

$$\rightarrow P(s_1, \dots, s_N) = \frac{1}{Z} e^{-\beta H(s_1, \dots, s_N)}$$

$$\langle s_i s_{i+r} \rangle = e^{-r/\xi}$$

lang. correlación

Agarra subcadena de los sitios múltiples de ℓ

$$s'_i = s_{\ell i}$$

$$\begin{aligned} P(s'_1, \dots, s'_{N/\ell}) &= P(s_\ell, s_{2\ell}, \dots, s_N) \\ &= \frac{1}{Z} \sum_{\{s_i, i \neq k\ell\}} e^{-\beta H(s_1, \dots, s_N)} \\ &= \frac{1}{Z'} e^{-\beta' H(s'_1, \dots, s'_{N/\ell})} \end{aligned}$$

Decimación

lo vemos
después

⇒ La subcadena es como una copia de la cadena entera
pero a temperatura T'

$$\langle s'_i s'_{i+r} \rangle = e^{-r/\xi'} \quad \text{long. correlación a temp. } T'$$

$$\langle s_{li} s_{li+l_r} \rangle = e^{-l_r/\xi}$$

$$\Rightarrow \frac{r}{\xi'} = \frac{l_r}{\xi} \quad \Rightarrow \boxed{\xi' = \frac{\xi}{l}}$$

$$T' = R_l(T)$$

Transformación del grupo de renormalización



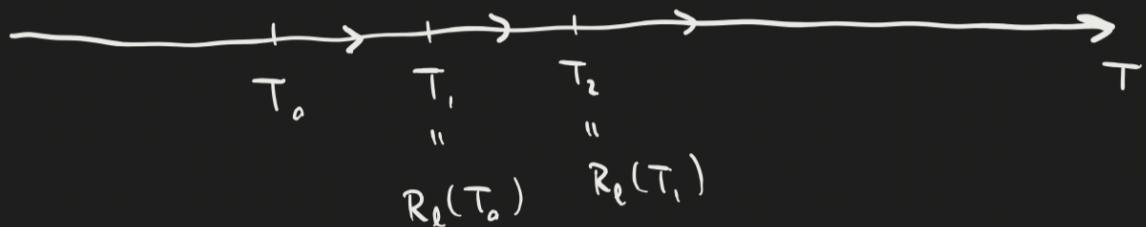
$$R_{\ell_2} \circ R_{\ell_1} = R_{\ell_1 \ell_2}$$

$$\hookrightarrow \tilde{R}_\ell = R_\ell \circ \ell$$

Flujo del grupo de renormalización

$$R_\ell \circ R_{1/\ell} = R_\ell \cdot \frac{1}{\ell}$$

$\approx R_1$ = identidad



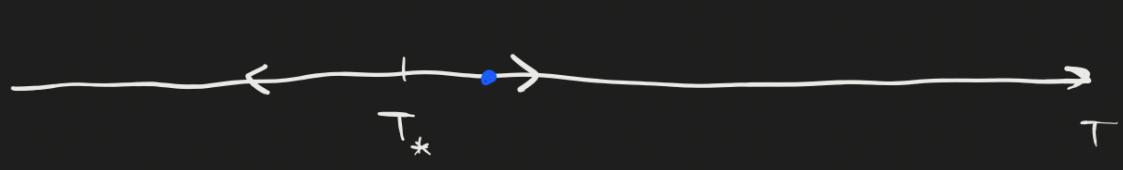
Puntos fijos: $\tau' = \tau$ ($R_\ell(\tau) = \tau$)

$$\Rightarrow \begin{cases} \tau' = \tau \\ \xi' = \frac{\xi}{\ell} \end{cases} \quad \Rightarrow \quad \xi = \frac{\xi}{\ell}$$

$$\Rightarrow \xi = 0, \infty$$



Pto fijo estable
 $\xi = 0$

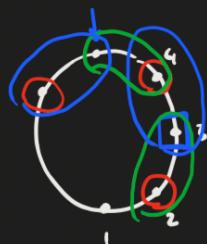


Pto fijo inestable
 $\xi = \infty$

\Rightarrow Ptos fijos inestables son ptos críticos

$$\boxed{\lambda = 2}$$

$$s'_i = s_{2i}$$



$$P(s'_1, \dots, s'_{N/2}) = P(s_2, s_4, \dots, s_N)$$

$$H(s_1, \dots, s_n) = -J \sum_{i=1}^{n/2} (s_{2i+1} s_{2i} + s_{2i+2}) \\ - J \sum_{i=1}^n s_i s_{i+1}$$

