SEGUNDO RECU

1. (a) $\log \mathcal{Z} = \sum\limits_{\substack{i \\ \uparrow \\ \text{estados} \\ \text{monopart}}} \log(1 + z e^{-\beta\epsilon_i})$

$$= g \sum_{n=0}^{\infty} \log\left[1 + z e^{-\beta\hbar\omega(n+\frac{1}{2})}\right]$$

$$\underset{\substack{\uparrow \\ \text{Euler-Maclaurin}}}{\simeq} g \left\{ \int_0^{\infty} du \, \log\left(1 + z e^{-\beta\hbar\omega u}\right) \right.$$

$$\left. - \frac{1}{24} \frac{z}{1+z} \beta\hbar\omega \right\}$$

$$\underset{\substack{\uparrow \\ x \equiv \beta\hbar\omega u}}{=} g \left\{ \frac{kT}{\hbar\omega} \underbrace{\int_0^{\infty} dx \, \log(1 + z e^{-x})}_{\parallel \ \leftarrow \text{ partes}} - \frac{1}{24} \frac{\hbar\omega}{kT} \frac{z}{1+z} \right\}$$

$$x \log(1 + z e^{-x}) \Big|_0^{\infty}$$

$$+ \int_0^{\infty} dx \, \frac{x z e^{-x}}{1 + z e^{-x}}$$

$$\parallel$$

$$\int_0^{\infty} dx \, \frac{x}{z^{-1} e^{x} + 1}$$

$$\parallel$$

$$\Gamma(2) f_2(z) = f_2(z)$$

$$\Rightarrow \boxed{\log \mathcal{Z} = g \left\{ \frac{kT}{\hbar\omega} f_2(z) - \frac{1}{24} \frac{\hbar\omega}{kT} \frac{z}{1+z} \right\}}$$

Reemplazando g y ω por lo que valen,

$$g \frac{kT}{\hbar\omega} = \frac{2 A e\cancel{B}}{h\cancel{c}} \frac{kT}{\hbar \cancel{eB}} m\cancel{c}$$

$$= 2A \frac{2\pi m kT}{h^2} = 2 \frac{A}{\lambda^2}$$

$$g \hbar\omega = \frac{2AeB}{hc} \frac{\hbar eB}{mc}$$

$$= \frac{\cancel{2}AeB}{c} \frac{eB}{2\pi mc} = \frac{A}{\pi} \frac{(eB)^2}{mc^2}$$

$$\Rightarrow \log \mathcal{Z} = 2 \frac{A}{\lambda^2} f_2(z) - \frac{1}{24\pi} A \frac{1}{kT} \frac{(eB)^2}{mc^2} \frac{z}{1+z}$$

$\rightarrow$ Si $B = 0$, $\log \mathcal{Z} = 2 \frac{A}{\lambda^2} f_2(z)$ gas ideal de fermiones.

(b) $$M = kT\left(\frac{\partial}{\partial B}\log Z\right)_{T,A,z} =$$

$$= -\cancel{kT}\,\frac{1}{12\pi}\,A\,\frac{1}{\cancel{kT}}\,\frac{e^2}{mc^2}\,B\,\frac{z}{1+z}$$

$$= -\frac{A}{12\pi}\,\frac{e^2}{mc^2}\,B\,\frac{z}{1+z}$$

Orden 1 en B $\Rightarrow$ Calculamos z a orden 0 en B

$$N = 2\,\frac{A}{\lambda^2}\,f_1(z)$$

$$f_1(z) = \int_0^\infty dx\,\frac{1}{z^{-1}e^x+1} = \int_0^\infty dx\,\underbrace{\frac{ze^{-x}}{1+ze^{-x}}}_{= -\frac{d}{dx}\log(1+ze^{-x})}$$

$$= -\log(1+ze^{-x})\Big|_0^\infty = \log(1+z)$$

$$\Rightarrow N = 2\,\frac{A}{\lambda^2}\log(1+z) \Rightarrow \frac{N\lambda^2}{2A} = \log(1+z)$$

$$\Rightarrow 1+z = e^{\frac{N\lambda^2}{2A}} \Rightarrow z = e^{\frac{N\lambda^2}{2A}} - 1$$

$$\Longrightarrow \quad \frac{z}{1+z} = \frac{e^{\frac{N\lambda^2}{2A}} - 1}{e^{\frac{N\lambda^2}{2A}}} = 1 - e^{-\frac{N\lambda^2}{2A}}$$

$$\Longrightarrow \quad \boxed{M = -\frac{A}{12\pi}\frac{e^2}{mc^2}\left(1 - e^{-\frac{N\lambda^2}{2A}}\right) B}$$

$$\Longrightarrow \quad \boxed{\chi = -\frac{A}{12\pi}\frac{e^2}{mc^2}\left(1 - e^{-\frac{N\lambda^2}{2A}}\right)}$$

$\chi < 0 \quad \Longrightarrow$ Diamagnetismo

$h \to 0 \quad \Rightarrow \quad \lambda \to 0 \quad \Longrightarrow \quad \chi \to 0$ Efecto puramente cuántico.

