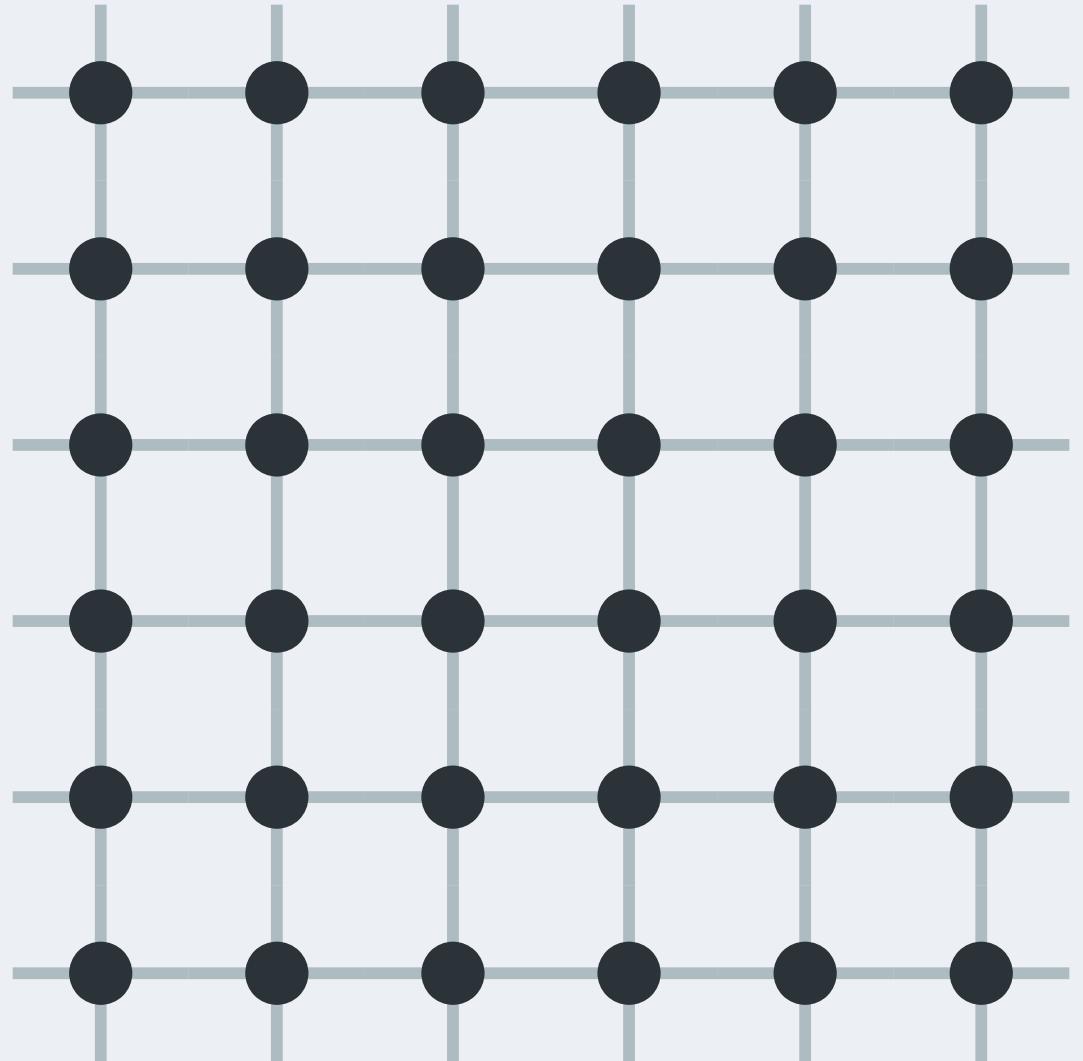
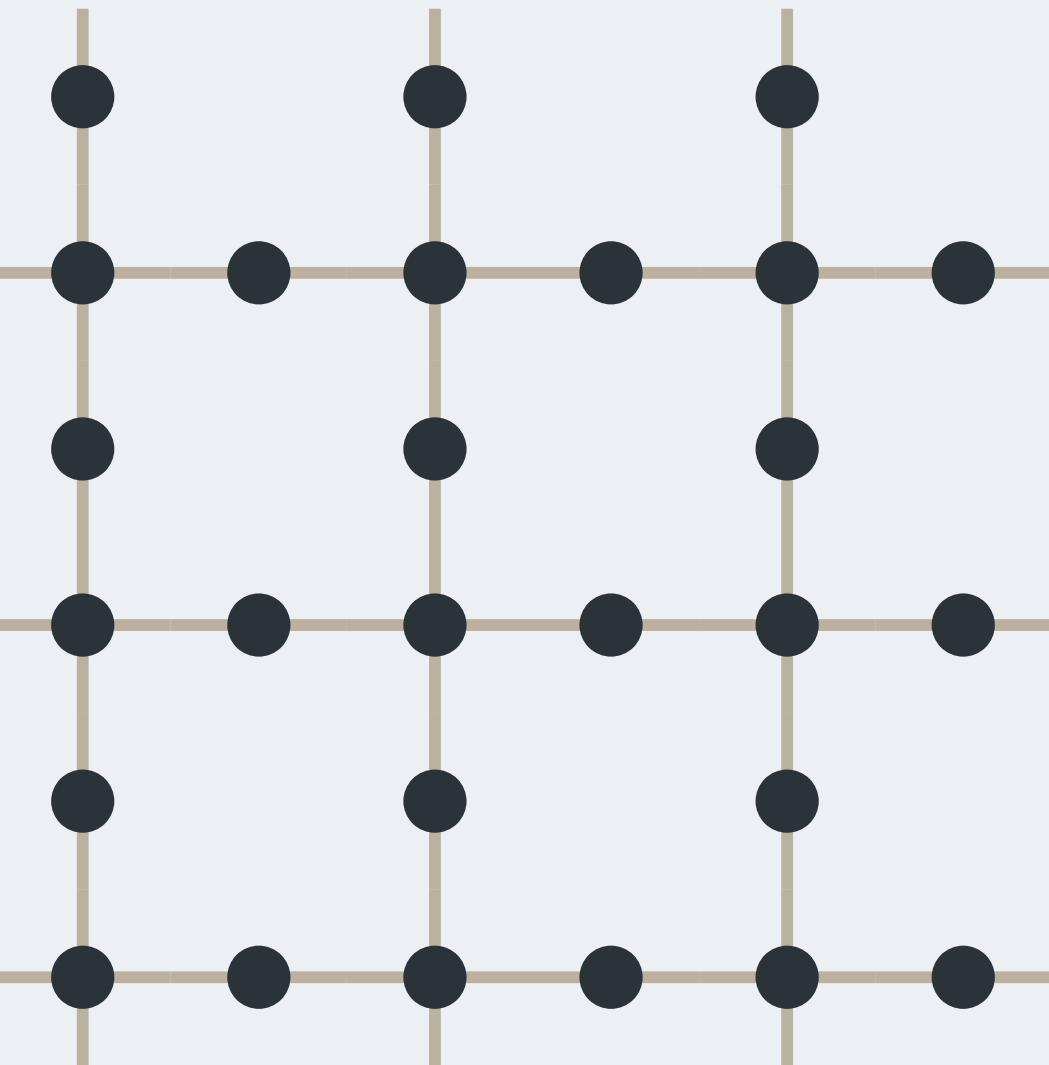
