon_f} + \dots$$

Entonces para temperaturas bajas  $T \ll T_f = \epsilon_f/k$ ,  $C_V$  es lineal en  $T$



Finalmente la energía libre de Helmholtz

$$\frac{A}{N} = \frac{3}{5}\epsilon_f \left( 1 - \frac{5\pi^2}{12} \left( \frac{kT}{\epsilon_f} \right)^2 \right)$$

de donde ( $A=U-TS$ )

$$\frac{S}{Nk} = \frac{\pi^2}{2} \left( \frac{kT}{\epsilon_f} \right)$$

$\Rightarrow S \rightarrow 0$  con  $T \rightarrow 0$  de acuerdo con la tercera ley

$$\frac{A}{N} = \mu - \frac{PV}{N}$$

## Ecuacion de Estado para situaciones no extremas

Hemos visto que ocurre cuando

$$\frac{\lambda^3}{v} \ll 1$$

y

$$\frac{\lambda^3}{v} \gg 1$$

Que ocurre si  $\frac{\lambda^3}{v} \lesssim 1$  hay que considerar toda la serie.

Proponemos un desarrollo en terminos del virial



$$\frac{PV}{NkT} = \sum_{l=1}^{\infty} (-1)^l a_l \left( \frac{\lambda^3}{g\nu} \right)^{l-1}$$

$$\frac{P}{kT} = \frac{g}{\lambda^3} f_{\frac{5}{2}}(z)$$

$$\frac{\lambda^3}{g\nu} = f_{\frac{3}{2}}(z) \Rightarrow \lambda^3 = g \frac{N}{V} f_{\frac{3}{2}}(z)$$

A determinar

De donde

$$\frac{PV}{NkT} = \frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)}$$

Entonces

$$f_{\frac{3}{2}}(z) = \sum_{l=1}^{\infty} \frac{(-1)^{l+1} z^l}{l^{\frac{3}{2}}} = z - \frac{z^2}{2^{\frac{3}{2}}} + \frac{z^3}{3^{\frac{3}{2}}} - \frac{z^4}{4^{\frac{3}{2}}} \dots$$

$$\left[ \frac{P}{kT} = \frac{N}{V} g f_{\frac{5}{2}}(z) = \frac{1}{\nu} g f_{\frac{5}{2}}(z) = \frac{g}{\lambda^3} f_{\frac{5}{2}}(z) \right]$$

$$\sum_{l=1}^{\infty} (-1)^l a_l \left( \frac{\lambda^3}{g\nu} \right)^{l-1} = \frac{f_{\frac{5}{2}}(z)}{f_{\frac{3}{2}}(z)}$$

Resultando

$$\left[ \sum_{l=1}^{\infty} (-1)^{l-1} a_l \left( \frac{\lambda^3}{g\nu} \right)^{l-1} \right] f_{\frac{3}{2}}(z) = f_{\frac{5}{2}}(z)$$

Reemplazando  $f_{\frac{3}{2}}(z)$  y  $f_{\frac{5}{2}}(z)$  por sus expresiones

$$f_{\frac{3}{2}}(z) = \sum_{l=1}^{\infty} \frac{(-1)^{l+1} z^l}{l^{\frac{3}{2}}} = z - \frac{z^2}{2^{\frac{3}{2}}} + \frac{z^3}{3^{\frac{3}{2}}} - \frac{z^4}{4^{\frac{3}{2}}} \dots$$

$$f_{\frac{5}{2}}(z) = \sum_{l=1}^{\infty} \frac{(-1)^{l+1} z^l}{l^{\frac{5}{2}}} = z - \frac{z^2}{2^{\frac{5}{2}}} + \frac{z^3}{3^{\frac{5}{2}}} - \frac{z^4}{4^{\frac{5}{2}}} \dots$$

luego

$$\left[ \sum_{l=1}^{\infty} (-1)^{l-1} a_l \left( z - \frac{z^2}{2^{\frac{3}{2}}} + \frac{z^3}{3^{\frac{3}{2}}} - \frac{z^4}{4^{\frac{3}{2}}} \dots \right)^{l-1} \right] \cdot \left[ z - \frac{z^2}{2^{\frac{3}{2}}} + \frac{z^3}{3^{\frac{3}{2}}} - \dots \right]$$
$$= z - \frac{z^2}{2^{\frac{5}{2}}} + \frac{z^3}{3^{\frac{5}{2}}} - \dots$$

Cosechando terminos del mismo orden en  $z$

$$a_1 z = z \Rightarrow a_1 = 1$$

$$\left\{ [a_1] + \left[ -a_2 \left( z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} - \frac{z^4}{4^{3/2}} \dots \right) \right] + \left[ -a_3 \left( z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} - \frac{z^4}{4^{3/2}} \right)^2 \right] - \dots \right\} \bullet$$

$$\bullet \left\{ z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} - \frac{z^4}{4^{3/2}} \dots \right\} = \left\{ z - \frac{z^2}{2^{5/2}} + \frac{z^3}{3^{5/2}} - \frac{z^4}{4^{5/2}} \dots \right\}$$

$\Rightarrow$

$$1) a_1 z = z$$

$$2) -a_1 \frac{z^2}{2^{3/2}} - a_2 z^2 = -\frac{z^2}{2^{5/2}}$$

$$3) a_1 \frac{z^3}{3^{3/2}} + a_2 2 \frac{z^3}{2^{3/2}} - a_3 z^3 = \frac{z^3}{3^{5/2}}$$

$$\left\{ [a_1] + \left[ -a_2 \left( z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} - \frac{z^4}{4^{3/2}} \dots \right) \right] + \left[ -a_3 \left( z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} - \frac{z^4}{4^{3/2}} \right)^2 \right] - \dots \right\} \bullet$$

$$\bullet \left\{ z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} - \frac{z^4}{4^{3/2}} \dots \right\} = \left\{ z - \frac{z^2}{2^{5/2}} + \frac{z^3}{3^{5/2}} - \frac{z^4}{4^{5/2}} \dots \right\}$$

$\Rightarrow$

$$1) a_1 z = z$$

$$2) -a_1 \frac{z^2}{2^{3/2}} - a_2 z^2 = -\frac{z^2}{2^{5/2}}$$

$$3) a_1 \frac{z^3}{3^{3/2}} + a_2 2 \frac{z^3}{2^{3/2}} - a_3 z^3 = \frac{z^3}{3^{5/2}}$$

$$\left\{ [a_1] + \left[ -a_2 \left( z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} - \frac{z^4}{4^{3/2}} \dots \right) \right] + \left[ -a_3 \left( z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} - \frac{z^4}{4^{3/2}} \right)^2 \right] - \dots \right\} \bullet$$

$$\bullet \left\{ z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} - \frac{z^4}{4^{3/2}} \dots \right\} = \left\{ z - \frac{z^2}{2^{5/2}} + \frac{z^3}{3^{5/2}} - \frac{z^4}{4^{5/2}} \dots \right\}$$

$\Rightarrow$

$$1) a_1 z = z$$

$$2) -a_1 \frac{z^2}{2^{3/2}} - a_2 z^2 = -\frac{z^2}{2^{5/2}}$$

$$3) a_1 \frac{z^3}{3^{3/2}} + a_2 2 \frac{z^3}{2^{3/2}} - a_3 z^3 = \frac{z^3}{3^{5/2}}$$

$$a_2 z^2 + a_1 \frac{z^2}{2^{\frac{3}{2}}} = -\frac{z^2}{2^{\frac{5}{2}}} = a_2 z^2 + \frac{z^2}{2^{\frac{3}{2}}} \Rightarrow$$

$$a_2 = -\frac{1}{2^{\frac{3}{2}}} + \frac{1}{2^{\frac{5}{2}}} = -0.17678$$

$$\frac{z^3}{3^{\frac{5}{2}}} = a_1 \frac{z^3}{3^{\frac{3}{2}}} + a_2 2 \frac{z^3}{2^{\frac{3}{2}}} + a_3 z^3 \Rightarrow$$

$$a_3 = -\frac{1}{3^{\frac{3}{2}}} + \frac{1}{3^{\frac{5}{2}}} + 0.17678 \frac{1}{2^{1/2}} = -0.0032977$$

De donde

$$\frac{PV}{NkT} = \sum_{l=1}^{\infty} (-1)^{l-1} a_l \left( \frac{\lambda^3}{g\nu} \right)^{l-1} = 1 + 0.17678 \left( \frac{\lambda^3}{g\nu} \right) - 0.0033 \left( \frac{\lambda^3}{g\nu} \right)^2$$

Como  $P = \frac{2}{3}(U/V) \Rightarrow U = \frac{3}{2}PV \Rightarrow C_V = \left[ \frac{\partial}{\partial T} \frac{3}{2}PV \right]$

Que resulta

$$C_V = \left[ \frac{\partial}{\partial T} \frac{3}{2}PV \right] = \frac{3}{2}Nk \left[ \frac{\partial}{\partial T} \left( T \sum_{l=1}^{\infty} (-1)^l a_l \left( \frac{\lambda^3}{g\nu} \right)^{l-1} \right) \right]$$

entonces con  $\lambda^3 = \left[ \frac{2\pi\hbar^2}{mkT} \right]^{3/2} \Rightarrow$

$$\frac{\partial}{\partial T} \lambda^3 = -\frac{3}{2} \left[ \frac{2\pi\hbar^2}{mk} \right]^{3/2} \left[ \frac{1}{T} \right]^{5/2} = -\frac{3}{2} \frac{1}{T} \lambda^3$$

$$C_V = \frac{3}{2}Nk \left[ \sum_{l=1}^{\infty} (-1)^l a_l \left( \frac{\lambda^3}{g\nu} \right)^{l-1} - \frac{3}{2} \sum_{l=1}^{\infty} (-1)^l a_l (l-1) \left( \frac{\lambda^3}{g\nu} \right)^{l-1} \right]$$

Entonces



$$\frac{C_V}{Nk} = \frac{3}{2} \left[ \sum_{l=1}^{\infty} (-1)^l a_l \left( \frac{\lambda^3}{g\nu} \right)^{l-1} + \frac{3}{2} \sum_{l=1}^{\infty} (-1)^l a_l (-l+1) \left( \frac{\lambda^3}{g\nu} \right)^{l-1} \right]$$

$$\frac{C_V}{Nk} = \frac{3}{2} \left[ \sum_{l=1}^{\infty} (-1)^l a_l \left( \frac{\lambda^3}{g\nu} \right)^{l-1} \left( \frac{5}{2} - \frac{3}{2}l \right) \right]$$

$$\frac{C_V}{Nk} = \frac{3}{2} \left[ 1 - (-0.17678) \left( \frac{5}{2} - \frac{3}{2} \cdot 2 \right) \left( \frac{\lambda^3}{g\nu} \right) - 2(-0.0033) \left( \frac{\lambda^3}{g\nu} \right)^2 \right]$$

$$\frac{C_V}{Nk} = \frac{3}{2} \left[ 1 - 0.08839 \left( \frac{\lambda^3}{g\nu} \right) + 0.0066 \left( \frac{\lambda^3}{g\nu} \right)^2 + \dots \right]$$

Pero  $\lambda$  va como  $(1/T^{1/2})$

## Aproximacion de gas de Fermi para estudiar un nucleo

Haremos la aproximacion de particulas independientes

Para cada particula

$$-\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} \Psi + \frac{\partial^2}{\partial y^2} \Psi + \frac{\partial^2}{\partial z^2} \Psi \right) = E\Psi$$

Condiciones de contorno de Dirichlet

→  $\Psi(x,y,z) = 0$  , en toda la frontera

Se puede escribir

$$\Psi(x,y,z) = X(x)Y(y)Z(z)$$

Con

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial i^2} \Psi_i(i) = E_i \Psi_i(i)$$

donde  $\Psi_i(i) = X(x), Y(y), \dots$

Para la energía

$$E = E_x + E_y + E_z$$

Supongamos un cubo de lado  $a$

Las condiciones de contorno son:  $X(x) = 0$  para  $x \leq a$  y  
 $x \geq a$

Resolviendo

$$E_x = \frac{\pi^2 \hbar^2 n_x^2}{2ma^2}$$

con  $n_x = 1, 2, 3 \dots$  (condiciones de contorno)

luego

$$E = \frac{\pi^2 \hbar^2}{2ma^2} [n_x^2 + n_y^2 + n_z^2]$$

Calculamos entonces el numero de estados  $N(E)dE$

Tenemos un grid 3dimensional y calculamos la cantidad de puntos entre dos cascaras esfericas entre  $r$  y  $r + dr$

$$\text{Escribimos } E = \frac{\pi^2 \hbar^2}{2ma^2} [r^2]$$

Como  $n_i$  son positivos  $\Rightarrow$  octante  $\Rightarrow$  como la densidad de puntos es 1 (un punto por cubo de dimensiones 1X1X1)

$$N(r)dr = \frac{1}{8} 4\pi r^2 dr = \frac{1}{2} \pi r^2 dr$$

como  $dE = \frac{\pi^2 \hbar^2}{2ma^2} 2r dr$ , entonces

$$N(E)dE = \frac{m^{3/2} V}{2^{1/2} \pi^2 \hbar^3} E^{1/2} dE$$

Si tomamos un nucleo (en esta aproximacion) podemos

calcular el nivel de Fermi

Tomemos en cuenta el spin unicamente

$$N = 2 \int_0^{e_f} N(E) dE$$

$$\begin{aligned} N &= 2 \int_0^{e_f} \frac{m^{3/2} V}{2^{1/2} \pi^2 \hbar^3} E^{1/2} dE = 2^{1/2} \frac{m^{3/2} V}{\pi^2 \hbar^3} \int_0^{e_f} E^{1/2} dE \\ &= 2^{1/2} \frac{m^{3/2} V}{\pi^2 \hbar^3} \frac{2}{3} E_f^{3/2} = 2^{3/2} \frac{m^{3/2} V}{3 \pi^2 \hbar^3} E_f^{3/2} \end{aligned}$$

de donde

$$E_f = \left( \frac{N}{V} \right)^{2/3} \frac{\hbar^2}{2m} 3^{2/3} \pi^{4/3} = \rho^{2/3} \frac{\hbar^2}{2m} 3^{2/3} \pi^{4/3}$$

Si lo aplicamos a un "nucleo"

Pensamos en un sistema esferico de radio  $R = r_0 A^{1/3}$

$$r_0 = 1.25 \cdot 10^{-13} \text{ cm}$$

Para un nucleo pesado ( $N \simeq 0.6A$ )

Entonces  $\rho = \frac{0.6A}{\frac{4}{3}\pi r_0^3 A} = \frac{0.45}{\pi r_0^3}$  de donde  $E_f \simeq 34 \text{ MeV}$

( $1 \text{ eV} \approx 11600 \text{ K}$  )

La energia de union media del ultimo neutron es  
 $\approx 6 \text{ MeV} = E_b \Rightarrow$

La profundidad del pozo es  $E_f + E_b \simeq 40 \text{ MeV}$

(en realidad es  $\approx 50 \text{ MeV}$  )

## Respecto del termino de simetria

El nucleo esta compuesto por protones y neutrones

Lo pensamos con 2 "gases" independientes

Para cada uno de ellos tendremos un nivel de Fermi

$$E_{f_n} \propto \left(\frac{N}{A}\right)^{2/3}$$

$$E_{f_p} \propto \left(\frac{Z}{A}\right)^{2/3}$$

Faltaria ver como son los pozos para cada especie (efecto de Coulomb para protones)

Ademas

$$\bar{E} = \frac{3}{5}E_f$$

$$E_t = \frac{3}{5}NE_f$$

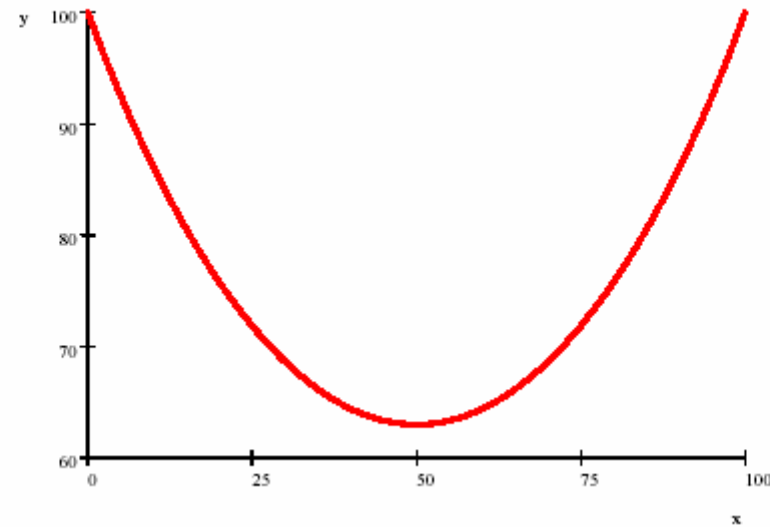
Si los pozos fuesen iguales

$$E(Z,A) \propto A^{-2/3}[N^{5/3} + Z^{5/3}] \propto A^{-2/3}[N^{5/3} + (A - N)^{5/3}]$$



sea  $A = 100$

$$100^{-2/3} (x^{5/3} + (100 - x)^{5/3})$$



Luego le gusta la simetria.