2. (a) $\log \mathcal{Z} = \frac{V}{\lambda^d} g_{\frac{d}{2}}(z) \simeq \frac{V}{\lambda^d} z$

$N = \frac{V}{\lambda^d} z \qquad$ si $z < 1$

$N = N_0 + \frac{V}{\lambda^d} \qquad$ si $z = 1$

$$z < 1 \implies z = \frac{\lambda^d}{\sigma} \implies \frac{\sigma}{\lambda^d} > 1$$

Entonces,

$$z = \begin{cases} \dfrac{\lambda^d}{\sigma} & \text{si } \dfrac{\sigma}{\lambda^d} > 1 \\[2ex] 1 & \text{si } \dfrac{\sigma}{\lambda^d} < 1 \end{cases}$$

$$\boxed{f = \begin{cases} 0 & \text{si } \dfrac{\sigma}{\lambda^d} > 1 \\[2ex] 1 - \dfrac{\sigma}{\lambda^d} & \text{si } \dfrac{\sigma}{\lambda^d} < 1 \end{cases}}$$

$$p = \frac{kT}{V} \log \mathcal{Z} = \frac{kT}{\lambda^d} z$$

$$\boxed{p = \begin{cases} \dfrac{kT}{\sigma} & \text{si } \dfrac{\sigma}{\lambda^d} > 1 \\[2ex] \dfrac{kT}{\lambda^d} & \text{si } \dfrac{\sigma}{\lambda^d} < 1 \end{cases}}$$

$$\underset{\|}{\Phi} = - kT \log \mathcal{Z} = - kT \frac{V}{\lambda^d} z$$

$U - TS - \mu N$

$$\Rightarrow TS = U - \mu N - \Phi$$

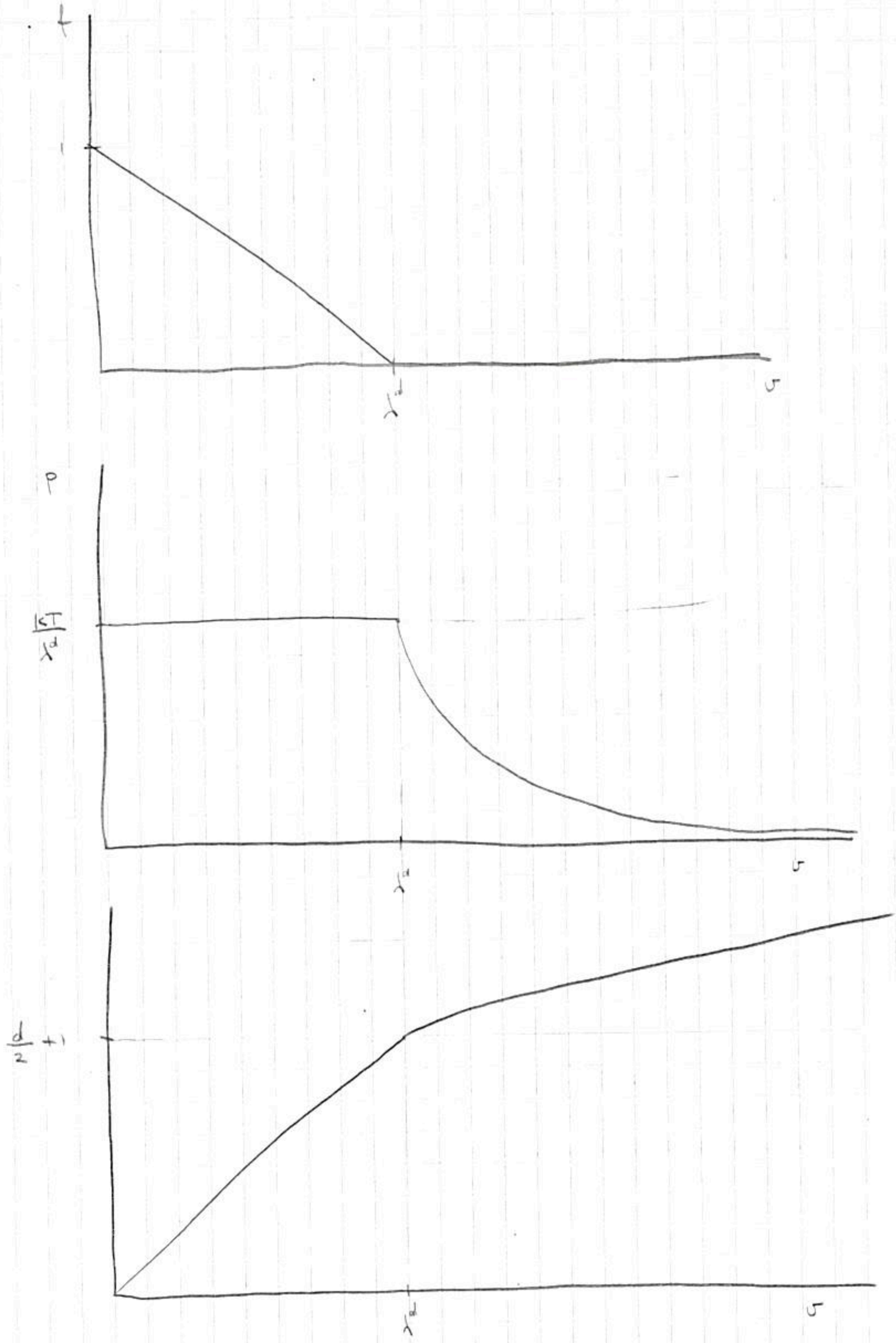
$$U = - \frac{\partial}{\partial \beta} \log \mathcal{Z} = \frac{d}{2} kT \frac{V}{\lambda^d} z$$