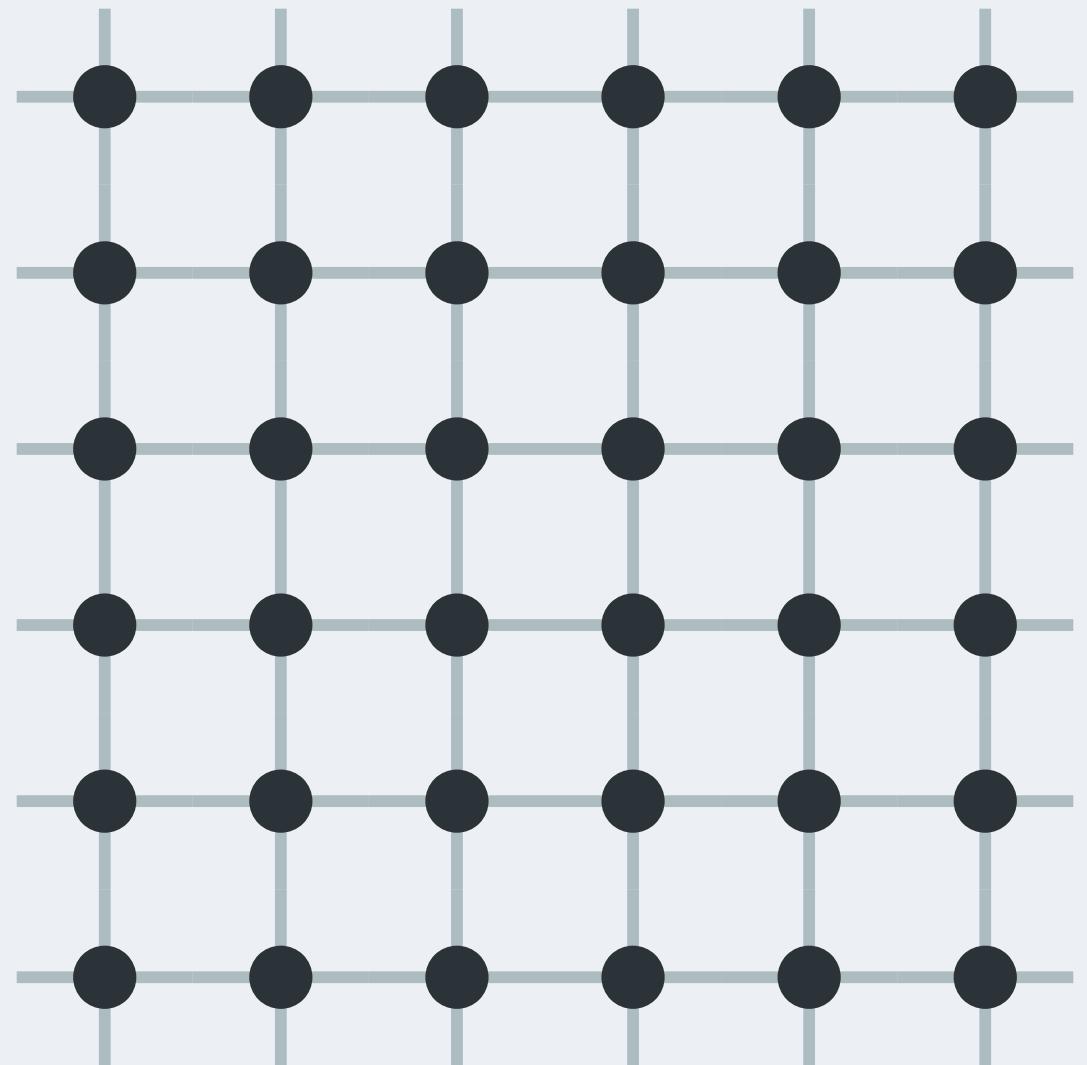