Luego podemos escribir el apartamiento del minimo

$$f_4(Z, A) \propto E(Z, A) - E(Z, A)_{\min} = C' A^{-2/3} \left\{ N^{5/3} + Z^{5/3} - 2 \left( \frac{A}{2} \right)^{5/3} \right\}$$

Lo cual se puede reescribir como:

sea

$$D = \frac{N-Z}{2} = N - \frac{A}{2} = \frac{A}{2} - Z$$

entonces

$$f_4(Z, A) \propto A^{-2/3} \left\{ \left[ \frac{A}{2} + D \right]^{\frac{5}{3}} + \left[ \frac{A}{2} - D \right]^{\frac{5}{3}} - 2 \left( \frac{A}{2} \right)^{\frac{5}{3}} \right\}$$

Para cada termino

$$\begin{aligned} \left[ \frac{A}{2} + D \right]^{\frac{5}{3}} &\simeq \left[ \frac{A}{2} \right]^{\frac{5}{3}} + \frac{5}{3} D \left[ \frac{A}{2} \right]^{\frac{2}{3}} + \frac{5}{3} \frac{2}{3} \frac{D^2}{2} \left[ \frac{A}{2} \right]^{\frac{-1}{3}} \\ \left[ \frac{A}{2} - D \right]^{\frac{5}{3}} &\simeq \left[ \frac{A}{2} \right]^{\frac{5}{3}} - \frac{5}{3} D \left[ \frac{A}{2} \right]^{\frac{2}{3}} + \frac{5}{3} \frac{2}{3} \frac{D^2}{2} \left[ \frac{A}{2} \right]^{\frac{-1}{3}} \end{aligned}$$

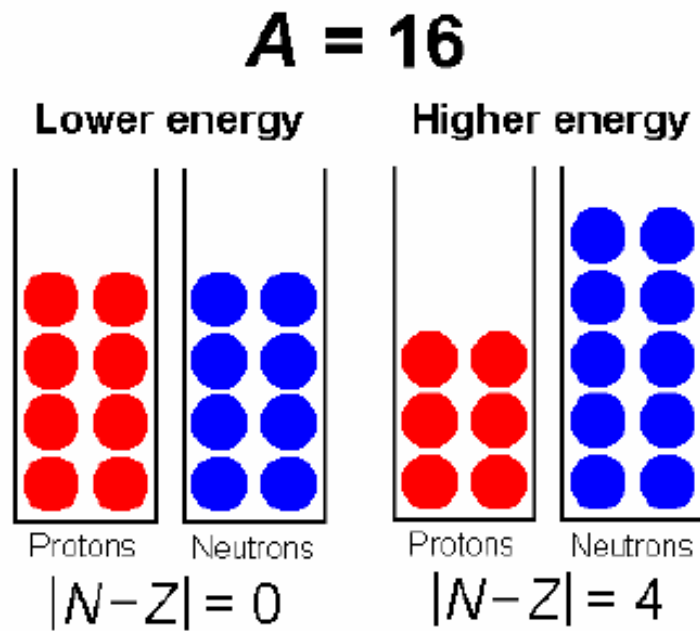
$$f_4(Z, A) \propto A^{-2/3} \left\{ 2 \left[ \frac{A}{2} \right]^{\frac{5}{3}} + \frac{5}{3} \frac{2}{3} D^2 \left[ \frac{A}{2} \right]^{\frac{-1}{3}} - 2 \left( \frac{A}{2} \right)^{\frac{5}{3}} \right\}$$

$$f_4(Z, A) \propto A^{-1} \frac{5}{3} \frac{2}{3} D^2 \left[ \frac{1}{2} \right]^{-1/3} \propto A^{-1} D^2 = \frac{\left( \frac{A}{2} - Z \right)^2}{A}$$

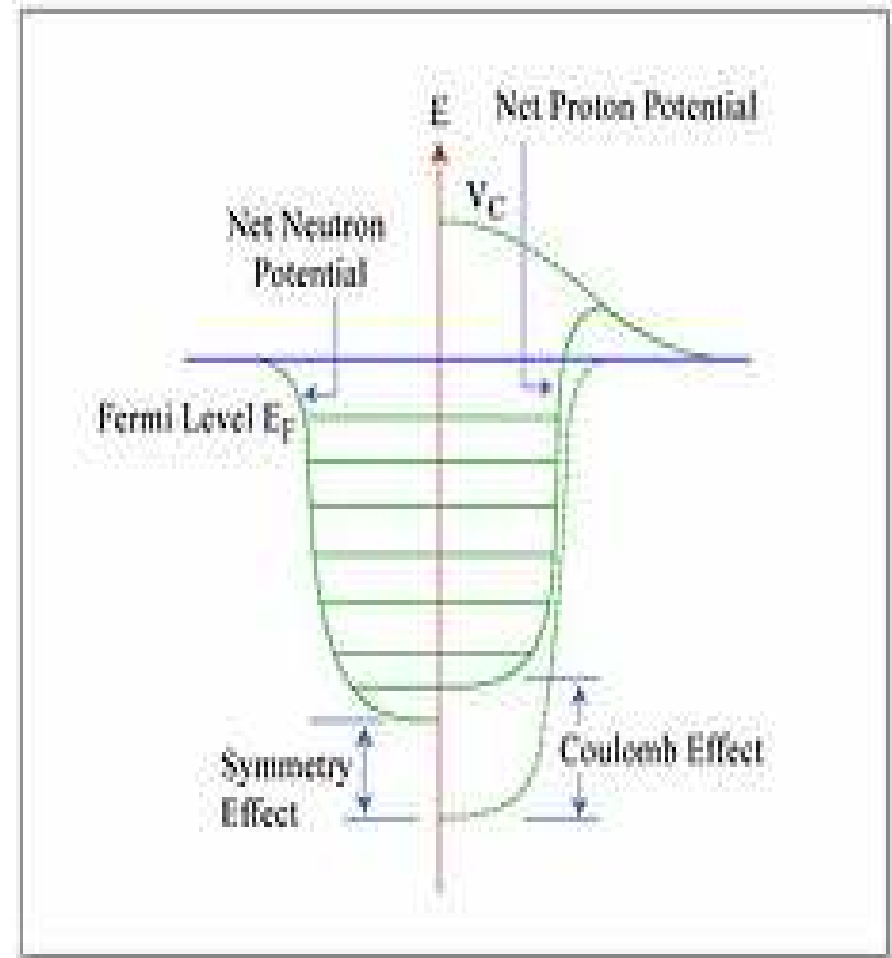
Sin embargo ademas hay problemas con las "profundidades parciales"

Porque  $f_4$ ?

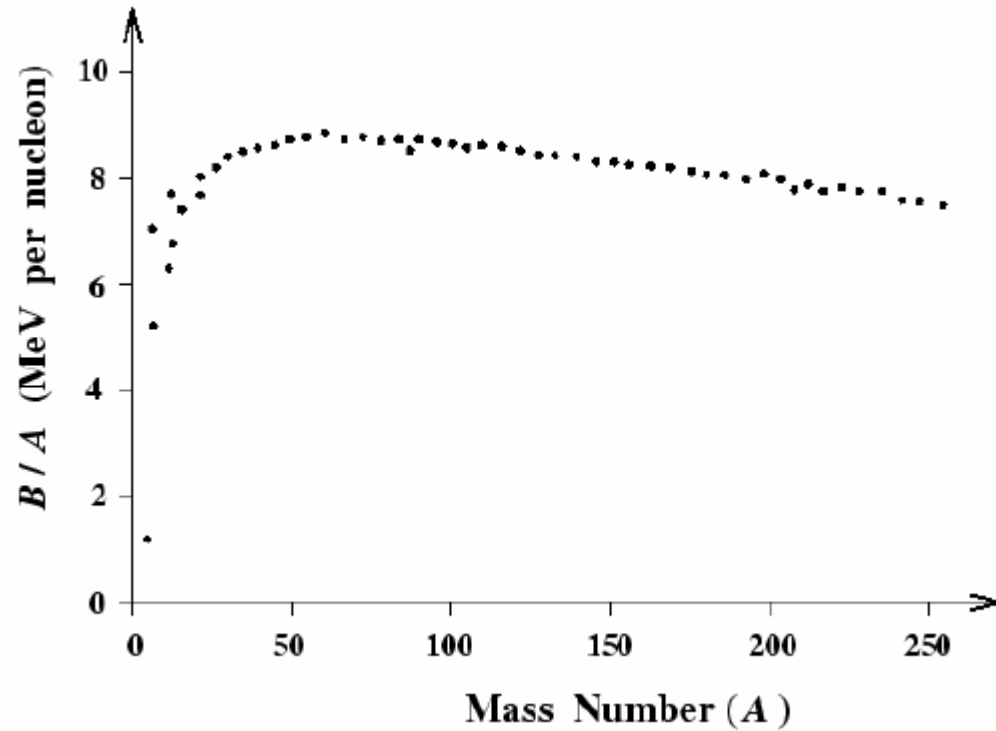
# Simetria



# Efecto Coulomb



Cuando se estudia la energía de union de los nucleos el comportamiento es :



**Lo que se describe mediante:**

# Formula semiempirica de masas

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A, Z)$$

Termino de volumen  
asociado a una fuerza  
de rango corto

Termino de  
Coulomb

$$\frac{3}{5} \frac{1}{4\pi\epsilon_0} \frac{Q^2}{R}$$

Termino de  
Simetria ( $f_4$ )

$$\delta(A, Z) = \begin{cases} +\delta_0 & Z, N \text{ even (A even)} \\ 0 & A \text{ odd} \\ -\delta_0 & Z, N \text{ odd (A even)} \end{cases}$$

Termino de superficie

here

For most nuclei with  $A > 20$  this simple formula does a very good job of determining the binding energies - usually better than 0.5%.

For example we estimate the binding energy per nucleon of  $^{80}_{35}\text{Br}$  (Bromine), for which  $Z=35$ ,  $A=80$  ( $N = 80 - 35 = 45$ ) and insert into the above formulae to get

$$\begin{aligned}
 \text{Volume term:} & \quad (15.56 \times 80) = 1244.8 \text{ MeV} \\
 \text{Surface term:} & \quad (-17.23 \times (80)^{2/3}) = -319.9 \text{ MeV} \\
 \text{Coulomb term:} & \quad \left( \frac{0.697 \times 35^2}{(80)^{1/3}} \right) = -198.4 \text{ MeV} \\
 \text{Asymmetry term:} & \quad \left( \frac{23.285 \times (45 - 35)^2}{80} \right) = -29.1 \text{ MeV} \\
 \text{Pairing term:} & \quad \left( \frac{-12.0}{(80)^{1/2}} \right) = -1.3 \text{ MeV}
 \end{aligned}$$

Note that we *subtract* the pairing term since both  $(A-Z)$  and  $Z$  are odd. This gives a total binding energy of 696.1 MeV. The measured value is 694.2 MeV.

# Energia de Union

- Termino de Volumen

$$R = r_0 A^{1/3}$$

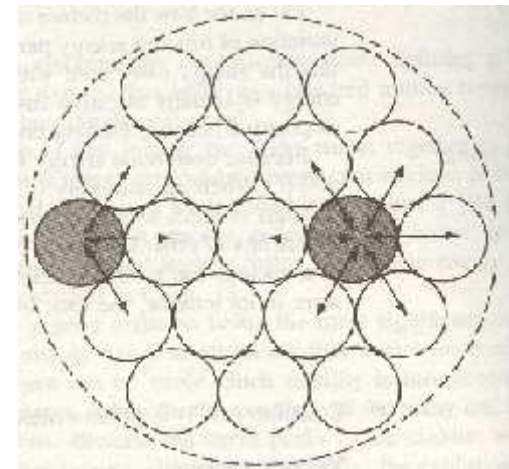
$$B_V = a_V A \quad \text{volumen} = (4/3)\pi R^3 = (4/3)\pi r_0^3 A$$

- Termino de superficie

- Nucleones en la superficie estan menos unidos

$$4\pi R^2 = 4\pi r_0^2 A^{2/3}$$

$$B_S = -a_S A^{2/3}$$





# Energia de Union

- Coulomb
- Para un nucleo de carga Ze,  $B_c$  es:

$$B_c = \frac{1}{2} \int_0^R \rho V(r) d^3r, \quad B_c = q\phi$$

$$= \frac{1}{2} \int_0^R \rho \frac{2}{3} \pi \rho (3R^2 - r^2) 4\pi r^2 dr = \frac{1}{3} \pi \rho^2 4\pi \int_0^R (3R^2 - r^2) r^2 dr$$

$$= \frac{4}{3} \pi^2 \frac{(Ze)^2}{\left(\frac{4}{3} \pi R^3\right)^2} \frac{4}{5} R^5 = \frac{3}{5} \frac{(Ze)^2}{R}$$

$$B_c = \frac{3}{5} \frac{(Ze)^2}{R} = \frac{3}{5} \frac{(Ze)(Ze)}{R} \equiv \frac{3}{5} e^2 \frac{Z(Z-1)}{R} = \frac{3}{5} e^2 \frac{Z(Z-1)}{r_0 A^{1/3}} \quad (\text{for } Z=1, B_c = 0)$$

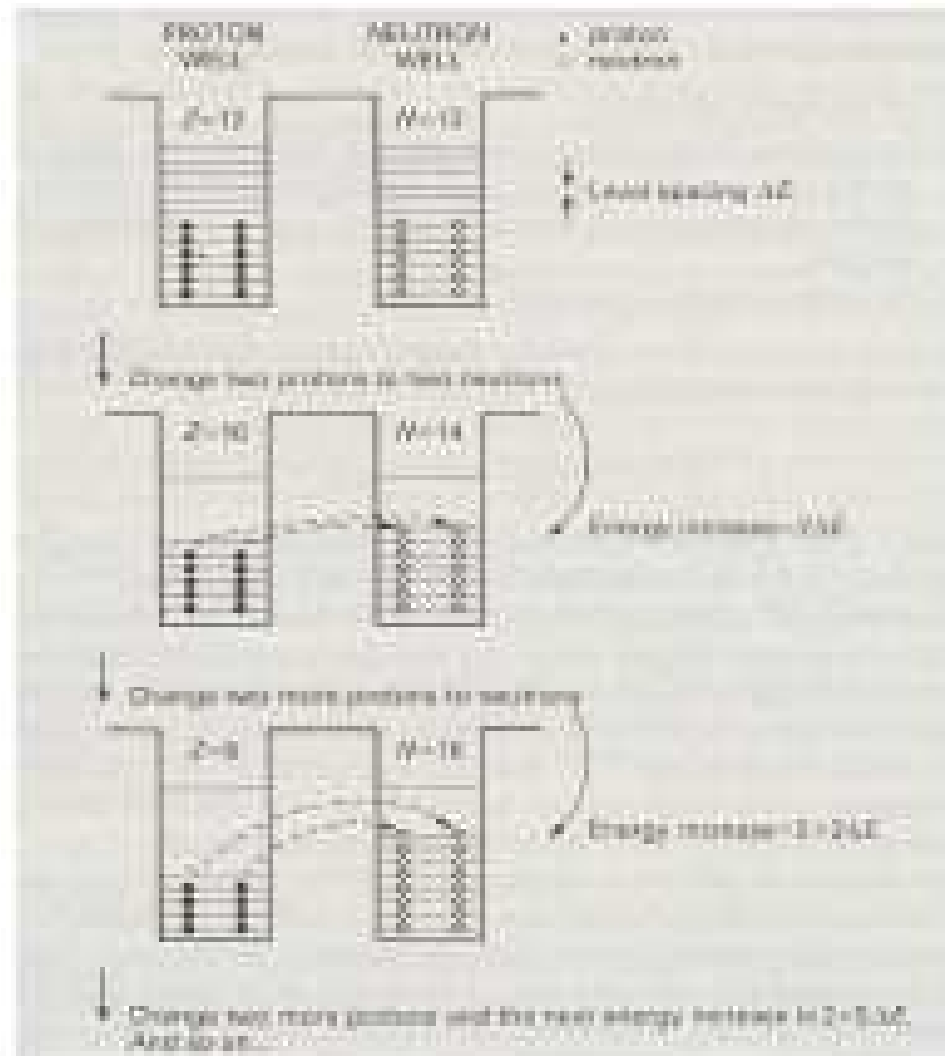
$$= -a_c \frac{Z^2}{A^{1/3}}$$

Pero la aproximacion es muy burda y  $a_c$  debe ser det. Exp.

# Energia de Union

## Symmetry Term

$$B_{\text{sym}} = -a_{\text{sym}} \frac{(Z - N)^2}{A}$$



## De las estrellas



All stars, as far as we know, are born from the gravitational collapse of the core of a molecular cloud.

