1. (a) $\left(\frac{\partial S}{\partial T}\right)_x = \frac{C_x}{T} \Longrightarrow S(T,x) = C_x \ln T + g(x)$

$\left(\frac{\partial S}{\partial x}\right)_T = -\left(\frac{\partial f}{\partial T}\right)_x = -\alpha x \Longrightarrow g'(x) = -\alpha x \Longrightarrow g(x) = -\frac{1}{2}\alpha x^2 + c$

Maxwell:

$dF = -S\,dT + f\,dx$

$\Longrightarrow -\left(\frac{\partial S}{\partial x}\right)_T = \left(\frac{\partial f}{\partial T}\right)_x$

$\Longrightarrow$ Escribiendo $c = -C_x \ln \theta$,

$$\boxed{S(T,x) = C_x \ln \frac{T}{\theta} - \frac{1}{2}\alpha x^2}$$

(b) $\left(\frac{\partial F}{\partial T}\right)_x = -S = \frac{1}{2}\alpha x^2 - C_x \ln \frac{T}{\theta}$

$$\Longrightarrow F(T,x) = \frac{1}{2}\alpha T x^2 - C_x\left[T(\ln T - 1) - T\ln\theta\right] + g(x)$$

$$= \frac{1}{2}\alpha T x^2 - C_x T\left(\ln\frac{T}{\theta} - 1\right) + g(x)$$

$\left(\frac{\partial F}{\partial x}\right)_T = f = \alpha T x \Longrightarrow \alpha T x + g'(x) = \alpha T x \Longrightarrow g'(x) = 0$

$\Longrightarrow g(x) = c$

Eligiendo origen de energías,

$$\boxed{F(T,x) = T\left[\frac{1}{2}\alpha x^2 - C_x\left(\ln\frac{T}{\theta} - 1\right)\right]}$$

$$\Rightarrow U(T,x) = F + TS = \boxed{C_x T = U(T,x)}$$

(c) $$S(T,x) - S(T_i, 0) = C_x \ln\frac{T}{T_i} - \frac{1}{2}\alpha x^2$$

Adiabático y cuasiestático $\Rightarrow \Delta S = 0$

$$\Rightarrow C_x \ln\frac{T}{T_i} = \frac{1}{2}\alpha x^2 \Rightarrow \boxed{T = T_i\, e^{\frac{\alpha x^2}{2C_x}}}$$

La temperatura aumenta, no importa si contraemos o estiramos el resorte. Esto es porque

$$dW = f\,dx = \frac{\alpha T}{2}\,dx^2$$

Si empezamos en $x = 0$, $dx^2 > 0$ siempre $\Rightarrow dW > 0 \underset{\text{Adiabático}}{\Rightarrow} dU > 0 \underset{U = C_x T}{\Rightarrow} dT > 0$

(d) Adiabático: $Q = 0$

Libre: $W = 0$ $\Bigg\} \Rightarrow \Delta U = 0 \Rightarrow \Delta T = 0$

$\uparrow$

$U = C_v T$

$$\Delta S = S(T, 0) - S(T, x) = \boxed{\frac{1}{2}\alpha x^2 = \Delta S}$$

$\Delta S > 0$ porque el proceso es adiabático y no cuasiestático.

2. (a) Estados: 1 iguales

2 ventaja Nadal

3 " Djokovic

4 gana Nadal

5 " Djokovic

$$\boxed{M = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & p & q & 0 & 0 \\ 2 & q & 0 & 0 & p & 0 \\ 3 & p & 0 & 0 & 0 & q \\ 4 & 0 & 0 & 0 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 \end{array}}$$

4 y 5 absorbentes $\Rightarrow$ No ergódica.