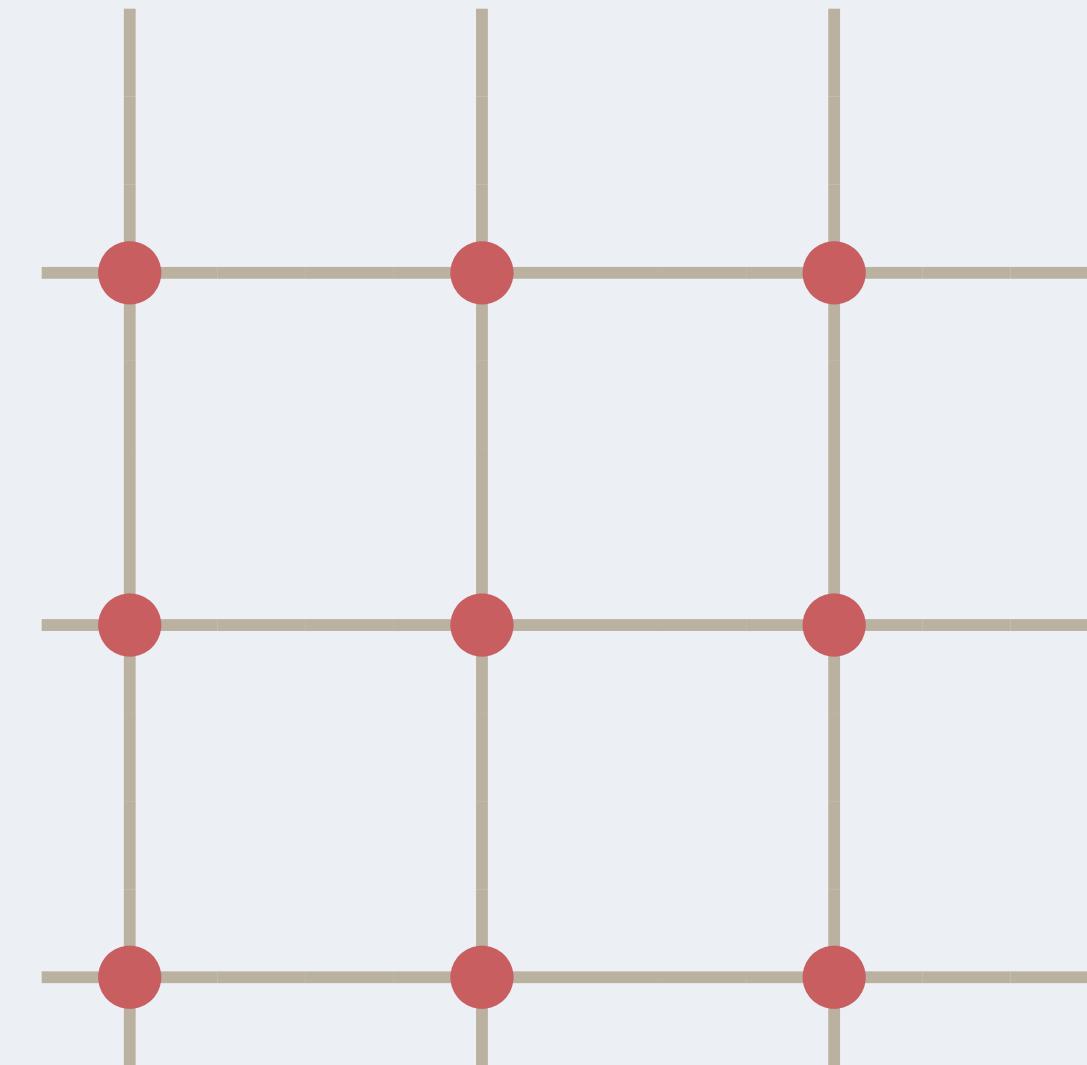
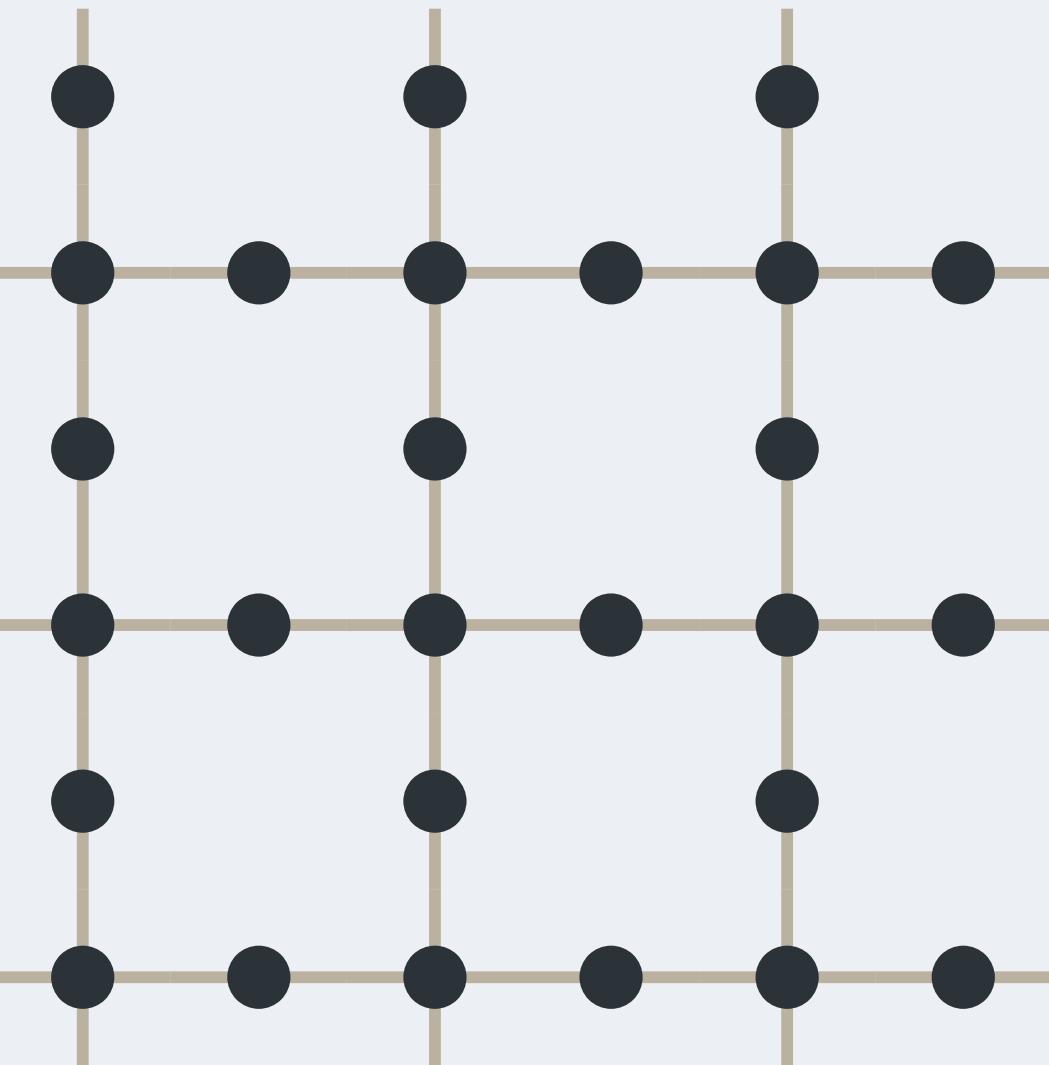