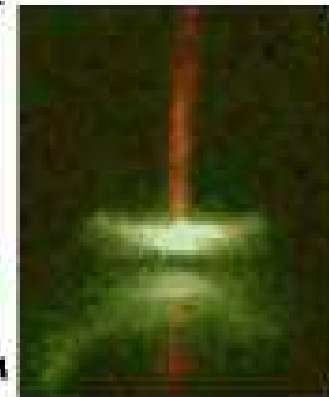
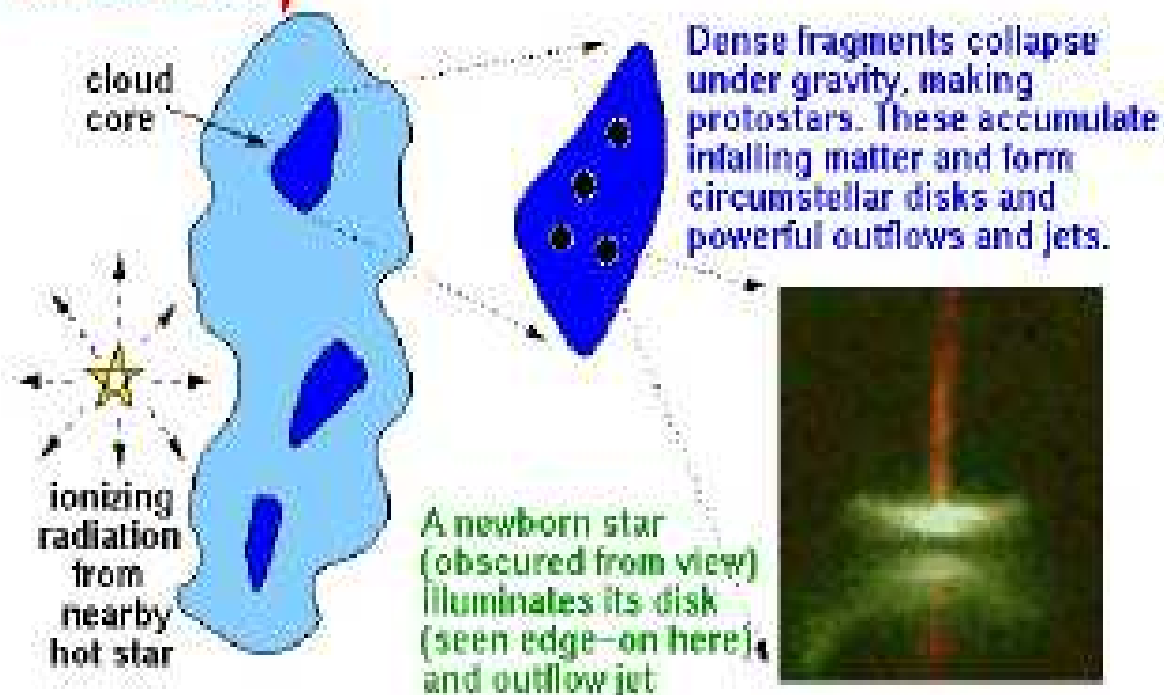


Molecular clouds are cold, dark, giant condensations of dust and molecular gas which serve as "stellar nurseries".

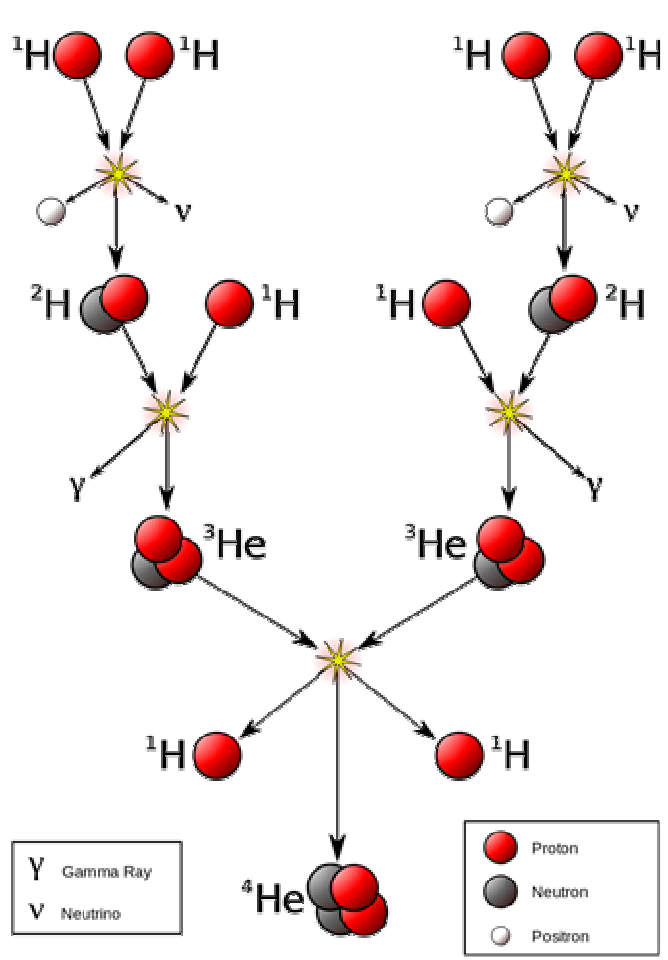
All stars are born in molecular clouds, including our Sun. Molecular clouds are the "stuff" we're made of!

Because of their dusty content, visible light cannot penetrate into a molecular cloud. Thus, infrared and submillimeter observations are needed to "see" the star-forming process.

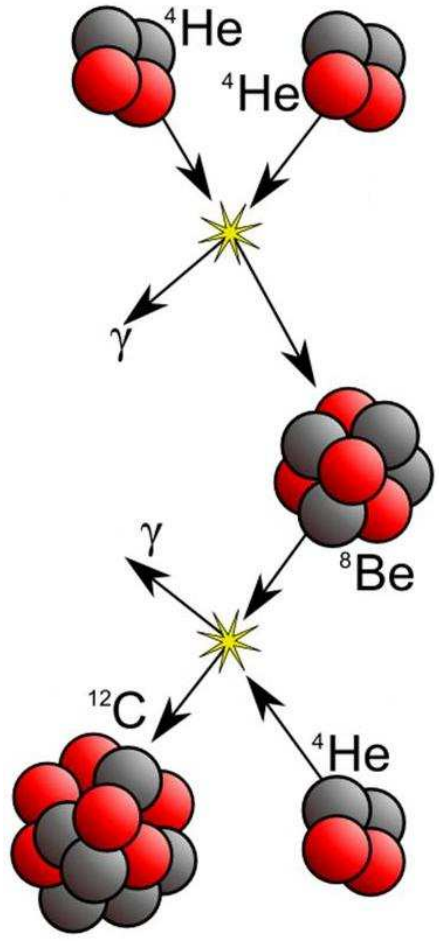
molecular cloud



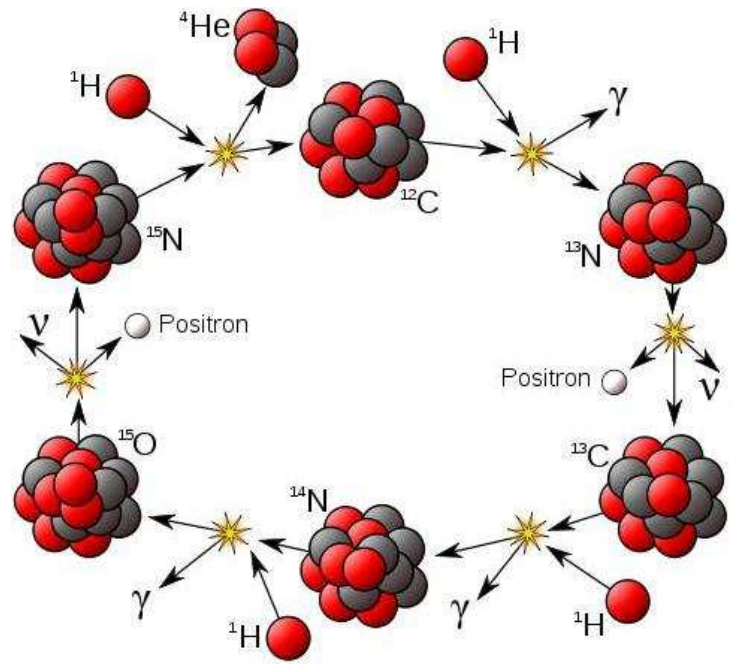
# A Closer Look at Nucleosynthesis



**H to He**



**He to C**



**CNO cycle**

In "normal stars"

# White dwarfs (enanas blancas)

## Algunas unidades

proton mass  $938.3 \text{ MeV}/c^2$  ( $1.6726 \times 10^{-27} \text{ kg}$ ) ( 1836 veces la masa del electron)

densidad nuclear  $2.7 \times 10^{14} \text{ g cm}^{-3} = 0.16 \text{ fm}^{-3}$

## El Sol

Mean diameter  $1.392 \times 10^6 \text{ km}$  (109 Earth diameters)

Circumference  $4.373 \times 10^6 \text{ km}$  (342 Earth diameters)

Surface area  $6.09 \times 10^{12} \text{ km}^2$  (11,900 Earths)

Volume  $1.41 \times 10^{18} \text{ km}^3$  (1,300,000 Earths)

Mass  $1.988435 \times 10^{30} \text{ kg}$  (332,946 Earths)

Density  $1.408 \text{ g/cm}^3$

Surface gravity  $273.95 \text{ ms}^{-2}$  (27.9 g)<sub>0</sub>

# Las Enanas Blancas

Estrellas de luminosidad muy baja

Están compuestas básicamente por Helio

Con una densidad de

$$10^7 \text{ g/cm}^3 \text{ (o sea } 10^7 \rho_{\odot}\text{)}$$

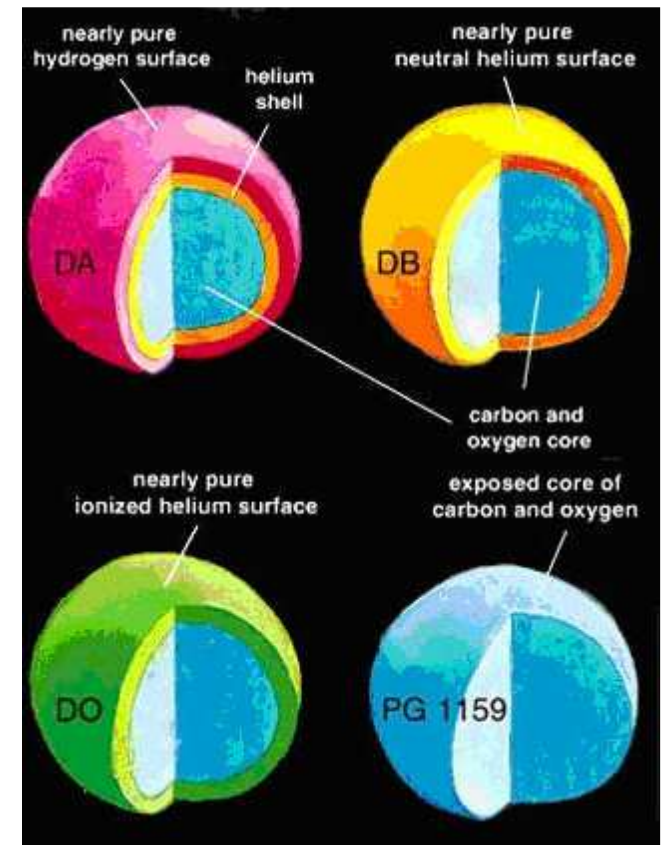
Masa

$$10^{33} \text{ g } (\approx M_{\odot})$$

Temperatura en el centro  
( $\approx 1000 \text{ eV}$ ) ( $\text{eV} \approx 11602 \text{ K}$ )

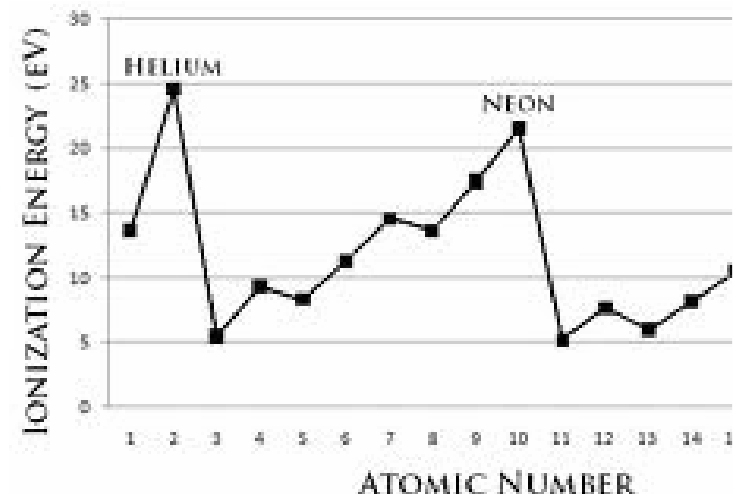
$$10^7 \text{ K } (\approx T_{\odot})$$

$$0.008 R_{\odot} \leq \text{Radio de una Enana} \leq 0.02 R_{\odot}$$



Material	Density in kg/m <sup>3</sup>	Notes
Water (fresh)	1,000	At STP
Osmium	22,610	Near room temperature
The core of the Sun	~150,000	
White dwarf star	$1 \times 10^9$ [1]	
Atomic nuclei	$2.3 \times 10^{17}$ [31]	Does not depend strongly on size of nucleus
Neutron star core	$8.4 \times 10^{16} - 1 \times 10^{18}$	
Black hole	$2 \times 10^{30}$ [32]	Critical density of an Earth-mass black hole

A esta energia los atomos de Helio estan completamente ionizados y por lo tanto tenemos un sopa de nucleos de Helio y electrones libres.





Calculamos las propiedades del "gas de electrones"

Para estos electrones

$$\epsilon_{F_e} = \frac{\hbar^2}{2m_e} \frac{1}{v^{2/3}}$$

$$\hbar = 1.05457 \times 10^{-34} \text{ J s}$$

$$m_e = 9.10938 \times 10^{-31} \text{ kg}$$

$$6.24150636309 \times 10^{12} \text{ MeV}$$

$$\begin{aligned} \epsilon_F &= \frac{(1.05457 \times 10^{-34})^2 \text{ kg}^2 \text{ m}^4 \text{ s}^2}{2 \cdot 9.10938 \times 10^{-31} \text{ kg s}^4} \left( \frac{10^7 \text{ g/cm}^3}{9.10938 \times 10^{-28} \text{ g}} \right)^{2/3} \\ &= \frac{\text{kg m}^4}{\text{m}^2 \text{ s}^2} \frac{(1.05457)^2}{2(9.10938)^{5/3}} \frac{10^4 10^{-68} (10^{35})^{2/3}}{10^{-31}} \end{aligned}$$

$$\frac{10^{-68}(10^7)^{2/3}(10^{28})^{2/3}10^4}{10^{-31}} \frac{(1.05457)^2}{2(9.10938)^{5/3}} = 3.0151 \times 10^{-12} J_s =$$

$$(6.24150636309 \times 10^{12})(0.30151 \times 10^{-11}) MeV =$$

$$\varepsilon_F = 18.819 MeV \sim 2 \times 10^7 eV \sim 10^{12} K.$$

Entonces si la correspondiente Temperatura de Fermi es del orden de  $T_F \approx 10^{12} K$ , la Temperatura de la estrella es  $10^7 K$  es mucho menor y por lo tanto los electrones están en un estado muy degenerado. → gas ideal a temperatura  $\approx 0$ .

