Guía 5, clase 1

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$$E = \frac{3}{2} kT g_s \frac{V}{\lambda^3} f_{5/2}(z)$$

$$f_{3/2}(z) = \frac{\lambda^3}{g_s v}$$

Sommerfeld: $\underset{\substack{\uparrow \\ z \gg 1}}{f_\nu(z) =} \frac{(\log z)^\nu}{\Gamma(\nu+1)} \left[1 + \frac{\pi^2}{6} \frac{\nu(\nu-1)}{(\log z)^2} + O\left((\log z)^{-4}\right) \right]$

T baja (V, N fijos) $\Rightarrow$ $\underline{\lambda^3 \gg v}$ $\Rightarrow$ $f_{3/2}(z) \gg 1$

$\Rightarrow$ $z \gg 1$ $\Rightarrow$ Aplica Sommerfeld:

$$f_{3/2}(z) = \frac{(\cancel{\beta}\mu)^{3/2}}{\Gamma(5/2)} \left[1 + \frac{\pi^2}{8} \left(\frac{kT}{\mu}\right)^2 + O\left(\left(\frac{kT}{\mu}\right)^4\right) \right]$$

$$\underset{\substack{\uparrow \\ \text{Ec. para } N}}{=} \frac{1}{g_s v} \underbrace{\left(\frac{h^2}{2\pi m}\right)^{3/2} \cancel{\left(\frac{1}{kT}\right)^{3/2}}}_{= \lambda^3}$$

$$\Rightarrow \mu = \underbrace{\left(\frac{\Gamma(5/2)}{g_s v}\right)^{2/3} \frac{h^2}{2\pi m}}_{= \mu(T=0) \equiv \epsilon_F} \left[1 + \frac{\pi^2}{8}\left(\frac{kT}{\mu}\right)^2 + O\left(\left(\frac{kT}{\mu}\right)^4\right)\right]^{-2/3}$$

$$(\log z)^{-4} = (\beta\mu)^{-4}$$

$$= \left(\frac{1}{\beta\mu}\right)^4$$

$$\underset{\substack{\uparrow \\ (1+x)^\alpha = 1 + \alpha x + O(x^2)}}{=} \epsilon_F \left[1 - \underbrace{\frac{2}{3}\frac{\pi^2}{8}}_{= \frac{\pi^2}{12}} \left(\frac{kT}{\mu}\right)^2 + O\left(\left(\frac{kT}{\mu}\right)^4\right)\right]$$

$$= \boxed{\epsilon_F \left[1 - \frac{\pi^2}{12} \left(\frac{kT}{\epsilon_F} \right)^2 + O(T^3) \right] = \mu}$$

$$\mu(T) = \mu(0) + O(T)$$

$$= \epsilon_F + O(T)$$

$$\Rightarrow \left(\frac{kT}{\mu} \right)^2 = \left[\frac{kT}{\epsilon_F (1 + O(T))} \right]^2$$

$$= \left(\frac{kT}{\epsilon_F} \right)^2 \left(\frac{1}{1 + O(T)} \right)^2$$

$$= \left(\frac{kT}{\epsilon_F} \right)^2 (1 + O(T))$$

$$= \left(\frac{kT}{\epsilon_F} \right)^2 + O(T^3)$$

$$\epsilon_F = \frac{h^2}{2\pi m}\left(\frac{\Gamma(5/2)}{g_s v}\right)^{2/3}$$

$$= \underbrace{2\pi\frac{\hbar^2}{m}}_{=\frac{\hbar^2}{2m}4\pi}\left(\frac{\Gamma(5/2)}{g_s v}\right)^{2/3} = \frac{\hbar^2}{2m}\left(\frac{\overbrace{(4\pi)^{3/2}}^{8\pi^{3/2}}\overbrace{\Gamma(5/2)}^{3\sqrt{\pi}/4}}{g_s v}\right)^{2/3}$$

$h = 2\pi\hbar$

$$= \boxed{\frac{\hbar^2}{2m}\left(\frac{6\pi^2}{g_s v}\right)^{2/3} = \epsilon_F}$$

$$E = \frac{3}{2}kT\, g_s \frac{V}{\lambda^3} f_{5/2}(z)$$

$$= \frac{3}{2}kT\, g_s\frac{V}{\lambda^3}\frac{(\beta\mu)^{5/2}}{\frac{5}{2}\Gamma(5/2)}\left[1 + \frac{5\pi^2}{8}\left(\frac{kT}{\mu}\right)^2 + O\left(\left(\frac{kT}{\mu}\right)^4\right)\right]$$

Sommerfeld

$\Gamma(\nu+1) = \nu\Gamma(\nu)$

$$\Gamma(\nu+1) = \nu\,\Gamma(\nu)$$

$$= \frac{3}{\cancel{2}} \underbrace{g_s V \left(\frac{2\pi m}{h^2}\right)^{3/2}}_{=\, N \epsilon_F^{-3/2}} \mu^{5/2} \frac{5}{\cancel{2}} \Gamma(5/2) \left[1 + \frac{5\pi^2}{8}\left(\frac{kT}{\mu}\right)^2 + O\left(\left(\frac{kT}{\mu}\right)^4\right)\right]$$

$$= \frac{3}{5} N \epsilon_F^{-3/2} \epsilon_F^{5/2} \underbrace{\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{\epsilon_F}\right)^2 + O(T^3)\right]^{5/2}}_{1 - \frac{5\pi^2}{24}\left(\frac{kT}{\epsilon_F}\right)^2 + O(T^3)} \times$$

$$\times \left[1 + \underbrace{\frac{5\pi^2}{8}\left(\frac{kT}{\mu}\right)^2}_{=\, \frac{5\pi^2}{8}\left(\frac{kT}{\epsilon_F}\right)^2 + O(T^3)} + O\left(\left(\frac{kT}{\mu}\right)^4\right)\right]$$

$$= \frac{3}{5} N \epsilon_F \left[1 - \frac{5\pi^2}{24}\left(\frac{kT}{\epsilon_F}\right)^2 + O(T^3)\right] \left[1 + \frac{5\pi^2}{8}\left(\frac{kT}{\epsilon_F}\right)^2 + O(T^3)\right]$$

$$= \frac{3}{5} N \epsilon_F \left[1 + 5\pi^2 \underbrace{\left(\frac{1}{8} - \frac{1}{24} \right)}_{= \frac{1}{12}} \left(\frac{kT}{\epsilon_F} \right)^2 + O(T^3) \right]$$

$$\Rightarrow \boxed{E = \frac{3}{5} N \epsilon_F \left[1 + \frac{5\pi^2}{12} \left(\frac{kT}{\epsilon_F} \right)^2 + O(T^3) \right]}$$

A $T = 0$, $E = \frac{3}{5} N \epsilon_F \neq 0$

energía de 1 part