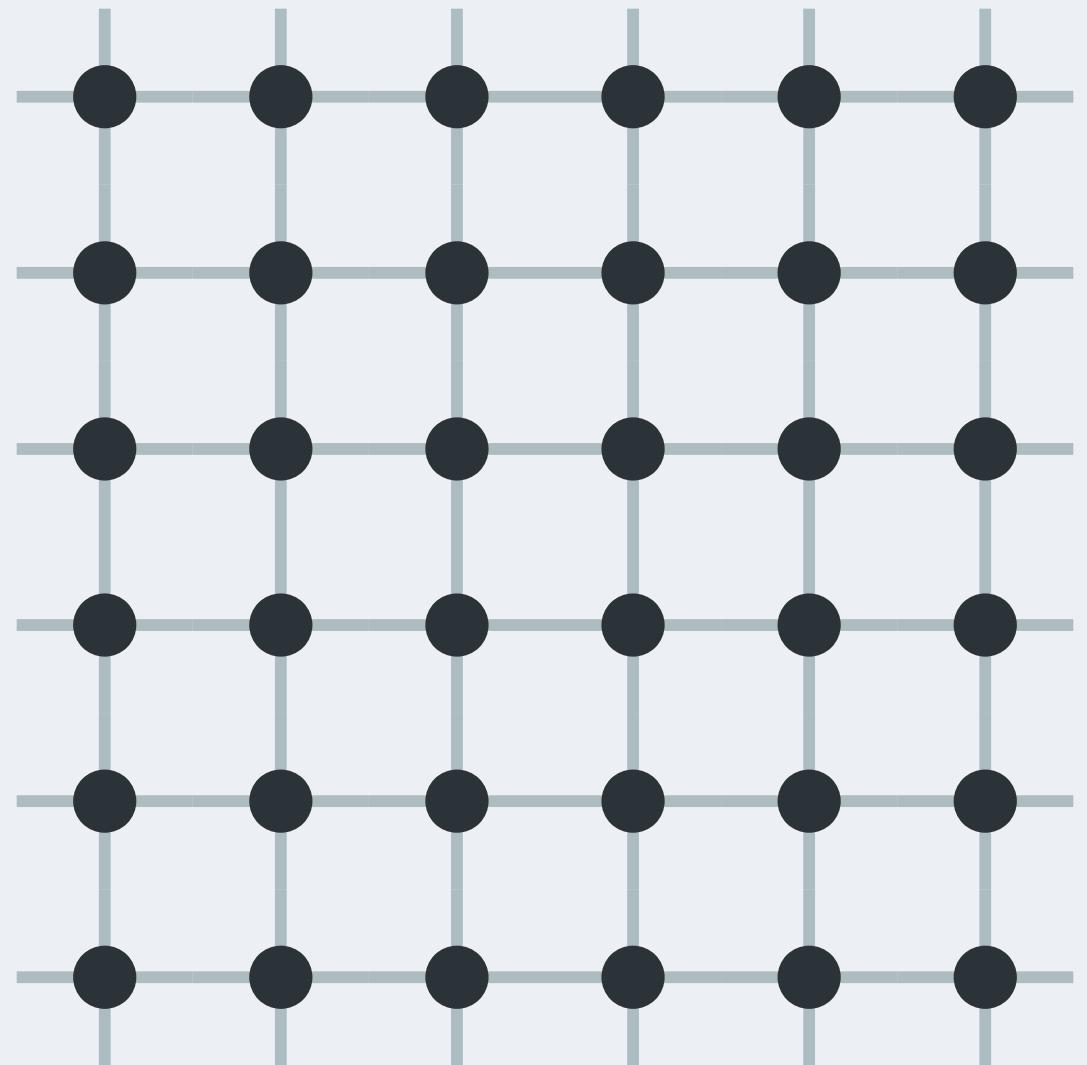


Hola





Aproximación