## Relativistic energy & momentum

Relacion de Einstein para la energia

$$E = mc^2$$

Que incluye a la energia cinetica y la energia asociada a la mas en reposo

La masa relativista es

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0$$

Momento relativista

$$p = \frac{m_0 v'}{\sqrt{1 - \frac{v'^2}{c^2}}} = \gamma m_0 v' \Rightarrow$$

$$p^2 c^2 = \frac{m_0^2 v'^2 c^2}{1 - \frac{v'^2}{c^2}} = \frac{m_0^2 \frac{v'^2}{c^2} c^4}{1 - \frac{v'^2}{c^2}} + \frac{m_0^2 c^4}{1 - \frac{v'^2}{c^2}} - \frac{m_0^2 c^4}{1 - \frac{v'^2}{c^2}} =$$

$$p^2 c^2 = \frac{m_0^2 \left( \frac{v'^2}{c^2} - 1 \right) c^4}{1 - \frac{v'^2}{c^2}} + \frac{m_0^2 c^4}{1 - \frac{v'^2}{c^2}} =$$

$$p^2 c^2 = -m_0^2 c^4 + m^2 c^4$$

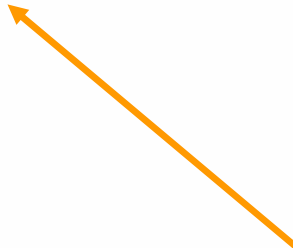
Luego

$$p^2 c^2 = (mc^2)^2 - m_0^2 c^4$$

donde  $m$  es la masa relativista y  $m_0$  es la masa en reposo

$$E = \sqrt{p^2 c^2 + m_0^2 c^4}$$

La energia cinetica relativista es



$$\begin{aligned} E_K &= mc^2 - m_0c^2 \\ &= m_0c^2(\gamma - 1) \\ &= m_0c^2 \left( \frac{1}{\sqrt{1 - \frac{v'^2}{c^2}}} - 1 \right) \end{aligned}$$

desarrollando

$$\begin{aligned} E_K &= m_0c^2 \left( \frac{1}{2} \frac{v'^2}{c^2} + \frac{3}{8} \frac{v'^4}{c^4} + \dots \right) = \\ &= m_0 \frac{1}{2} v'^2 + \frac{3}{8} m_0 \frac{v'^4}{c^2} + \dots \end{aligned}$$

Si  $v' \ll c \dots$

## fotón

Para el fotón la expresión  $p = \frac{m_0 v'}{\sqrt{1 - \frac{v'^2}{c^2}}} = \gamma m_0 v'$  tiene problemas,

entonces usamos

$$E = \sqrt{p^2 c^2 + m_0^2 c^4}$$

Que con  $m_0 = 0 \Rightarrow$

fotón

$$E = pc \Rightarrow$$

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$



## Mas propiedades del gas de electrones

$$m_e = 9.10938 \times 10^{-31} \text{ kg}$$

$$c = 2.9979 \times 10^8 \text{ m/s}$$

de donde

$$m_e c \approx 2.7 \times 10^{-23} \text{ kgm/s}$$

mientras que

$$p_F = \left( \frac{3N}{4\pi gV} \right)^{1/3} h \approx 1.05457 \times 10^{-34} \text{ Js} \left( \frac{3}{8\pi} \right)^{1/3} \left( \frac{N}{V} \right)^{1/3}$$

→

con

$$\left( \frac{N}{V} \right)^{1/3} = \left( \frac{10^7 \text{ g/cm}^3}{9.10938 \times 10^{-28} \text{ g}} \right)^{1/3} \approx (10^{34} / \text{cm}^3)^{1/3} \approx 10^{34/3} \frac{1}{\text{cm}} = 10^{40/3} \frac{1}{\text{m}}$$

$$\left( \frac{3}{8\pi} \right)^{1/3} = 0.49237$$

resulta

$$p_F \approx 10^{-22} \text{ kgm/s}$$

$$m_e c \approx 2.7 \times 10^{-23} \text{ kgm/s}$$

$$p_F \approx 10^{-22} \text{ kgm/s}$$

como son comparables resulta que el gas lo podemos considerar relativistico

De esta forma el sistema es : un gas de  $N$  electrones en el estado fundamental, a muy alta densidad (los tratamos relativisticamente) moviendose en un fondo neutralizador de  $N/2$  nucleos de Helio. Son estos los que "proveen" el campo gravitatorio que balancea la presion del gas de Fermi.

# Un par de aproximaciones

Sea la densidad de electrones

electrones por nucleon

densidad central

$$n_e = Y_e \frac{\rho_c}{m_H}$$

Sea un gas de electrones no relativista degenerado

$$P = K_{NR} n^{5/3} = \frac{h^2}{5m} \left[ \frac{3}{8\pi} \right]^{2/3} n^{5/3}$$

Cual es la presion necesaria para mantener a la Estrella ?


### Contraccion gravitatoria

Sea una masa:

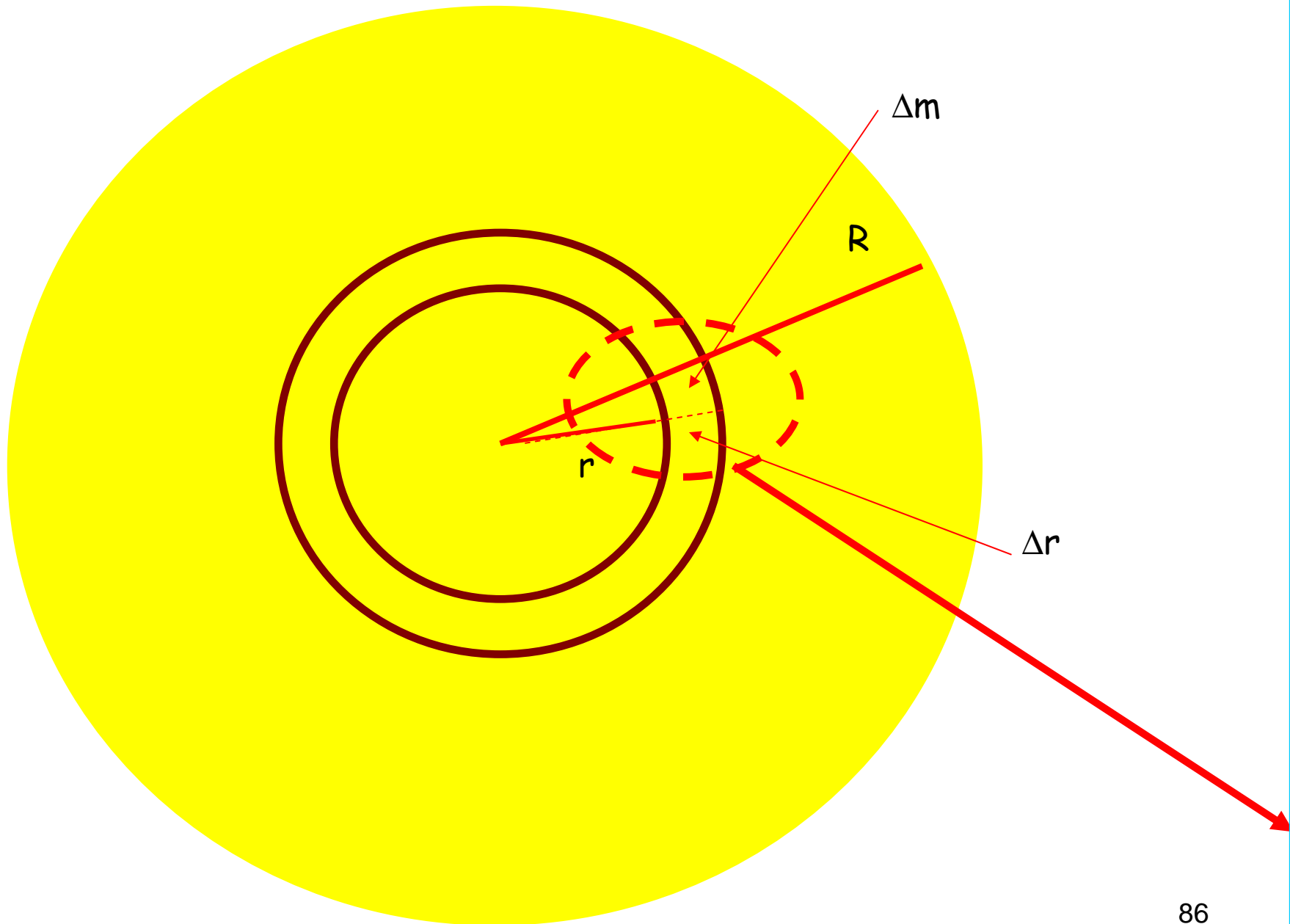
$$m(r) = \int_0^r \rho(r') 4\pi r'^2 dr'$$

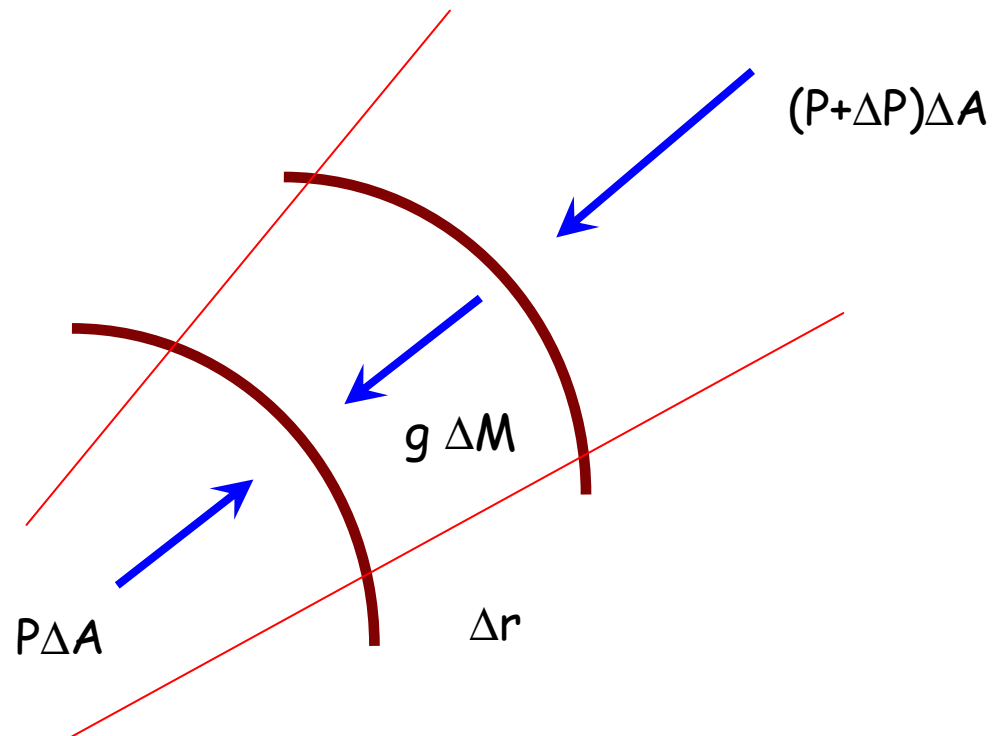
Sobre la cual actua una aceleracion gravitatoria

$$g(r) = \frac{Gm(r)}{r^2}$$

Para un elemento de volumen 

# Simple sistema y el equilibrio hidrostático





Para un elemento de volumen

$$\left[ P(r) + \frac{dP(r)}{dr} \Delta r - P(r) \right] \Delta A = \frac{dP(r)}{dr} \Delta r \Delta A$$

la aceleración del elemento de volumen es

$$\Delta M = \rho(r) \Delta r \Delta A$$

$$-\frac{d^2 r}{dt^2} = g(r) + \frac{1}{\rho(r)} \frac{dP}{dr}$$

En equilibrio hidrostático  $-\frac{d^2 r}{dt^2} = 0$ , entonces



$$\frac{dP}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$

Para un equilibrio global

$$\int_0^R \frac{4\pi r^3}{3} \frac{dP}{dr} dr = -\int_0^R \frac{4\pi r^3}{3} \frac{Gm(r)\rho(r)}{r^2} dr$$
$$\int_0^R 4\pi r^3 \frac{dP}{dr} dr = -\int_0^R 4\pi r^2 \frac{Gm(r)\rho(r)}{r} dr$$

El termino de la derecha es la energia potencial gravitatoria del sistema

$$-\int_{m=0}^{M=M} \frac{Gm(r)}{r} dm = E_{GR}$$

El termino de la izquierda es (integrando por partes)

$$\int_0^R 4\pi r^3 \frac{dP}{dr} dr = [P(r)4\pi r^3]_0^R - 3 \int_0^R P(r)4\pi r^2 dr = 0 - 3\langle P \rangle V$$

De donde

$$\langle P \rangle = -\frac{1}{3} \frac{E_{GR}}{V}$$

- Si estamos a baixa densidad. los electrones e iones forman un gas clasico y

$$P = nKT$$

- A altas densidades los electrones forman un gas cuantico degenerado

$$\varepsilon_p = mc^2 + p^2/2m$$

con una presion dada por

$$P = \frac{h^2}{5m} \left[ \frac{3}{8\pi} \right]^{2/3} n_e^{5/3} = n \frac{p_F^3}{5m}$$

En este caso

$$\left[ \frac{3n}{8\pi} \right]^{1/3} h = p_F \ll mc$$

Como  $p_F = \left[ \frac{3n}{8\pi} \right]^{1/3} h$ . lo anterior es equivalente a

$$n \ll \left( \frac{mc}{h} \right)^3$$

-Si las densidades son muy altas

$$n \gg \left( \frac{mc}{h} \right)^3$$

el gas se hace ultrarelativista entonces

$$\varepsilon_p = pc$$

de donde

$$E = \int_0^{p_F} pc g_s \frac{V}{h^3} 4\pi p^2 dp = N \frac{3}{4} p_F c$$


$$N = \frac{8\pi V}{h^3} p_F^3$$

y la presión se aproxima a

$$P = \frac{hc}{4} \left[ \frac{3}{8\pi} \right]^{1/3} n_e^{4/3}$$

## Presion ejercida por el gas de electrones

Los estados del gas son (tomar en cuenta que  $s = \pm \frac{1}{2}$  )

$$\epsilon_{ps} = \left[ (pc)^2 + (m_e c^2)^2 \right]^{1/2}$$


El momento de Fermi es

$$\frac{2V}{h^3} \left( \frac{4}{3} \pi p_F^3 \right) = N = \frac{8\pi V}{h^3} \int_0^{p_f} p^2 dp$$

de donde

$$p_F = \hbar \left( \frac{3\pi^2}{v} \right)^{1/3} = \left( \frac{3}{8\pi v} \right)^{1/3} h$$

La energia del estado fundamental es

$$E_0 = 2 \sum_{|\mathbf{p}| \leq p_F} \left[ (pc)^2 + (m_e c^2)^2 \right]^{1/2}$$
$$= \frac{2V}{h^3} \int_0^{p_F} dp 4\pi p^2 \left[ (pc)^2 + (m_e c^2)^2 \right]^{1/2}$$

Si calculamos la presion

$$P = \frac{1}{3} \frac{N}{V} \left\langle p \frac{d\epsilon}{dp} \right\rangle$$

$$P = \frac{1}{3} \frac{N}{V} \left\langle p \frac{d\epsilon}{dp} \right\rangle$$

$$\text{con } \epsilon = \left[ (pc)^2 + (m_e c^2)^2 \right]^{1/2} = m_e c^2 \left\{ 1 + \left[ \frac{p}{m_e c} \right]^2 \right\}^{1/2} \Rightarrow$$

$$v = \frac{d\epsilon}{dp} = \frac{1}{2} \frac{2m_e c^2 p / m_e^2 c^2}{\left\{ 1 + \left[ \frac{p}{m_e c} \right]^2 \right\}^{1/2}} = \frac{p/m_e}{\left\{ 1 + \left[ \frac{p}{m_e c} \right]^2 \right\}^{1/2}}$$

De esta forma

$$P_0 = \frac{8\pi}{3h^3} \int_0^{p_F} \frac{p/m_e}{\left\{ 1 + \left[ \frac{p}{m_e c} \right]^2 \right\}^{1/2}} p p^2 dp$$



Se define  $p = mc(\sinh\theta)$   
con  $\sinh\theta = (e^x - e^{-x})/2$   
con  $\cosh\theta = (e^x + e^{-x})/2$   
 $dp = mc(\cosh\theta) d\theta$   
que cumplen  $\cosh^2\theta - \sinh^2\theta = 1$



Entonces la integral es

$$P_0 = \frac{8\pi}{3h^3} \int_0^{p_F} \frac{m^4 c^5 \sinh^4\theta}{\{\cosh^2\theta\}^{1/2}} \cosh\theta d\theta = \frac{8\pi}{3h^3} m^4 c^5 \int_0^{p_F} \sinh^4\theta d\theta$$

Que resulta dar

$$\begin{aligned} P_0 &= \frac{\pi m^4 c^5}{3h^3} A(x) = \frac{\pi m^4 c^5}{3h^3} [x(x^2 + 1)^{1/2}(2x^2 - 3) + 3 \sinh^{-1} x] = \\ &= ((\pi m^4 c^5)/(3h^3)) \cdot A(x) \end{aligned}$$

Donde

$$x = \sinh \theta_F = \frac{p_F}{mc}$$

Se puede considerar el siguiente proceso :

Cuando se equilibra este sistema

Como sabemos, el gas de fermi necesita "paredes" para estar confinado

En este caso tenemos el potencial gravitatorio!

Sea un contenedor esferico que realiza un cambio de volumen  $V$  (si  $dV > 0 \Rightarrow \Delta E < 0$ )  $\rightarrow$

$$dE_0 = -P_0\left(\frac{1}{V}\right)dV = -P_0(R)4\pi R^2 dR$$

Que pasa si esto ocurre para un sistema que se sostiene confinado por accion de la gravedad?

Haciendo varias suposiciones (el sistema no es homogeneo, etc) calculamos el cambio de energia potencial gravitatoria debido a un cambio de volumen como

$$dE_g = \frac{dE_g}{dR} dR = \alpha \frac{GM^2}{R^2} dR$$

$\alpha$  aparece para tener en cuenta efectos de no homogeneidad.