$$\Rightarrow S = \frac{1}{\cancel{T}} \left\{ \frac{d}{2} k\cancel{T} \frac{V}{\lambda^d} z - k\cancel{T} (\log z) N + k\cancel{T} \frac{V}{\lambda^d} z \right\}$$

$$= k \left\{ \left(\frac{d}{2} + 1 \right) \frac{V}{\lambda^d} z - N \log z \right\}$$

$\Rightarrow$ Entropia por partícula $\quad s = k \left\{ \left(\frac{d}{2} + 1 \right) \frac{v}{\lambda^d} z - \log z \right\}$

$$\boxed{s = \begin{cases} k \left[\frac{d}{2} + 1 + \log \frac{v}{\lambda^d} \right] & \text{si} \quad \frac{v}{\lambda^d} > 1 \\ k \left(\frac{d}{2} + 1 \right) \frac{v}{\lambda^d} & \text{si} \quad \frac{v}{\lambda^d} < 1 \end{cases}}$$


f
1
λ^d
v
p
$\frac{kT}{\lambda^d}$
λ^d
v
$\frac{d}{2}+1$
λ^d
v

A presión $\frac{kT}{\lambda^d}$ podemos tener muchos valores distintos de v porque podemos tener distintas proporciones de fase normal y fase condensada.

Fase condensada no tiene entropía, por eso al disminuir v (y por lo tanto aumentar $\frac{1}{v}$) disminuye s.

(b)

$$Q_{cedido} = -Q_{absorbido} \underset{\text{cuasiestático}}{=} -\int T\,dS \underset{\text{Isotermo}}{=} -T\left(S_{fin} - S_{in}\right)$$

$$= T\left(S_{in} - S_{fin}\right)$$

$\Rightarrow$ Por partícula, $q_{cedido} = T\left(\underset{k\left(\frac{d}{2}+1\right)}{s_{in}} - \underset{0}{s_{fin}}\right)$

$$\Rightarrow \boxed{q_{cedido} = kT\left(\frac{d}{2}+1\right)}$$

3. $$H = - \sum_{i=1}^{N} \left[J s_{i-1} s_i + \mu B s_i \right]$$

$$= - \sum_{i=1}^{N} \left[J s_{i-1} s_i + \frac{\mu B}{2} \left(s_{i-1} + s_i \right) \right]$$

$\uparrow$ cerrada

$$= - \sum_{i=1}^{N/3} \Big[J \left(s_{3i-3} s_{3i-2} + s_{3i-2} s_{3i-1} + s_{3i-1} s_{3i} \right) + \frac{\mu B}{2} \left(s_{3i-3} + 2 s_{3i-2} + 2 s_{3i-1} + s_{3i} \right) \Big]$$

$$Q = \sum_{\{s_j\}} e^{-\beta H}$$

$$= \sum_{\{s_j\}} e^{\sum_{i=1}^{N/3} \left[K_1 \left(s_{3i-3} s_{3i-2} + s_{3i-2} s_{3i-1} + s_{3i-1} s_{3i} \right) + \frac{K_2}{2} \left(s_{3i-3} + 2 s_{3i-2} + 2 s_{3i-1} + s_{3i} \right) \right]}$$