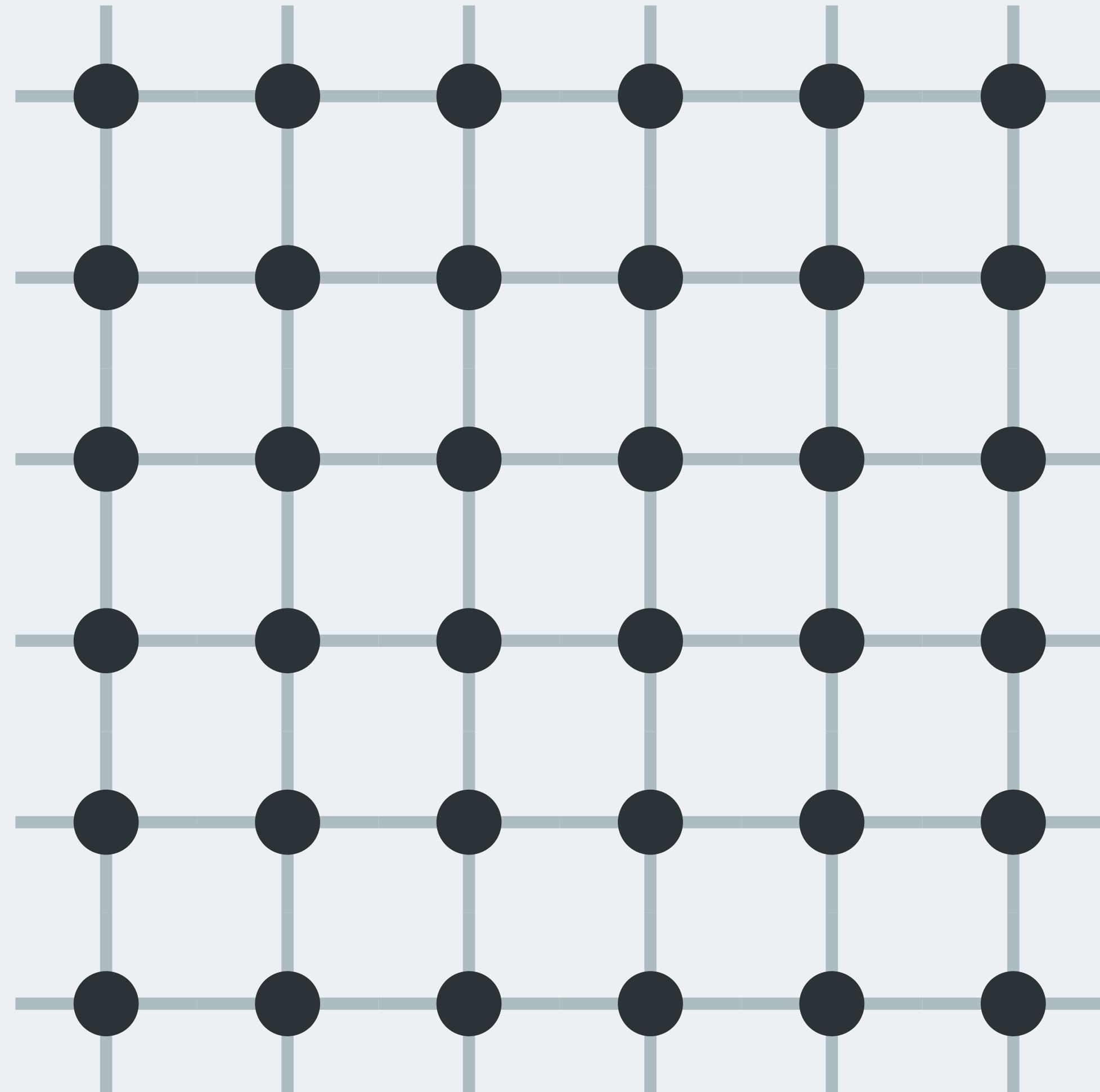


Aproximación