Para un sistema en equilibrio la energía total se mantiene constante entonces  $dE_0 = -dE_g$  dando

$$P_0(R)4\pi R^2 = \alpha \frac{GM^2}{R^2} \Rightarrow P_0(R) = \alpha \frac{GM^2}{4\pi R^4}$$

 Como la masa la concentran los  $H_e$  (núcleos)

$$M = N(m_e + 2m_p) \approx 2Nm_p$$

Utilizando la relacion para  $P_0$  con  $A(x) = A\left(\left[\frac{9\pi M}{8m_p}\right]^{1/3} \frac{\hbar/mc}{R}\right)$

se obtiene

$$A\left(\left[\frac{9\pi M}{8m_p}\right]^{1/3} \frac{\hbar/mc}{R}\right) = 6\pi\alpha\left(\frac{\hbar/mc}{R}\right)^3 \frac{GM^2/R}{mc^2}$$

Queda una relacion implicita (ver forma de  $A$ )

Como hay formas asintoticas para  $x \gg 1$  y  $x \ll 1$ , o sea:

$$P_0 = \frac{\pi m^4 c^5}{3h^3} A(x) = \frac{\pi m^4 c^5}{3h^3} [x(x^2 + 1)^{1/2}(2x^2 - 3) + 3 \sinh^{-1} x]$$

$$\begin{array}{l}
 [p_F/(mc)] \gg 1 \longrightarrow R \ll 10^8 \text{ cm} \\
 \phantom{[p_F/(mc)]} 0 \\
 [p_F/(mc)] \ll 1 \longrightarrow R \gg 10^8 \text{ cm}
 \end{array}$$

Se puede demostrar que:

$x_F = [p_F/(mc)] \gg 1$  corresponde al limite ultrarelativista

$x_F \ll 1$  corresponde al limite no relativista

$$\begin{array}{ccc}
 [p_F/(mc)] \gg 1 & \longrightarrow & R \ll 10^8 \text{ cm} \\
 0 & & \\
 [p_F/(mc)] \ll 1 & \longrightarrow & R \gg 10^8 \text{ cm}
 \end{array}$$

para  $R \gg 10^8 \text{ cm}$  y  $R \ll 10^8 \text{ cm}$  para las cuales se obtiene

$$R \gg 10^8 \text{ cm} \Rightarrow R \propto M^{-1/3}$$

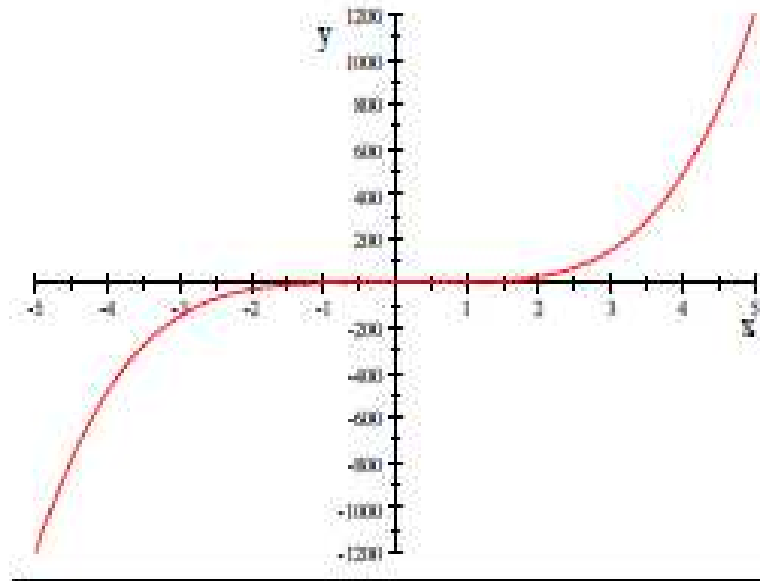
$$R \ll 10^8 \text{ cm} \Rightarrow R \approx \frac{\hbar}{mc} \left[ \frac{9\pi}{8} \frac{M}{m_p} \right]^{1/3} \left[ 1 - \left( \frac{M}{M_0} \right)^{2/3} \right]^{1/2}$$

con

$$M_0 = \frac{9}{64} (3\pi/\alpha^3)^{1/2} \frac{(\hbar c/G)^{3/2}}{m_p^2}$$

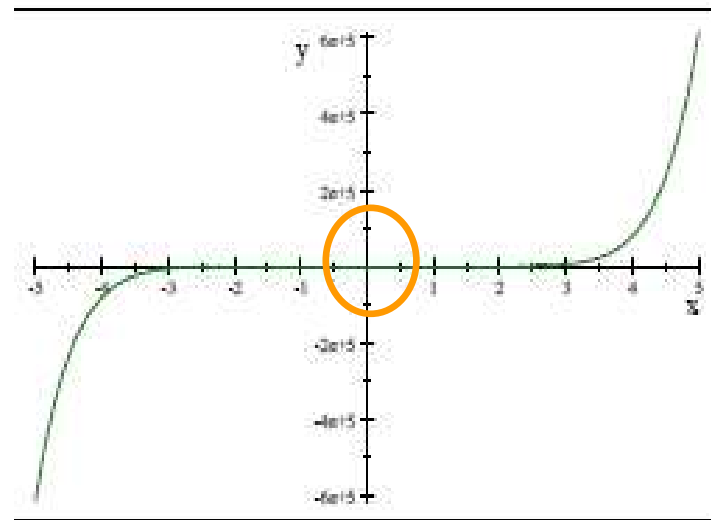
$A(x)$

$$x(x^2 + 1)^{1/3}(2x^2 - 3) + 3 \ln(x + \sqrt{1 + x^2})$$



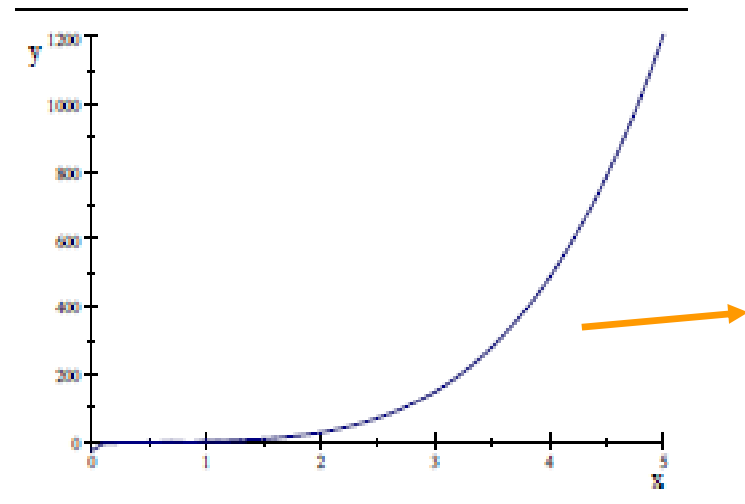
$X \ll 1$

$$\frac{1}{3}x^3 - \frac{4}{9}x^5 + \frac{1}{3}x^9$$



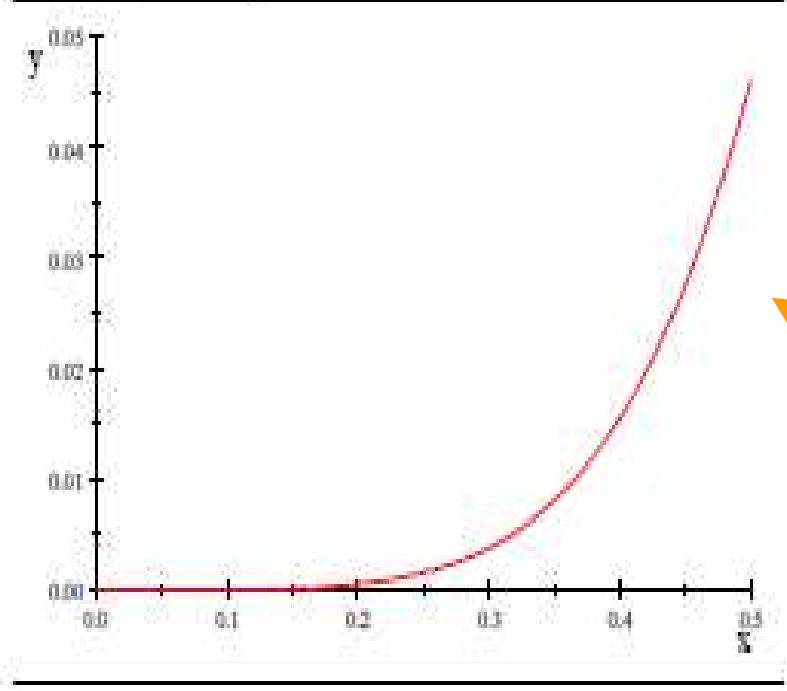
$X \gg 1$

$$2x^4 - 2x^2 + 3(\ln(2x) - \frac{1}{2x})$$



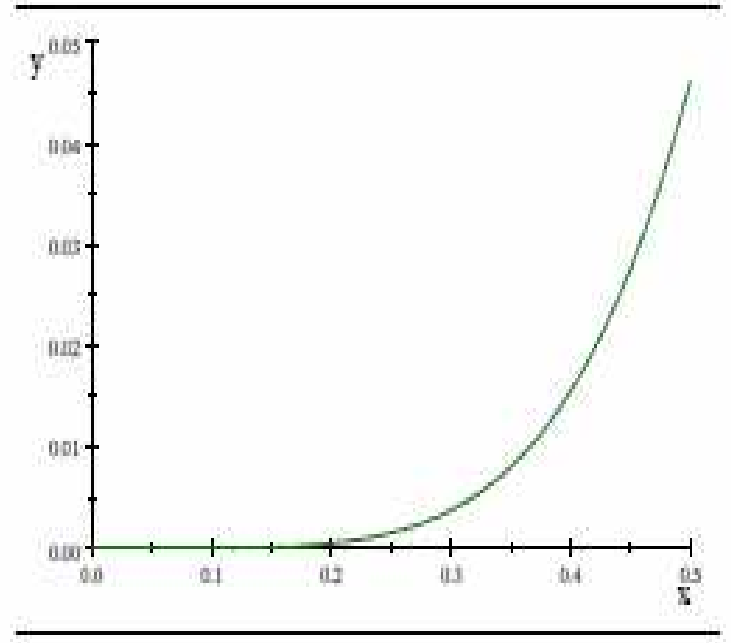


$$x(x^2+1)^{1/2}(2x^2-3) + 3\ln(x+\sqrt{1+x^2})$$



$x \ll 1$

$$\frac{8}{3}x^3 - \frac{4}{7}x^7 + \frac{1}{1}x^9$$



De donde

- a) al crecer la masa disminuye el radio
- b) para  $M > M_0$  no hay solución real

⇒ una enana blanca en equilibrio debe tener

$$M < M_0$$

Chandrasekhar calculo:

$$M_0 = \frac{5.75}{\mu_e^2} M_\odot$$

con  $\mu_e = M/Nm_H$  grado de ionización (usualmente del orden de 2) ⇒

$$M_0 \approx 1.44 \cdot M_\odot$$

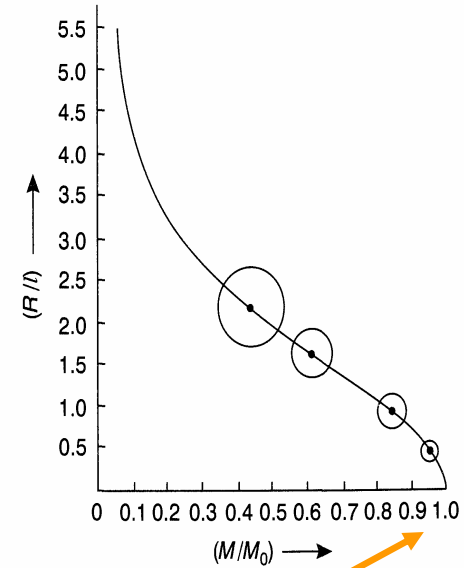
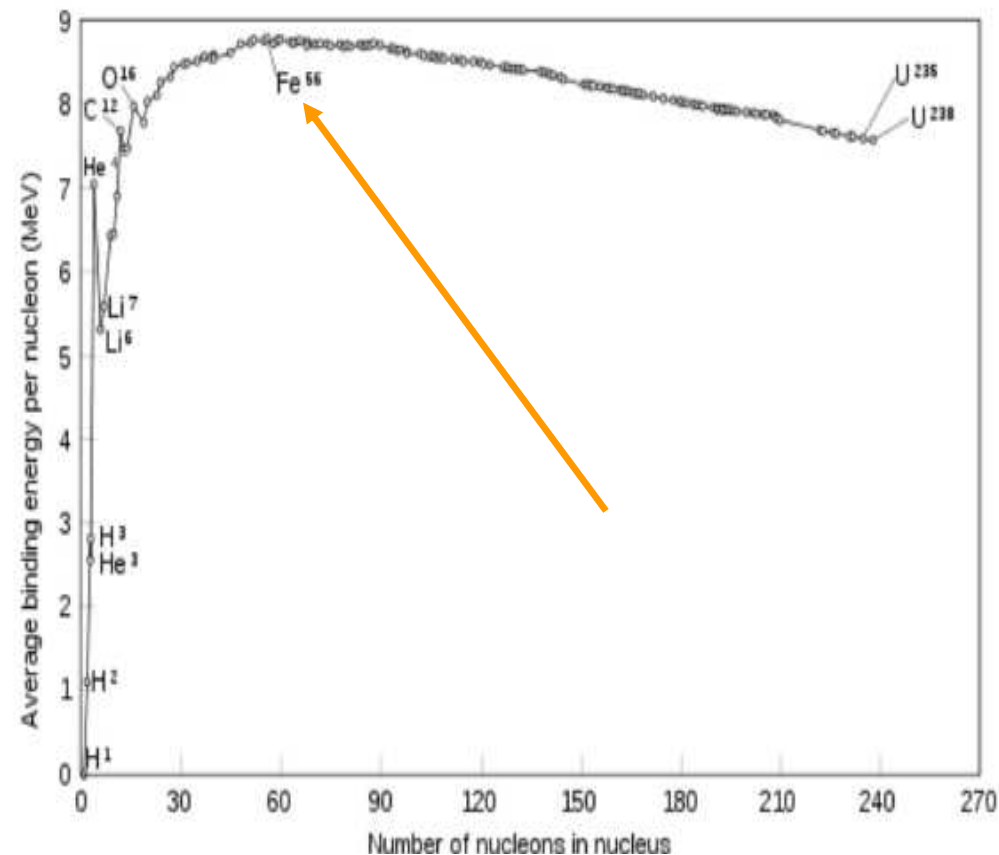
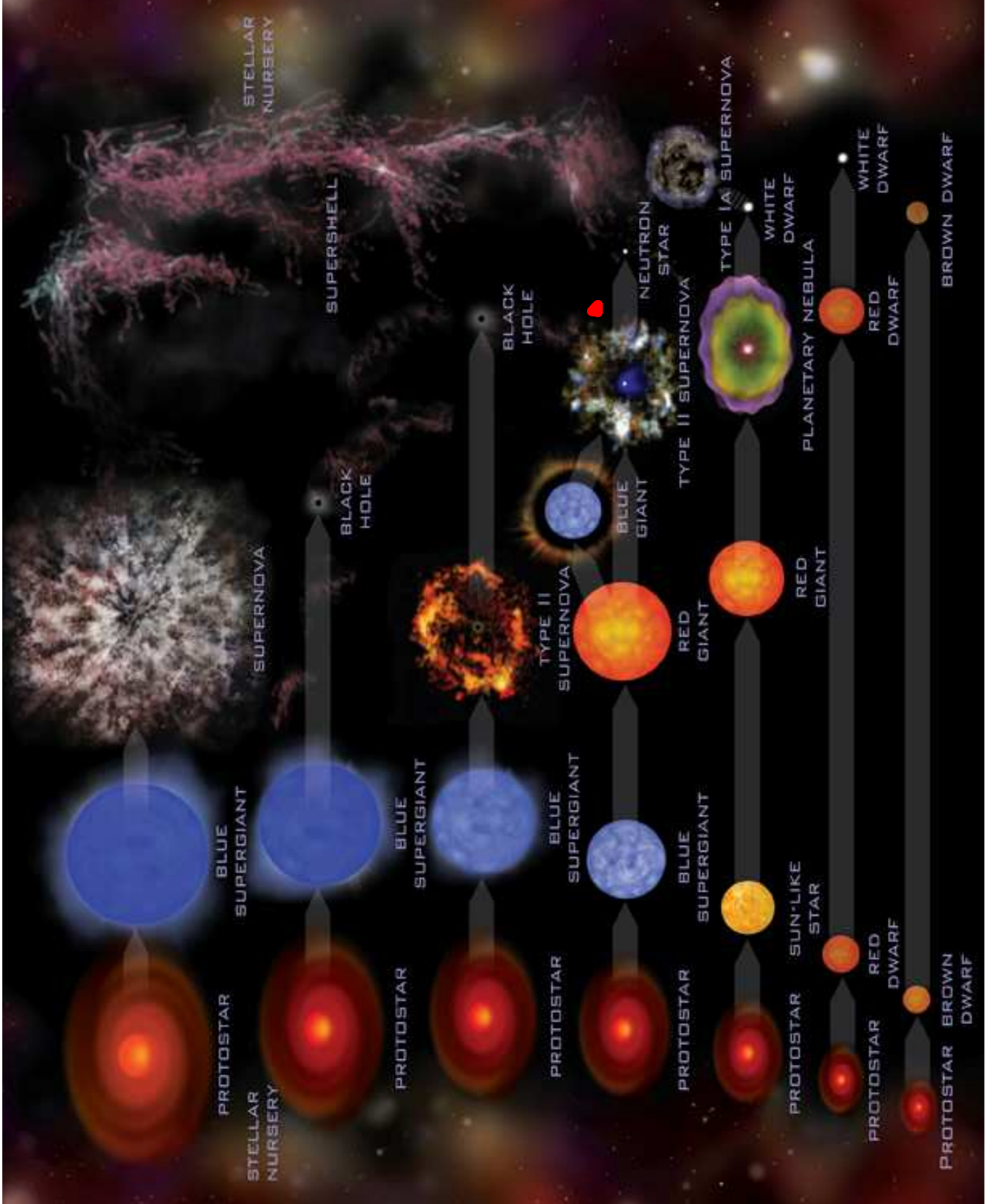


FIG. 8.9. The mass-radius relationship for white dwarfs (after Chandrasekhar, 1939). The masses are expressed in terms of the limiting mass  $M_0$  and the radii in terms of a characteristic length  $l$ , which is given by  $7.71\mu_e^{-1} \times 10^8$  cm  $\approx 3.86 \times 10^8$  cm.