$\uparrow$

$K_1 = \beta J$

$K_2 = \mu B$

$$\sum_{\{s_{3i-2},\, s_{3i-1}\}} e^{-\beta H} = \prod_{i=1}^{N/3} e^{\frac{K_2}{2} \left(s_{3i-3} + s_{3i} \right)} \times$$

$$\times \underbrace{\sum_{s,s'} e^{K_1 \left(s_{3i-3} s + s s' + s' s_{3i} \right) + K_2 \left(s + s' \right)}}_{\parallel}$$

$$2 e^{K_1} \cosh \left[K_1 \left(s_{3i-3} + s_3 \right) + 2K_2 \right] + 2 e^{-K_1} \cosh \left[K_1 (s_{3i} - s \right.$$

$$\Rightarrow Q = \sum_{\{s_{3i}\}} \prod_{i=1}^{N/3} e^{\frac{K_2}{2}(s_{3i-3}+s_{3i})} \, 2 \left\{ e^{K_1} \cosh\left[K_1(s_{3i-3}+s_{3i}) + 2K_2\right] + e^{-K_1} \cosh\left[K_1(s_{3i-3}-s_{3i})\right] \right\}$$

$$\underset{\substack{\uparrow \\ s_i' = s_{3i}}}{=} \sum_{\{s_i'\}} \prod_{i=1}^{N/3} \underbrace{e^{\frac{K_2}{2}(s_{i-1}'+s_i')} \, 2 \left\{ e^{K_1} \cosh\left[K_1(s_{i-1}'+s_i') + 2K_2\right] + e^{-K_1} \cosh\left[K_1(s_{i-1}'-s_i')\right] \right\}}_{\parallel}$$

$$e^{K_0' + K_1' s_{i-1}' s_i' + \frac{K_2'}{2}(s_{i-1}'+s_i')}$$

① $++$ $$e^{K_0' + K_1' + K_2'} = 2 e^{K_2} \left\{ e^{K_1} \cosh\left[2(K_1+K_2)\right] + e^{-K_1} \right\}$$

② $--$ $$e^{K_0' + K_1' - K_2'} = 2 e^{-K_2} \left\{ e^{K_1} \cosh\left[2(K_1-K_2)\right] + e^{-K_1} \right\}$$

③ $+-$ $$e^{K_0' - K_1'} = 2 \left\{ e^{K_1} \cosh(2K_2) + e^{-K_1} \cosh(2K_1) \right\}$$

$$\frac{① \times ②}{③^2} \longrightarrow \boxed{e^{4K_1'} = \frac{\left\{ e^{K_1} \cosh\left[2(K_1+K_2)\right] + e^{-K_1} \right\}\left\{ e^{K_1} \cosh\left[2(K_1-K_2)\right] + e^{-K_1} \right\}}{\left[e^{K_1} \cosh(2K_2) + e^{-K_1} \cosh(2K_1) \right]^2}}$$

$$\frac{①}{②} \longrightarrow \boxed{e^{2K_2'} = e^{2K_2} \, \frac{e^{K_1} \cosh\left[2(K_1+K_2)\right] + e^{-K_1}}{e^{K_1} \cosh\left[2(K_1-K_2)\right] + e^{-K_1}}}$$

(b) $K_2 = 0 \Rightarrow e^{2K_2'} = 1 \Rightarrow K_2' = 0.$

$$e^{4K_1'} = \frac{\left[e^{K_1}\cosh(2K_1) + e^{-K_1}\right]^2}{\left[e^{K_1} + e^{-K_1}\cosh(2K_1)\right]^2}$$

$$\Rightarrow e^{2K_1'} = \frac{e^{K_1}\cosh(2K_1) + e^{-K_1}}{e^{K_1} + e^{-K_1}\cosh(2K_1)}$$

$x \equiv e^{-2K_1}$

$$\Rightarrow x' = \frac{\frac{1}{\sqrt{x}} + \frac{\sqrt{x}}{2}\left(\frac{1}{x} + x\right)}{\frac{1}{2\sqrt{x}}\left(\frac{1}{x} + x\right) + \sqrt{x}}$$

$$= \frac{3x + x^3}{1 + 3x^2} \Rightarrow \boxed{x' = \underbrace{\frac{3x + x^3}{1 + 3x^2}}_{R(x)}}$$

$x = 0$ pto fijo

$x \neq 0$ punto fijo $\Leftrightarrow 1 = \frac{3 + x^2}{1 + 3x^2} \Rightarrow 1 + 3x^2 = 3 + x^2$

$2x^2 = 2$

$x = \pm 1$ (- no tiene sentido)

$\Longrightarrow$ 2 únicos ptos fijos, $\boxed{x = 0,\ 1}$

Estabilidad:

$$R'(x) = \frac{3 + 3x^2}{1 + 3x^2} - \frac{3x + x^3}{(1 + 3x^2)^2}\ 6x$$

$$R'(0) = 3 > 1 \Longrightarrow \boxed{x = 0 \text{ inestable}}$$

$$R'(1) = \frac{6}{4} - \frac{4}{4^2} \cdot 6 = 0 < 1 \Longrightarrow \boxed{x = 1 \text{ estable}}$$