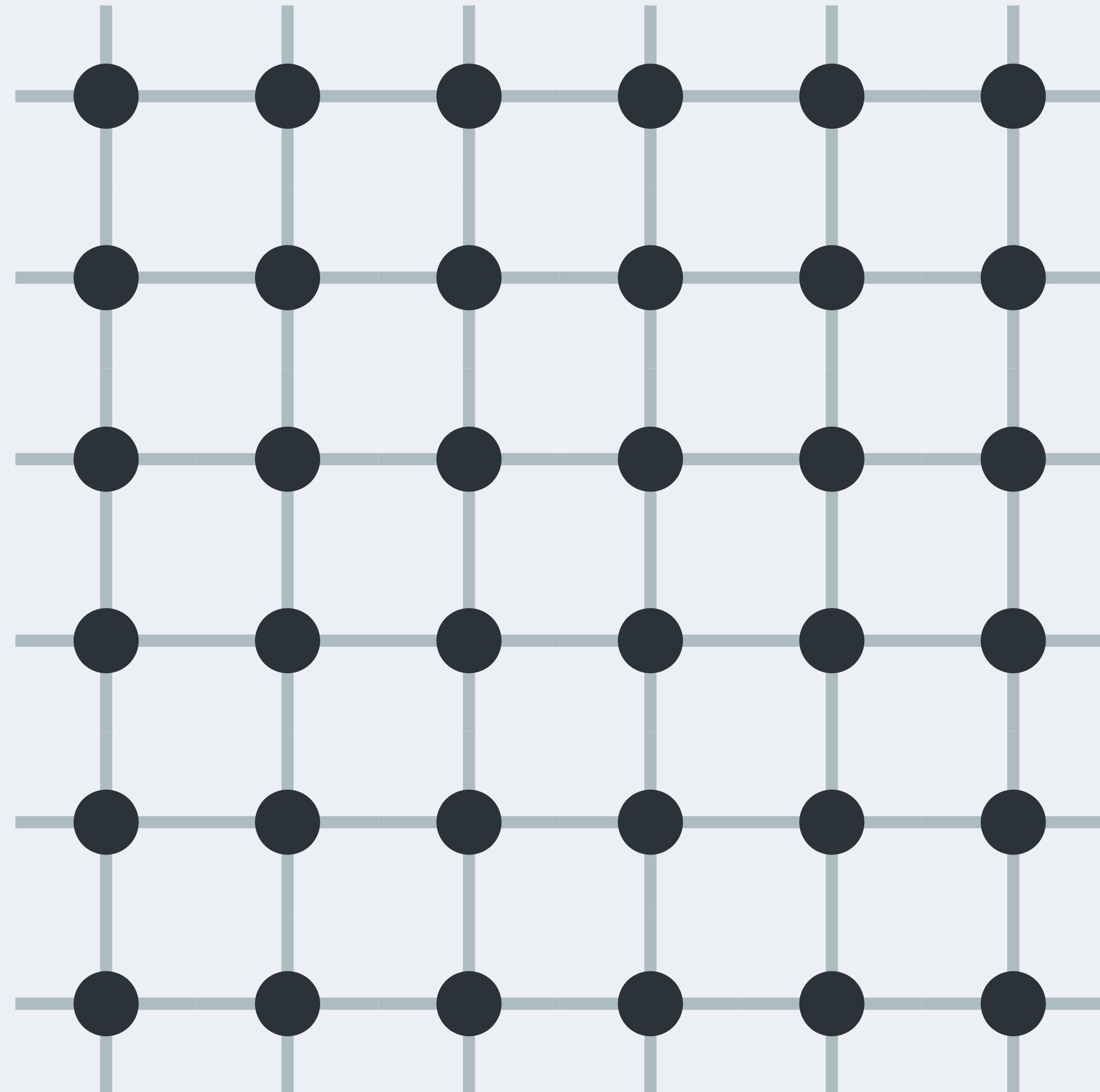


Diezmado

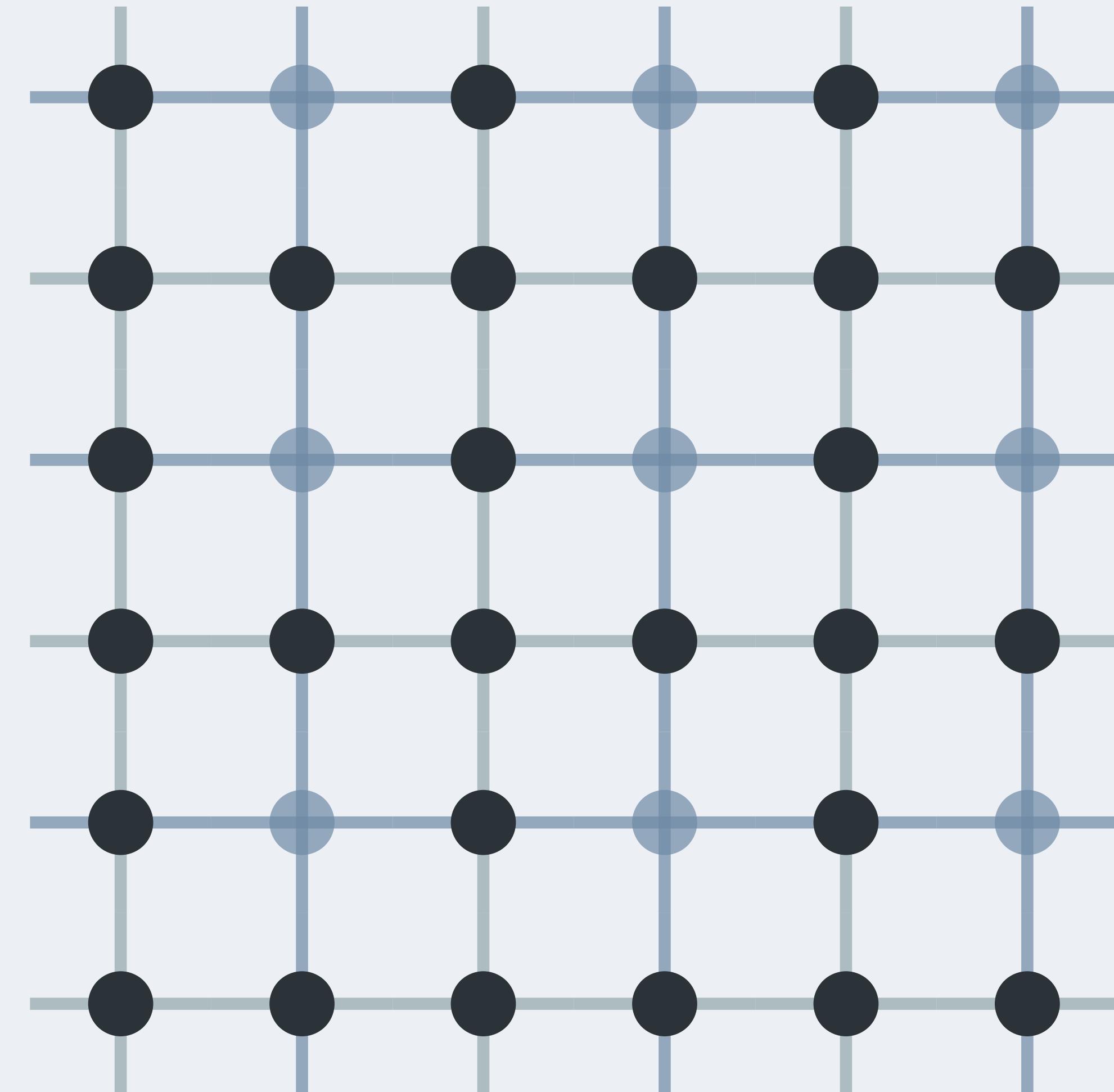