Desde cualquier estado transitorio se puede acceder a un absorbente

$\Rightarrow$ Absorbente

La matriz ya está en forma canónica.

$$Q = \begin{pmatrix} 0 & p & q \\ q & 0 & 0 \\ p & 0 & 0 \end{pmatrix} \qquad R = \begin{pmatrix} 0 & 0 \\ p & 0 \\ 0 & q \end{pmatrix}$$

(b) $$I - Q = \begin{pmatrix} 1 & -p & -q \\ -q & 1 & 0 \\ -p & 0 & 1 \end{pmatrix}$$

$$\Longrightarrow \det(I-Q) = \begin{vmatrix} 1 & -p \\ -q & 1 \end{vmatrix} - q \begin{vmatrix} -q & 1 \\ -p & 0 \end{vmatrix}$$

$$= 1 - pq - pq = 1 - 2pq$$

$$\Longrightarrow N = (I-Q)^{-1} = \boxed{\frac{1}{1-2pq} \begin{pmatrix} 1 & p & q \\ q & 1-pq & q^2 \\ p & p^2 & 1-pq \end{pmatrix} = N}$$

$$\Longrightarrow \boxed{\langle t \rangle = \frac{1}{1-2pq} (1 + p + q)}$$

Empezamos en el estado 1 (iguales)

Si $p = 1$, $\langle t \rangle = 2$ ✓

(Si Nadal gana seguro el game termina en 2 puntos)

Si $p = \frac{1}{2}$, $\langle t \rangle = \frac{1}{1 - \frac{2}{4}} \cdot 2 = \frac{2}{1/2} = 4$ ✓

(si ninguno gana seguro el game tarda más en terminar)

(c) $$B = NR = \frac{1}{1-2pq} \begin{pmatrix} 1 & p & q \\ q & 1-pq & q^2 \\ p & p^2 & 1-pq \end{pmatrix} \begin{pmatrix} 0 & 0 \\ p & 0 \\ 0 & q \end{pmatrix}$$

$$= \frac{1}{1-2pq} \begin{pmatrix} p^2 & q^2 \\ p(1-pq) & q^3 \\ p^3 & q(1-pq) \end{pmatrix}$$

$$\boxed{P(\text{Nadal gana el game}) = \frac{p^2}{1-2pq}}$$

Empezamos
en el estado
1 (iguales)

Si $p = 1$, $P(\text{Nadal}) = 1$ ✓

Si $p = \frac{1}{2}$, $P(\text{Nadal}) = \frac{\frac{1}{4}}{1 - \frac{2}{4}} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$ ✓

3. (a) $E = \hbar\omega n \pm \epsilon$

Eligiendo origen energías

(b) $Z = Z_1^N$ $\qquad Z_1 = Z_{1,\,tras}\, Z_{1,\,int}$

$$Z_{1,\,tras} = \sum_{n=0}^{\infty} e^{-\beta\hbar\omega n} = \frac{1}{1-e^{-\beta\hbar\omega}}$$

$$Z_{1,\,int} = e^{\beta\epsilon} + e^{-\beta\epsilon} = 2\cosh(\beta\epsilon)$$

$$\Rightarrow \boxed{Z(\beta, N) = \left[\frac{2\cosh(\beta\epsilon)}{1-e^{-\beta\hbar\omega}}\right]^N}$$

(c) $$\langle E_1\rangle = -\frac{\partial}{\partial\beta}\log Z_1 = -\frac{\partial}{\partial\beta}\log Z_{1,\,tras} - \frac{\partial}{\partial\beta}\log Z_{1,\,int}$$

$$= \frac{\hbar\omega\, e^{-\beta\hbar\omega}}{1-e^{-\beta\hbar\omega}} - \epsilon\, \mathrm{tgh}(\beta\epsilon)$$

$$\Rightarrow \boxed{\langle E_1\rangle = \frac{\hbar\omega}{e^{\beta\hbar\omega}-1} - \epsilon\, \mathrm{tgh}(\beta\epsilon)}$$