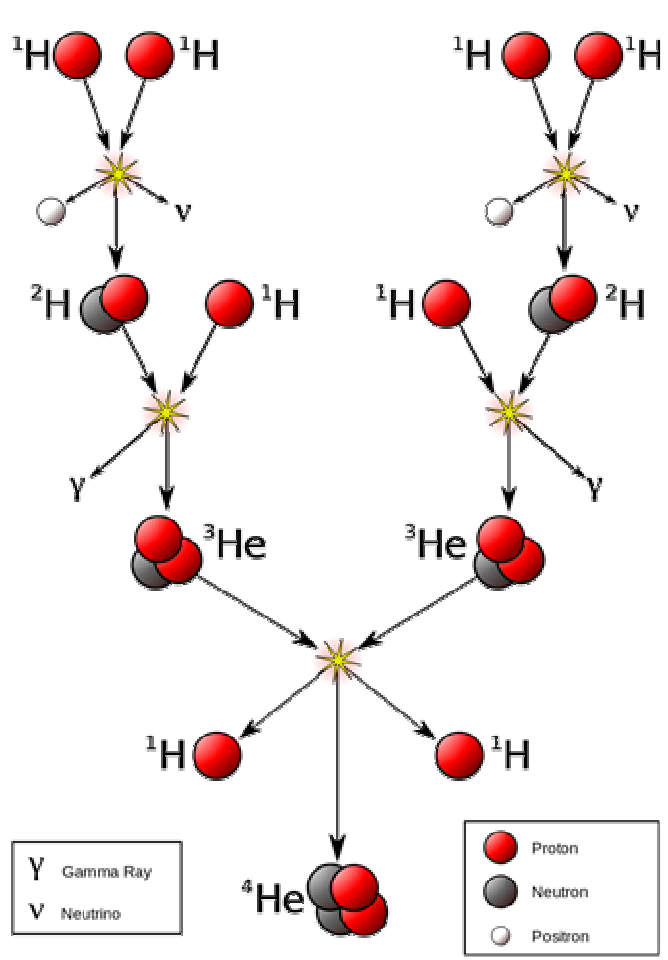
Que es lo que ocurre? porque este limite?

Si la masa supera el valor de Chandrasekhar la presion electronica no es suficiente para soportar la compresion gravitatoria y la estrella sigue colapsando.

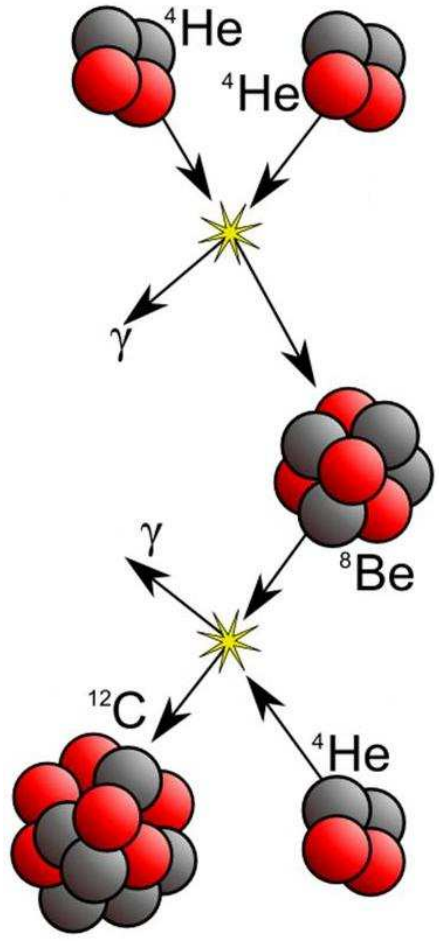




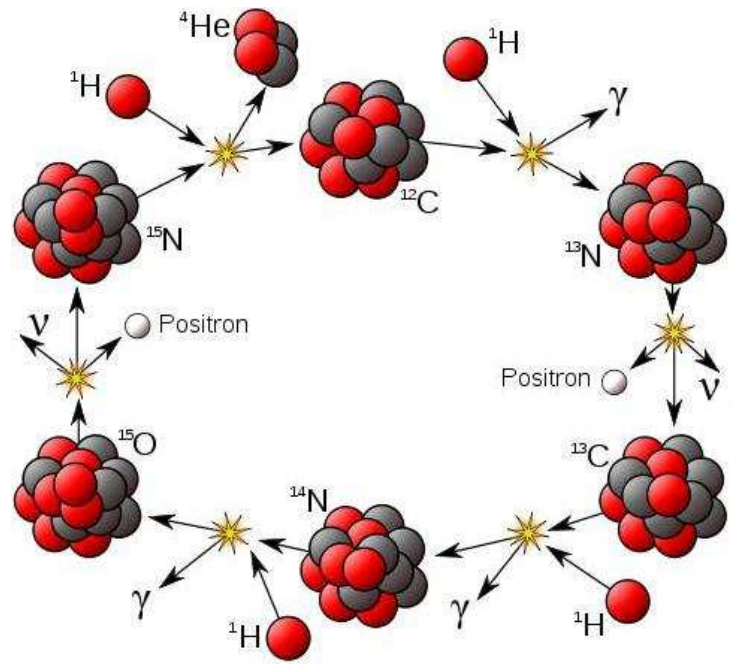
# A Closer Look at Nucleosynthesis



**H to He**

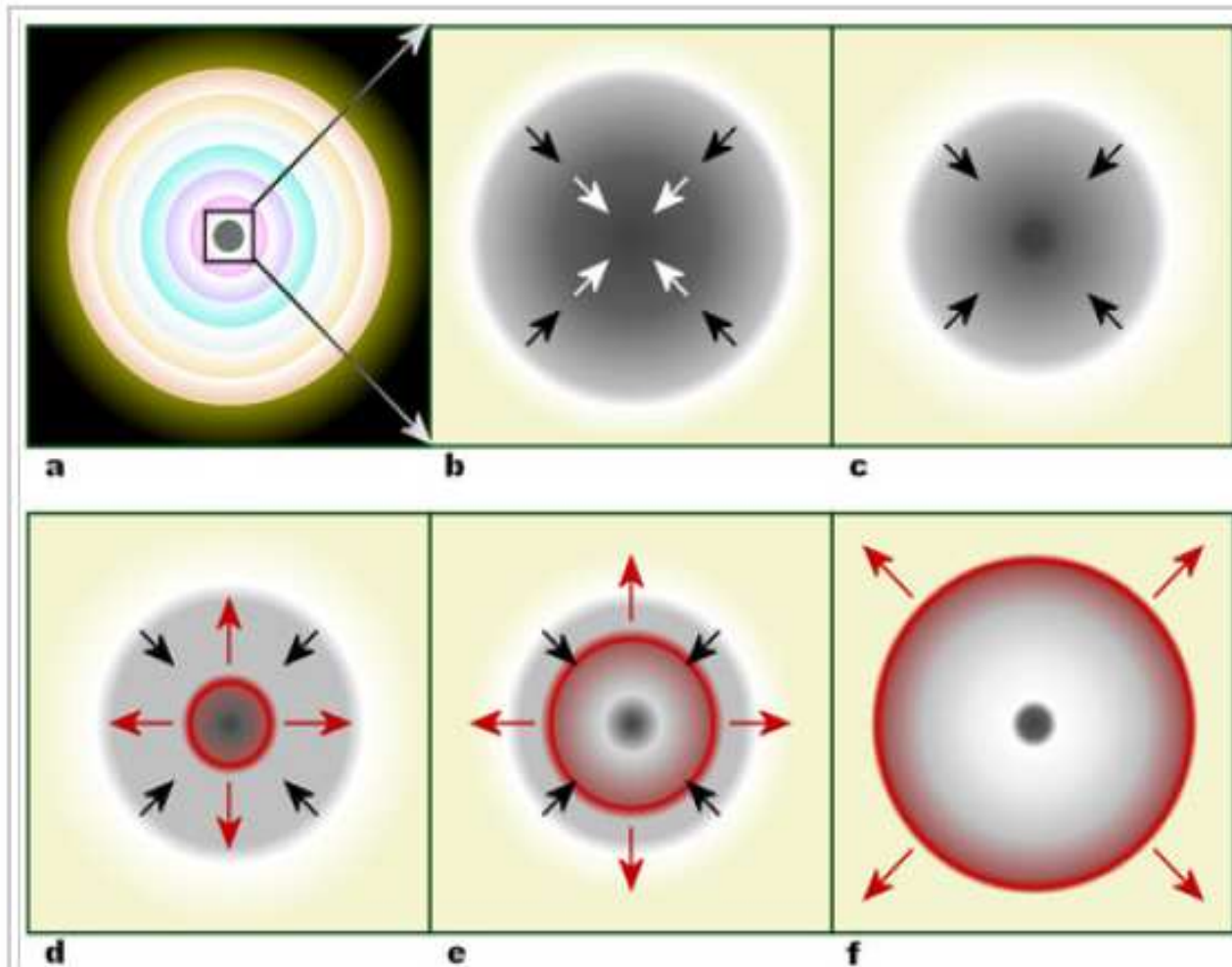


**He to C**



**CNO cycle**

In "normal stars"



Within a massive, evolved star (a) the onion-layered shells of elements undergo fusion, forming a nickel-iron core (b) that reaches Chandrasekhar-mass and starts to collapse. The inner part of the core is compressed into neutrons (c), causing infalling material to bounce (d) and form an outward-propagating shock front (red). The shock starts to stall (e), but it is re-invigorated by neutrino interaction. The surrounding material is blasted away (f), leaving only a degenerate remnant.

# Estrella de Neutrones

## Neutron Star

Neutron stars are the collapsed cores of some massive stars (8 → 20-30 solar masses)

1 solar mass < NS Mass < 3 solar masses

NS radius  $\approx 10^{-5}$  Solar Radius  $\approx 10$  Km

NS Density  $\approx 10^{15}$  g/cm<sup>3</sup> (water  $\approx 1.0$  g/cm<sup>3</sup>)

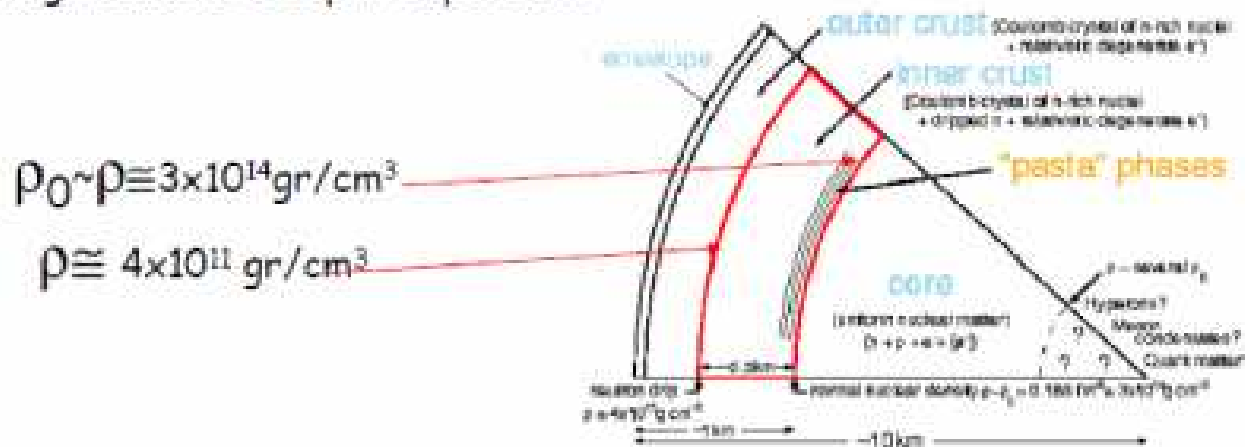
Electrically Neutral

Neutron star vs. Chicago

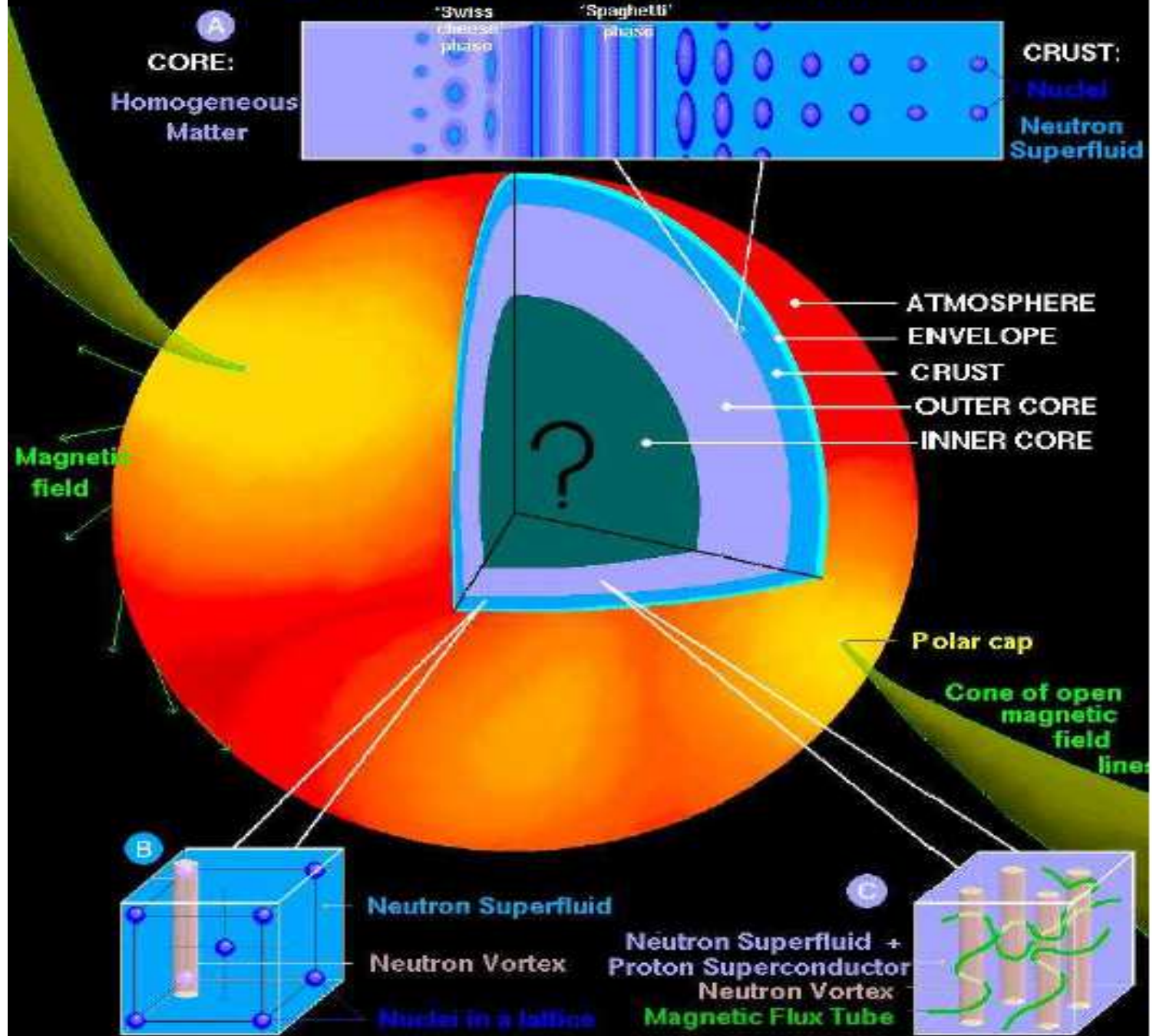


Mass=1.4 M<sub>sun</sub>, Radius=10 km  
 Spin rate up to 30,000 rpm  
 Density=10<sup>15</sup> g/cm<sup>3</sup>, Magnetic field=10<sup>13</sup> Gauss