$$= \frac{1}{Z} \sum_{s_1, s_3, \dots} e^{-\beta H(s_1, \dots, s_N)}$$

$$= \frac{1}{Z} \underbrace{\sum_{s_1, s_3, \dots}}_{\prod_{i=1}^{N/2} Z_i} e^{+\kappa \sum_{i=1}^{N/2} s_{2i+1} (\underline{s_{2i} + s_{2i+2}})}$$

$$Z_i = \sum_{s_{2i+1} = \pm 1} e^{\kappa s_{2i+1} (s_{2i} + s_{2i+2})} = e^{\kappa (s_{2i} + s_{2i+2})} + e^{-\kappa (s_{2i} + s_{2i+2})}$$

$\uparrow \quad \quad \quad \uparrow$
 $s_{2i+1} = 1 \quad \quad \quad s_{2i+1} = -1$

$$= 2 \cosh [\kappa (s'_i + s'_{i+1})]$$

$$\Rightarrow \boxed{P(s'_1, \dots, s'_{N/2}) = \frac{1}{Z} \prod_{i=1}^{N/2} 2 \cosh [K(s'_i + s'_{i+1})]}$$

$$e^{-\beta' H(s'_1, \dots, s'_{N/2})} = e^{K' \sum_{i=1}^{N/2} s'_i s'_{i+1}} = \prod_{i=1}^{N/2} e^{K' s'_i s'_{i+1}}$$

$$P(s'_1, \dots, s'_{N/2}) = \frac{1}{Z'} e^{-\beta' H(s'_1, \dots, s'_{N/2})}$$

←

Quiero ver

$$\frac{1}{Z'} \prod_{i=1}^{N/2} e^{K' s'_i s'_{i+1}} = \frac{2^{N/2}}{Z} \prod_{i=1}^{N/2} \cosh [K(s'_i + s'_{i+1})] \quad \leftarrow$$

$$\Rightarrow \boxed{e^{K' s'_i s'_{i+1}} = \alpha \cosh [K(s'_i + s'_{i+1})]} \quad P'(m) = \alpha \downarrow P(m)$$

← D1

$$1 = \sum_n p'(n) = \alpha \sum_n p(n) = \alpha$$

$$\underline{s'_i = s'_{i+1} = 1}$$

$$e^{k'} = \alpha \cosh(2k) \quad \leftarrow$$

$$\underline{s'_i = s'_{i+1} = -1} \rightarrow \text{Misma ec.}$$

$$\underline{s'_i = 1, s'_{i+1} = -1}$$

$$e^{-k'} = \alpha \quad \leftarrow$$

$$\underline{s'_i = -1, s'_{i+1} = 1} \rightarrow \text{Misma ec.}$$

Dividido ambas ecas

$$\boxed{e^{2k'} = \cosh(2k)} \quad \begin{matrix} \uparrow J \\ \downarrow J \end{matrix}$$

Ec. grupo
renormalización

$$\operatorname{tgh} k' = \frac{\sinh k'}{\cosh k'} = \frac{e^{k'} - e^{-k'}}{e^{k'} + e^{-k'}}$$

$$\begin{aligned}
 &= \frac{\sqrt{\cosh 2k} - \frac{1}{\sqrt{\cosh 2k}}}{\sqrt{\cosh 2k} + \frac{1}{\sqrt{\cosh 2k}}} \\
 &= \frac{\cosh 2k - 1}{\cosh 2k + 1} = \frac{\cosh^2 k + \sinh^2 k - 1}{\cosh^2 k + \sinh^2 k + 1} \\
 &\quad \text{cosh } 2k = \cosh^2 k + \sinh^2 k \quad \cosh^2 k
 \end{aligned}$$

$$= \frac{2 \sinh^2 k}{2 \cosh^2 k} = \operatorname{tgh}^2 k$$

Def. $x \equiv \tanh k$ \Rightarrow $\boxed{x' = x^2}$ | ec. del grupo de renormalización



Ptos fijos $x' = x$ \Rightarrow $x = x^2$ \Rightarrow $\boxed{x = 0, 1}$ Ptos fijos



\rightarrow Pto crítico $x = 1 \Rightarrow k = \infty \Rightarrow \beta = \infty \Rightarrow \boxed{T = 0}$

5. De hecho, es fácil estudiar decimaciones arbitrarias con la ayuda de la matriz de transferencia. Consideré la cadena de Ising unidimensional, cerrada, sin campo magnético, formada por N spines y con constante de acople adimensional $\beta J = K$.