$$c = \frac{d\langle E_1\rangle}{dT} = -\frac{1}{kT^2}\frac{d\langle E_1\rangle}{d\beta} = -\frac{1}{kT^2}\left\{-\frac{\hbar\omega}{\left(e^{\beta\hbar\omega}-1\right)^2}\,\hbar\omega\, e^{\beta\hbar\omega} - \frac{\epsilon^2}{\cosh^2(\beta\epsilon)}\right\}$$

$$tgh'(x) = 1 - tgh^2 x = \frac{1}{\cosh^2 x}$$

$$\Rightarrow \boxed{c(T) = \frac{1}{kT^2}\left[(\hbar\omega)^2\frac{e^{\frac{\hbar\omega}{kT}}}{\left(e^{\frac{\hbar\omega}{kT}}-1\right)^2} + \frac{\epsilon^2}{\cosh^2\left(\frac{\epsilon}{kT}\right)}\right]}$$

(d) $kT \gg \hbar\omega,\ \epsilon \Rightarrow \beta\hbar\omega;\ \beta\epsilon \ll 1$

$$\Rightarrow \langle E_1\rangle \simeq \hbar\omega\left[\frac{1}{\beta\hbar\omega} - \frac{1}{2} + \frac{\beta\hbar\omega}{12}\right] - \beta\epsilon^2$$

$$\simeq \frac{1}{\beta} - \frac{\hbar\omega}{2} + \beta\left[\frac{(\hbar\omega)^2}{12} - \epsilon^2\right]$$

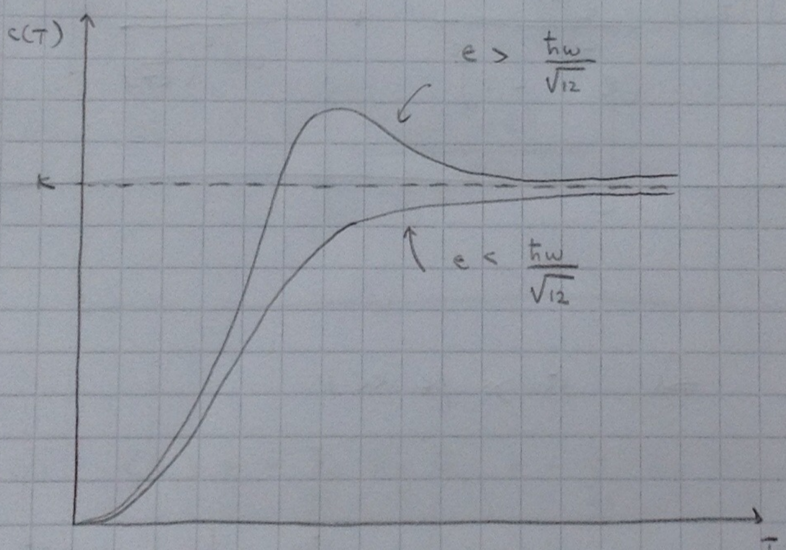
$$= \boxed{kT - \frac{\hbar\omega}{2} + \frac{1}{kT}\left[\frac{(\hbar\omega)^2}{12} - \epsilon^2\right] \simeq \langle E_1\rangle}$$

$$\Longrightarrow \boxed{c(T) \underset{\uparrow}{\simeq} k + \frac{1}{kT^2}\left[\epsilon^2 - \frac{(\hbar\omega)^2}{12}\right]}$$

Altas temperaturas

Bajas temperaturas $kT \ll \hbar\omega, \epsilon$

$$\Longrightarrow \boxed{c(T) \simeq \frac{1}{kT^2}\left[(\hbar\omega)^2 e^{-\frac{\hbar\omega}{kT}} + (2\epsilon)^2 e^{-\frac{2\epsilon}{kT}}\right]}$$



(e) $\mu = \left(\frac{\partial F}{\partial N}\right)_T$ $F = -kT \log Z = -NkT \log Z_1(T)$

$$\Longrightarrow \mu = -kT \log Z_1 = \boxed{-kT \log\left[\frac{2\cosh(\beta\epsilon)}{1 - e^{-\beta\hbar\omega}}\right] = \mu}$$