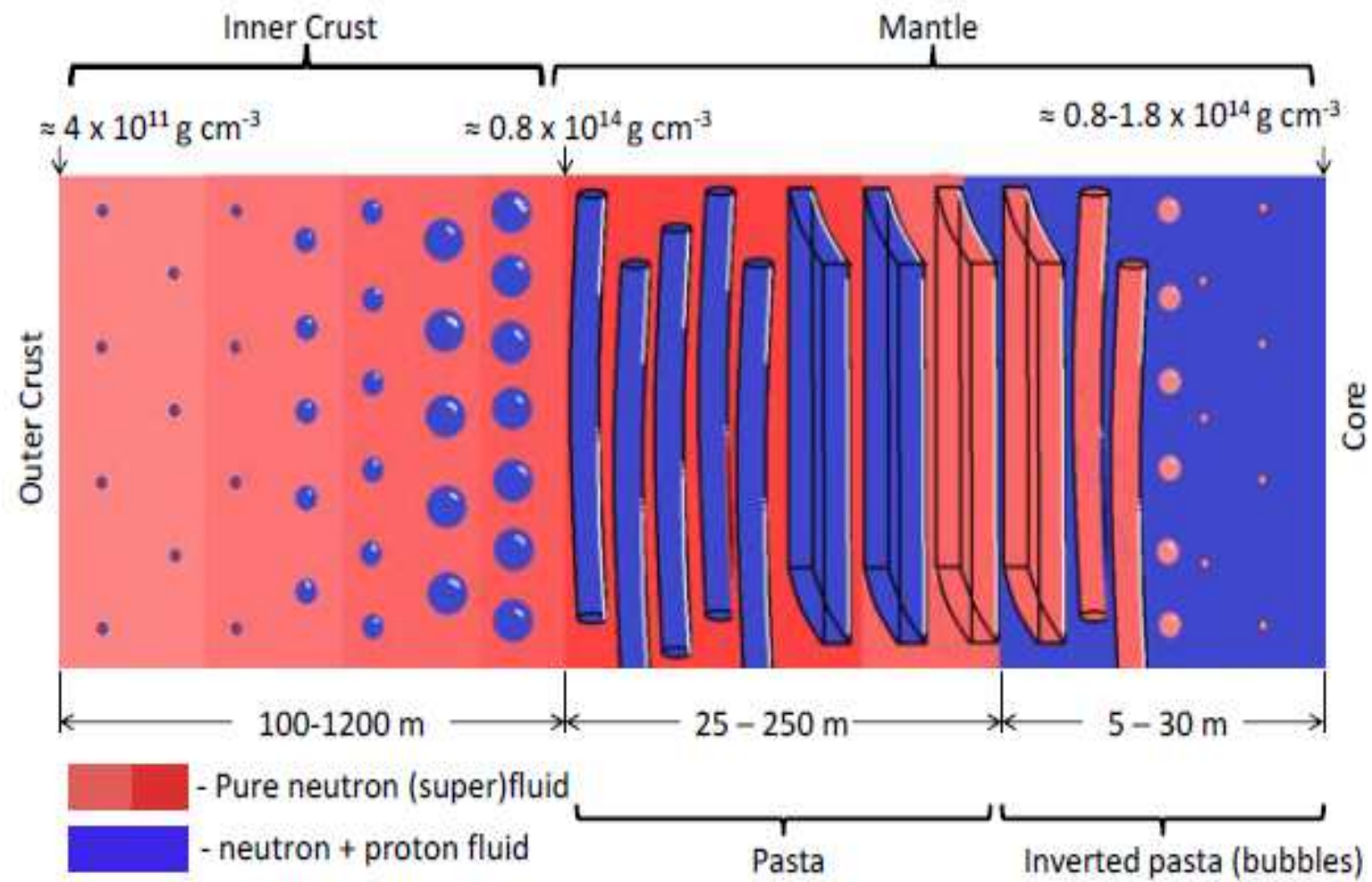
This magnitudes are depth dependent



# A NEUTRON STAR: SURFACE and INTERIOR







CMD Model

Potential

$$\begin{aligned}
 V_{np}(r) &= V_r [\exp(-\mu_r r)/r - \exp(-\mu_r r_c)/r_c] \\
 &\quad - V_a [\exp(-\mu_a r)/r - \exp(-\mu_a r_a)/r_a] \\
 V_{nn}(r) &= V_{pp}(r) = V_0 [\exp(-\mu_0 r)/r - \exp(-\mu_0 r_c)/r_c]
 \end{aligned}$$

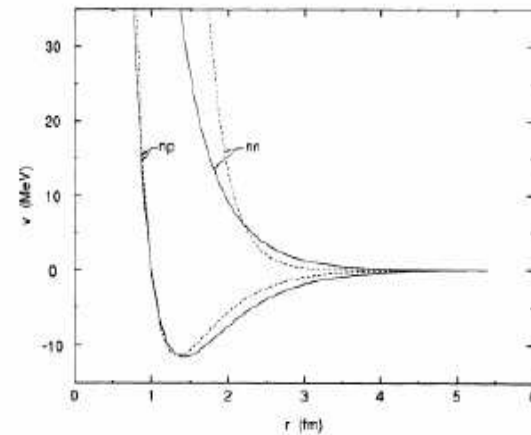
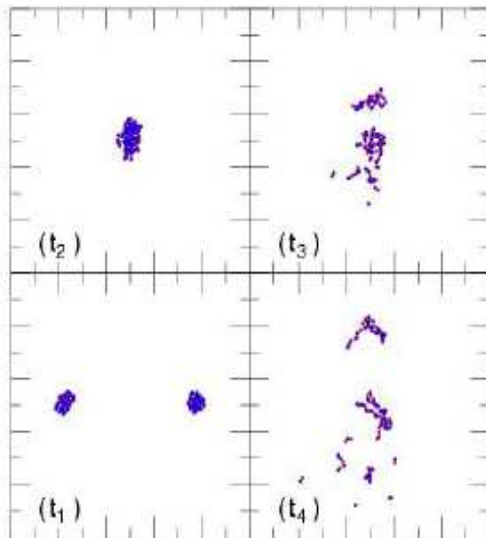
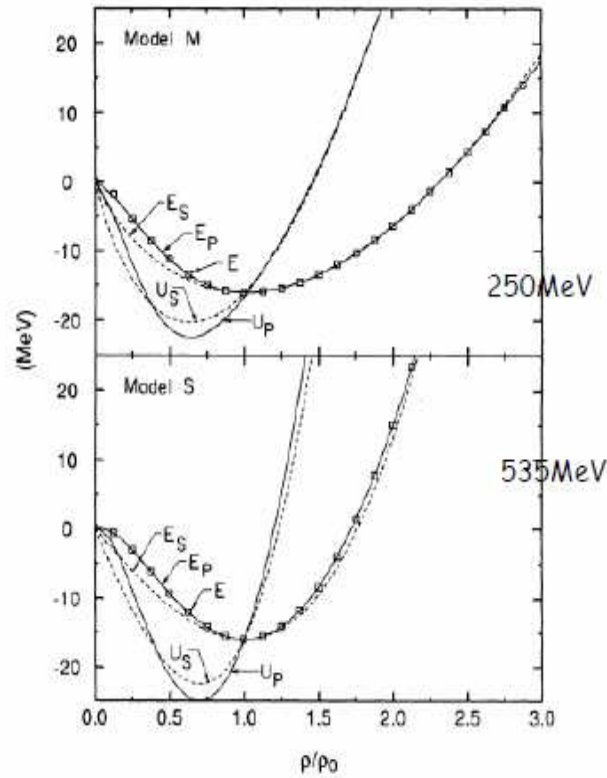


FIG. 1. The interparticle potentials  $v_{nn}$  and  $v_{np}$  for the  $M$  (solid) and  $S$  (dashed) models.

Dynamical evolution :

# CMD Model



PHYSICAL REVIEW C

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## Accuracy of the Vlasov-Nordheim approximation in the classical limit

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(Received 1 March 1990)

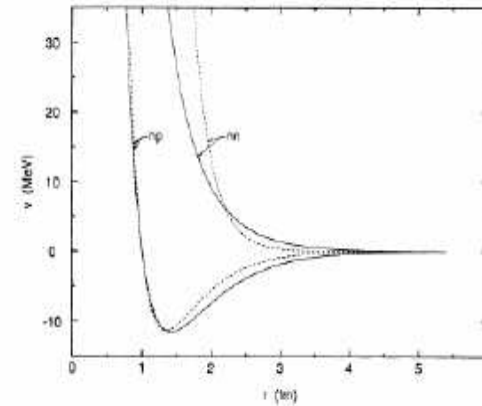
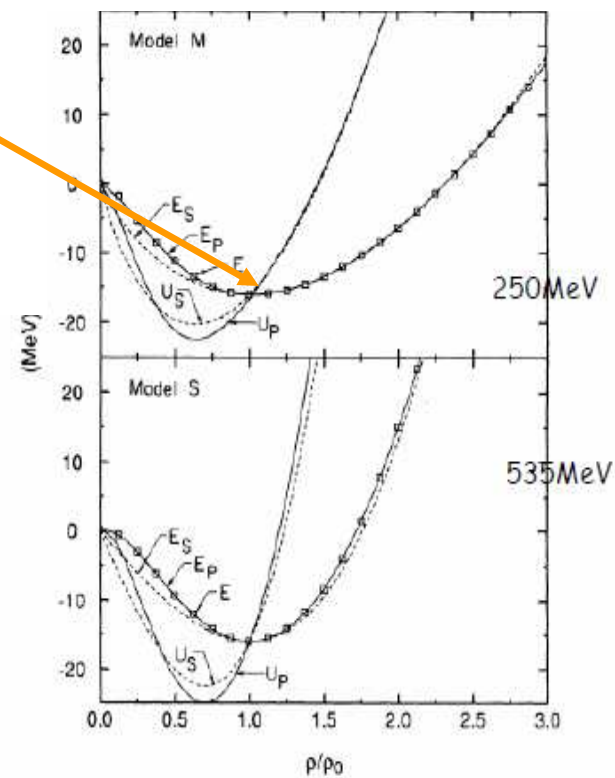
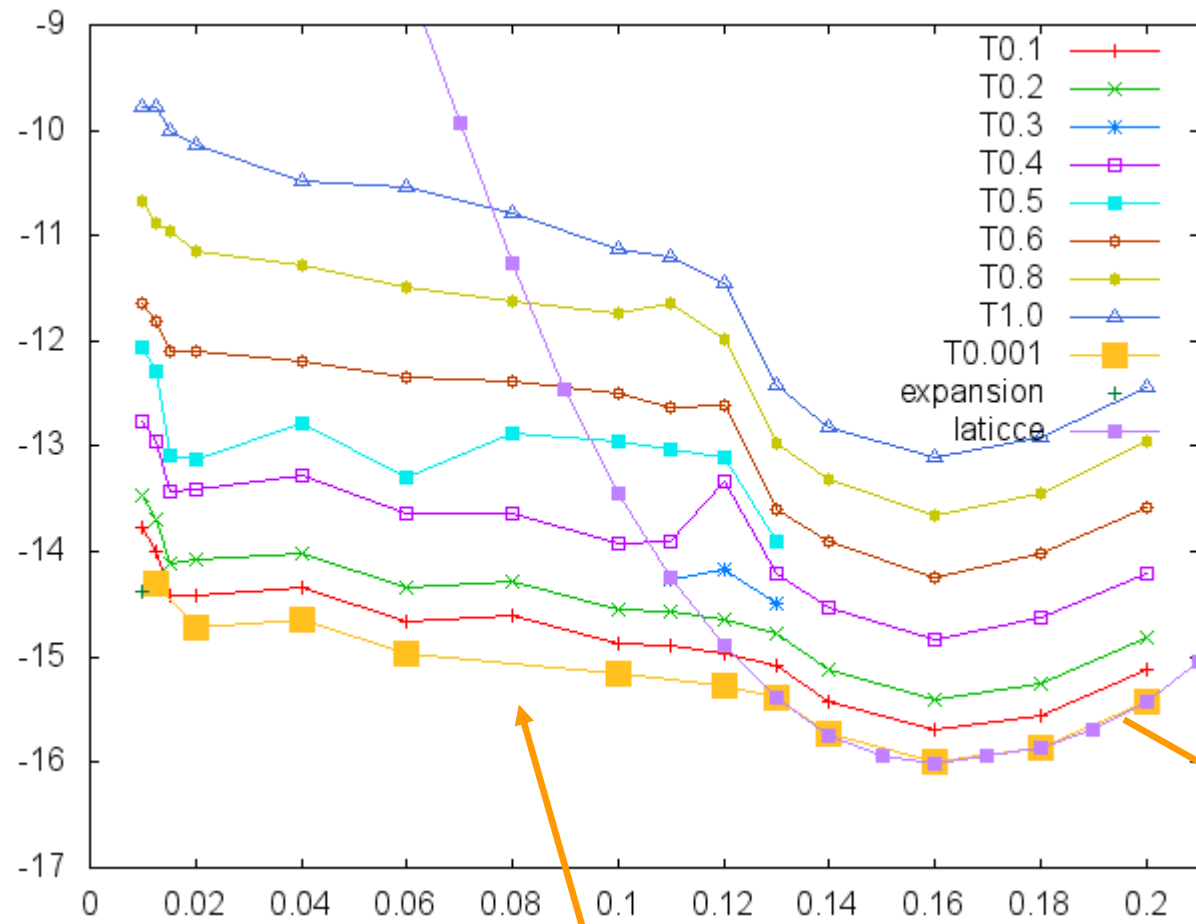


FIG. 1. The interparticle potentials  $v_{nn}$  and  $v_{np}$  for the  $M$  (solid) and  $S$  (dashed) models.

TABLE II. Energies of nuclei (MeV/nucleon) for the  $M$  and  $S$  models compared to experimental binding energies.

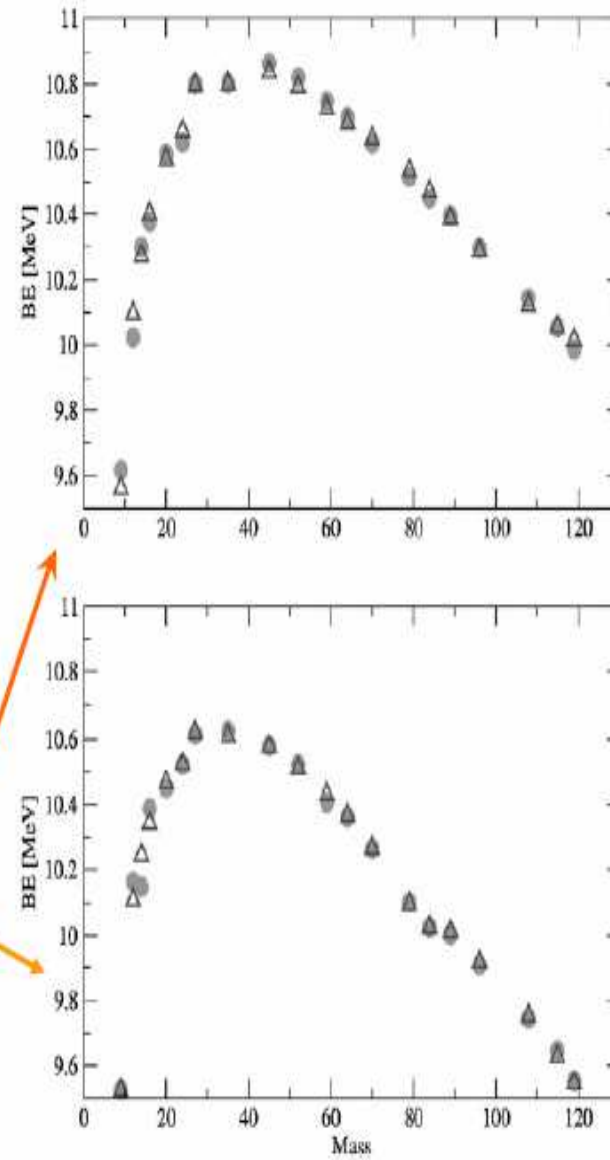
$A$	$Z$	$E_M$	$E_S$	$E_{exp}$
2	1	-5.76	-5.67	-1.11
3	1	-7.14	-7.26	-2.83
4	2	-7.91	-7.05	-7.07
5	2	-7.47	-6.22	-5.47
16	8	-10.39	-10.49	-7.98
40	20	-10.44	-10.60	-8.55
90	40	-9.93	-10.25	-8.71
139	57	-9.12	-9.58	-8.38
197	79	-8.38	-8.84	-7.92
200	82	0.14	0.71	7.07



# CMD Model

**Table 1.** Comparison of coefficients obtained for different models.

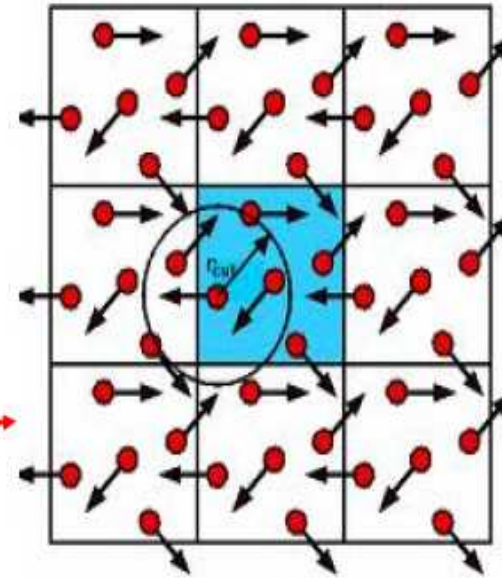
Coefficient	Stiff	Medium	Experimental
$C_v$	16.1	17.37	15.75
$C_r$	-11.73	-14.38	-17.8
$C_c$	-0.197	-0.226	-0.177
$C_{sym}$	-34.07	-25.08	-23.7



**Figure 1.** Energies obtained with the mass formula fit (triangles) for the stiff and medium models (top and bottom panels, respectively) together with the corresponding ground states calculated using frictional molecular dynamics (circles).