- (a) Si l es un divisor de N , pruebe que los spines ubicados en las posiciones $l, 2l, \dots, N$ (es decir, las posiciones múltiplos de l) se comportan como una nueva cadena de Ising con constante de acople adimensional K' , con

$$\tanh K' = (\tanh K)^l.$$

Ayuda: use la matriz de transferencia. Va a tener que calcular sus autovalores.

- (b) Encuentre los puntos fijos y estudie su estabilidad.

$$\begin{aligned}
 P(s_1, \dots, s_N) &= \frac{1}{Z} e^{-\beta H(s_1, \dots, s_N)} \quad \leftarrow \\
 &= \frac{1}{Z} e^{K \sum_{i=1}^N s_i s_{i+1}} \quad = \quad \frac{1}{Z} \prod_{i=1}^N e^{K s_i s_{i+1}} \\
 &\qquad \qquad \qquad \text{("} \quad \text{("} \\
 &\qquad \qquad \qquad q_{s_i s_{i+1}} \quad q_{s_i s_{i+1}}
 \end{aligned}$$

$$= \frac{1}{Z} q_{s_1 s_2} q_{s_2 s_3} \cdots q_{s_{N-1} s_N} q_{s_N s_1} \quad \leftarrow$$

$$\begin{aligned}
&= \frac{1}{Z} \left(q_{s_N s_1} \downarrow \cdots q_{s_{l-1} s_l} \downarrow \right) \\
&\quad \times \left(q_{s_1 s_{l+1}} \downarrow \cdots q_{s_{2l-1} s_{2l}} \downarrow \right) \\
&\quad \times \cdots \\
&\quad \times \left(q_{s_{N-l} s_{N-l+1}} \downarrow \cdots q_{s_{N-1} s_N} \downarrow \right) = P(s_1, \dots, s_N)
\end{aligned}$$

$$P(s_1, s_{2l}, \dots, s_N) = \frac{1}{Z} (q^l)_{s_N s_1} (q^l)_{s_1 s_{2l}} \cdots (q^l)_{s_{N-l} s_N}$$

$$s'_i = s_{l+i}$$

$$\rightarrow P(s'_1, \dots, s'_{N/l}) = \frac{1}{Z} \underbrace{(q^l)_{s'_{N/l} s'_1} (q^l)_{s'_1 s'_2} \cdots (q^l)_{s'_{N/l} s'_{N/l}}}_{}$$

$$= \boxed{\frac{1}{Z} \left(q^l \right)_{s'_1 s'_2} \dots \left(q^l \right)_{s'_{n/l} s'_1} = P(s'_1, \dots, s'_{n/l})}$$

$$\frac{1}{Z'} e^{-\int H(s'_1, \dots, s'_n)} = \underbrace{\frac{1}{Z'} q'_{s'_1 s'_2} \dots q'_{s'_{n/l} s'_1}}$$

$$\Rightarrow q' = \propto q^l \Rightarrow \lambda'_{\pm} = \propto \lambda_{\pm}^l$$

(λ_{\pm} autovalores de q)

$$\lambda_{\pm} = e^k \left(\cosh b \pm \sqrt{\sinh^2 b + e^{-4k}} \right) = e^k \pm e^k \sqrt{e^{-4k}}$$

\uparrow
 $b=0$

guia 7

$$= e^k \pm e^{-k} \Rightarrow \lambda_+ = 2 \cosh k$$
$$\lambda_- = 2 \sinh k$$

$$\Rightarrow 2 \cosh k' = \propto (2 \cosh k)^\ell$$
$$2 \sinh k' = \propto (2 \sinh k)^\ell$$

$$\Rightarrow \boxed{\tanh k' = (\tanh k)^\ell}$$