# Our work: use molecular dynamics to study neutron stars

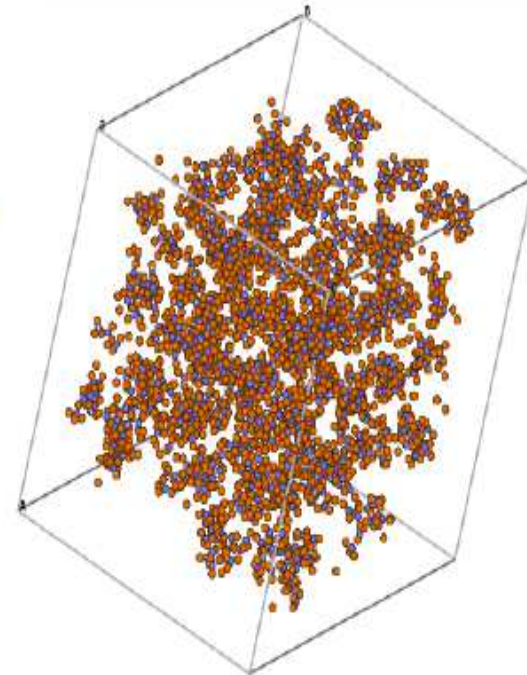
Solve the dynamics as before but...  
Simulate the infinite systems using Periodic boundary conditions



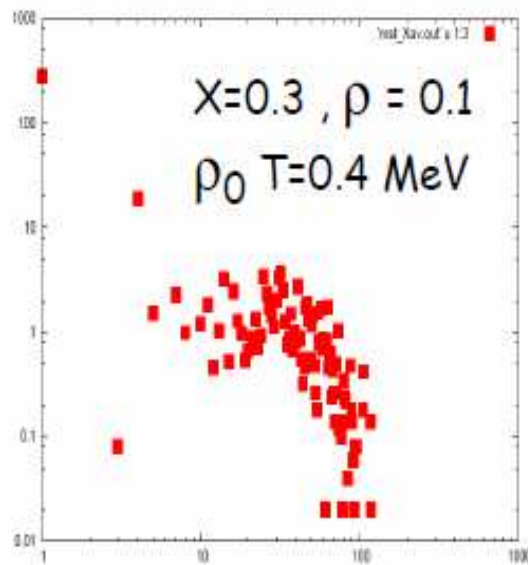
For Coulomb use Thomas Fermi approximation

$$V_C(r) \approx \frac{e^2}{r}$$

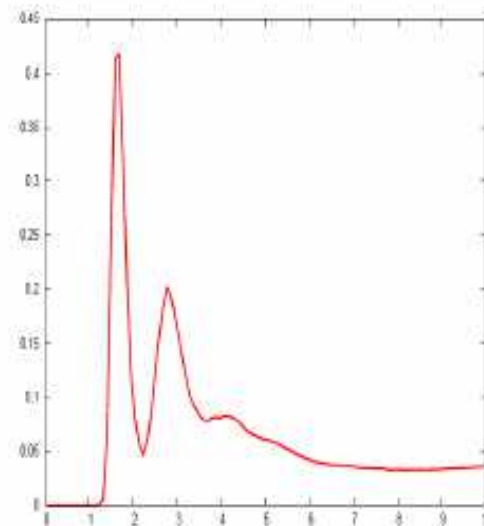
$$V_C^{TF}(r) \approx \frac{e^2}{r} \exp(-r/\lambda)$$



## Study:

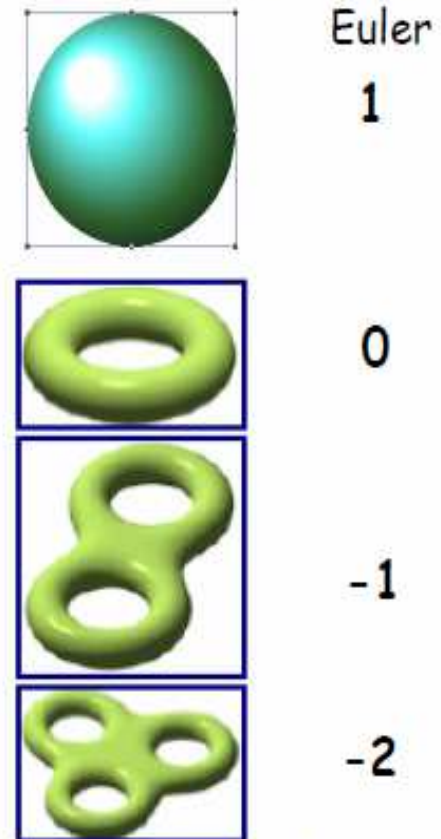


Fragments and  
their mass  
distribution



Pair correlation  
function

## Topology (why?)



Minkowski  
functionals

Results: Nuclear Pasta!

nice lasagna

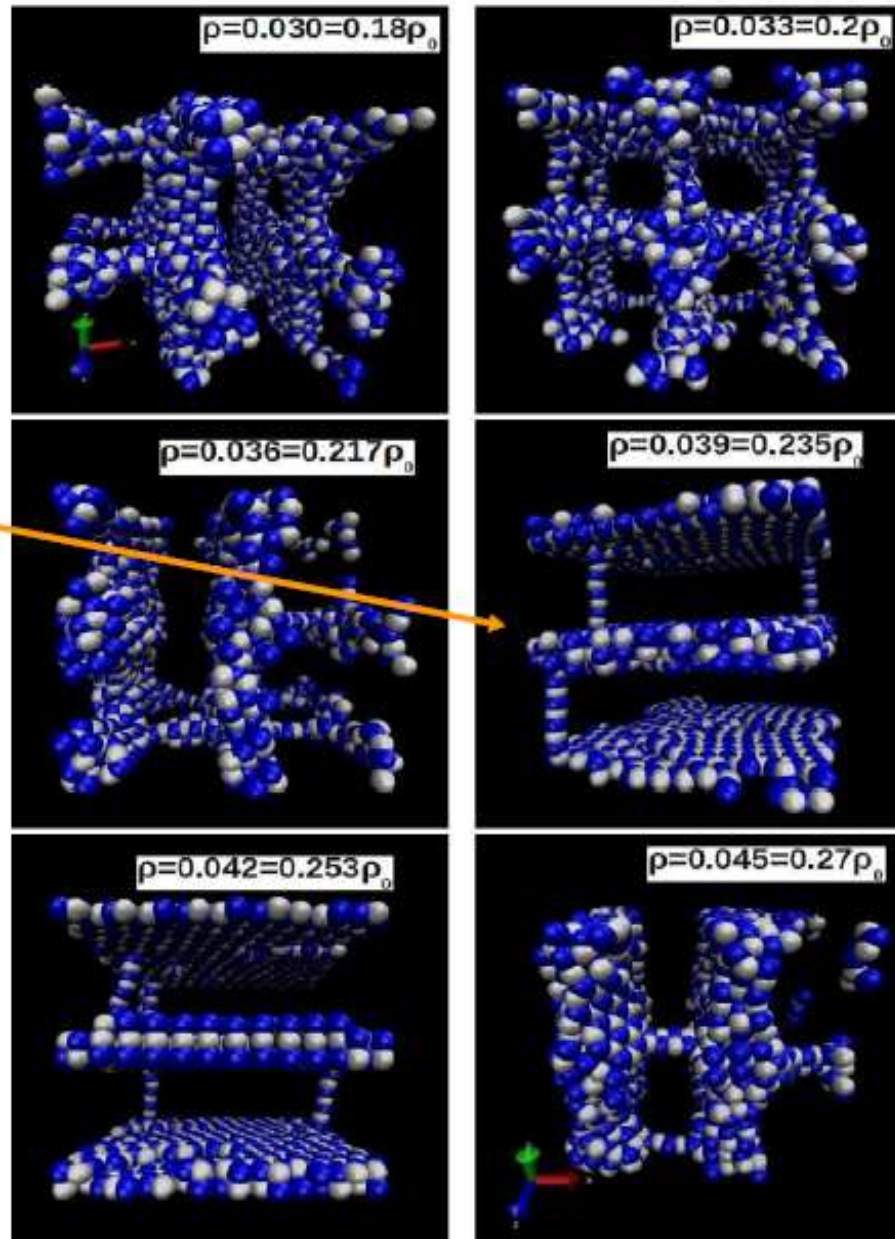
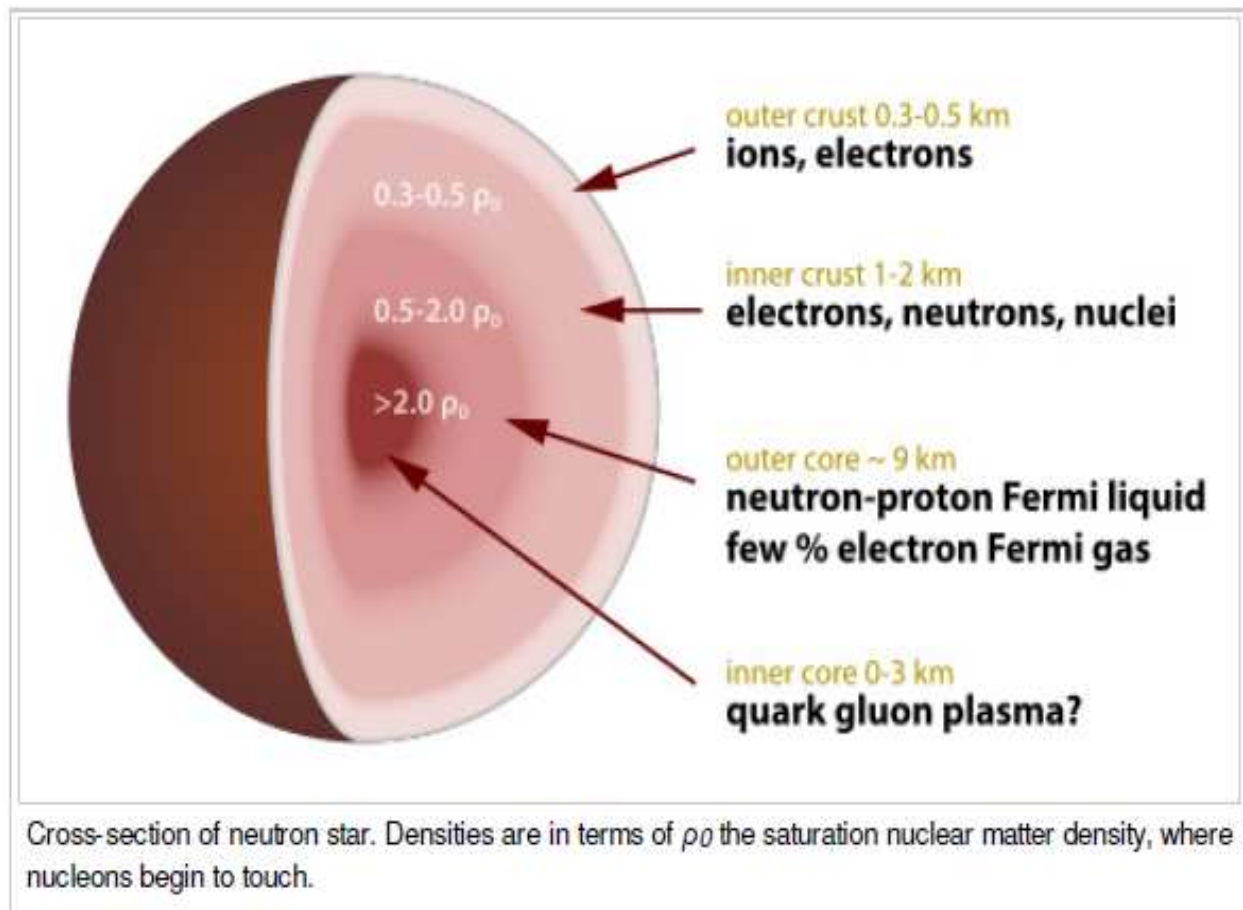


Figure 8. Configuration obtained with the screened Coulomb potential with  $x = 0.5$ ,  $T = 0.1 \text{ MeV}$  and densities between  $\rho = 0.03 \text{ fm}^{-3}$  to  $0.045 \text{ fm}^{-3}$



A **neutron star** is a type of stellar remnant that can result from the gravitational collapse of a massive star during a Type II, Type Ib or Type Ic supernova event. Such stars are composed almost entirely of neutrons, which are subatomic particles without electrical charge and with slightly larger mass than protons. Neutron stars are very hot and are supported against further collapse by quantum degeneracy pressure due to the Pauli exclusion principle. This principle states that no two neutrons (or any other fermionic particles) can occupy the same place and quantum state simultaneously.



Core-burning nuclear fusion stages for a 25-solar mass star

Process	Main fuel	Main products	25 M <sub>☉</sub> star <sup>[6]</sup>		
			Temperature (Kelvin)	Density (g/cm <sup>3</sup> )	Duration
hydrogen burning	hydrogen	helium	$7 \times 10^7$	10	$10^7$ years
triple-alpha process	helium	carbon, oxygen	$2 \times 10^8$	2000	$10^6$ years
carbon burning process	carbon	Ne, Na, Mg, Al	$8 \times 10^8$	$10^6$	$10^3$ years
neon burning process	neon	O, Mg	$1.6 \times 10^9$	$10^7$	3 years
oxygen burning process	oxygen	Si, S, Ar, Ca	$1.8 \times 10^9$	$10^7$	0.3 years
silicon burning process	silicon	nickel (decays into iron)	$2.5 \times 10^9$	$10^8$	5 days

When the core's mass exceeds the Chandrasekhar limit of about 1.4 solar masses, degeneracy pressure can no longer support it, and catastrophic collapse ensues.<sup>[10]</sup> The outer part of the core reaches velocities of up to 70,000 km/s (23% of the speed of light) as it collapses toward the center of the star.<sup>[11]</sup> The rapidly shrinking core heats up, producing high-energy gamma rays that decompose iron nuclei into helium nuclei and free neutrons via photodisintegration. As the core's density increases, it becomes energetically favorable for electrons and protons to merge via inverse beta decay,

producing neutrons and elementary particles called neutrinos. Because neutrinos rarely interact with normal matter they can escape from the core, carrying away energy and further accelerating the collapse, which proceeds over a timescale of milliseconds. As the core detaches from the outer layers of the star, some of these neutrinos are absorbed by the star's outer layers, beginning the supernova explosion.<sup>[12]</sup>

For Type II supernovae, the collapse is eventually halted by short-range repulsive neutron-neutron interactions, mediated by the strong force, as well as by degeneracy pressure of neutrons, at a density comparable to that of an atomic nucleus. Once collapse stops, the infalling matter rebounds, producing a shock wave that propagates outward. The energy from this shock dissociates heavy elements within the core. This reduces the energy of the shock, which can stall the explosion within the outer core.<sup>[13]</sup>

Cuenta para el radio

$$x(x^2 + 1)^{1/2}(2x^2 - 3) + 3 \sinh^{-1}x$$

$$x = \left(\frac{3n}{8\pi}\right)^{1/3} \left(\frac{h}{mc}\right) = \left(\frac{3}{8\pi}\right)^{1/3} \left(\frac{3}{4\pi}\right)^{1/3} (10^{33})^{1/3} 10^{-11} = 0.30544$$

$$\text{Si } R \gg 10^8 \rightarrow A(x) \approx \frac{8}{5}x^5$$

$$\text{Si } R \ll 10^8 \rightarrow A(x) \approx 2x^4 - 2x^2$$