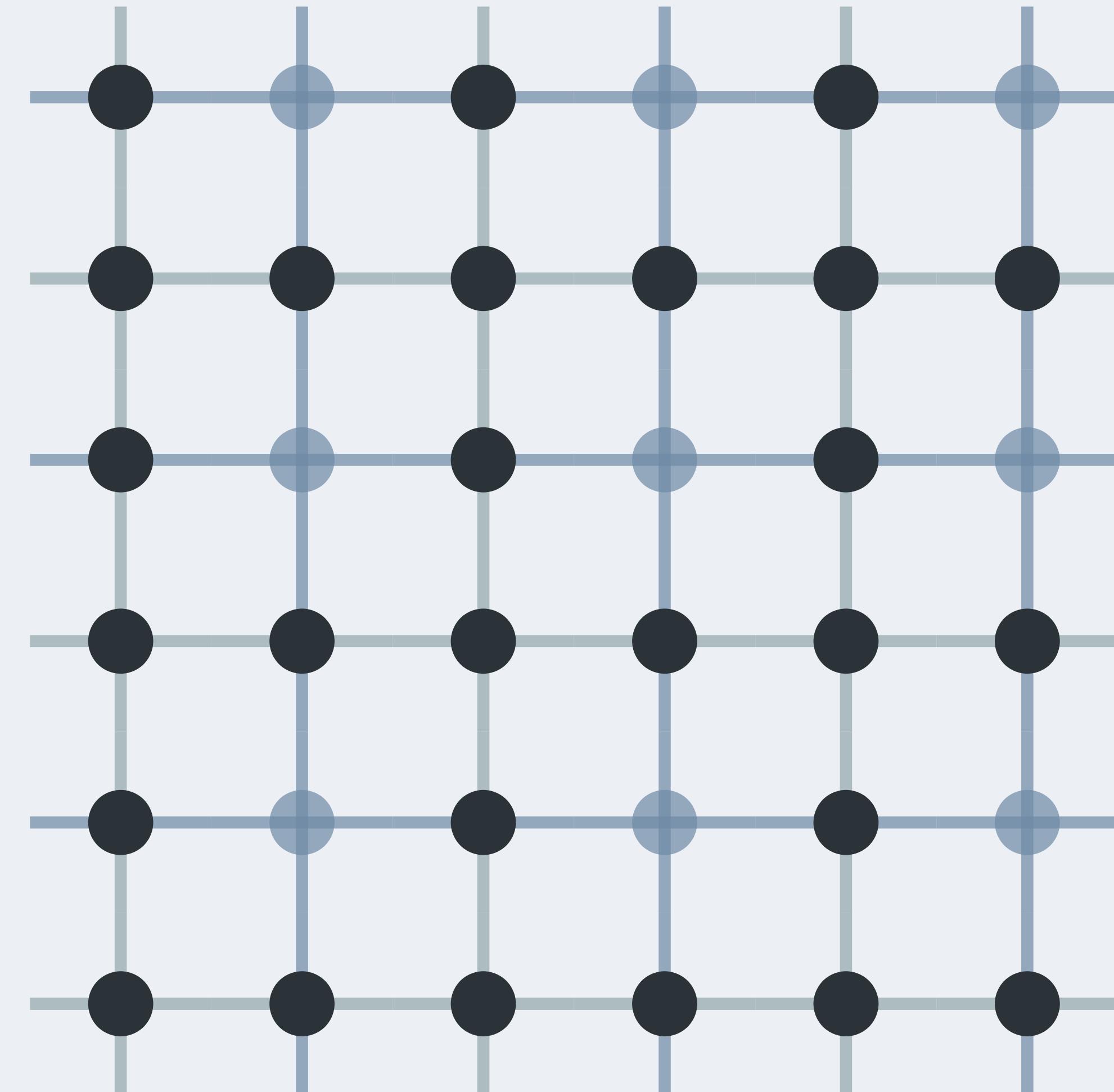
$$\beta H = -K \sum_{i,j=0}^{N-1} [s_{i,j}(s_{i+1,j} + s_{i,j+1})]$$



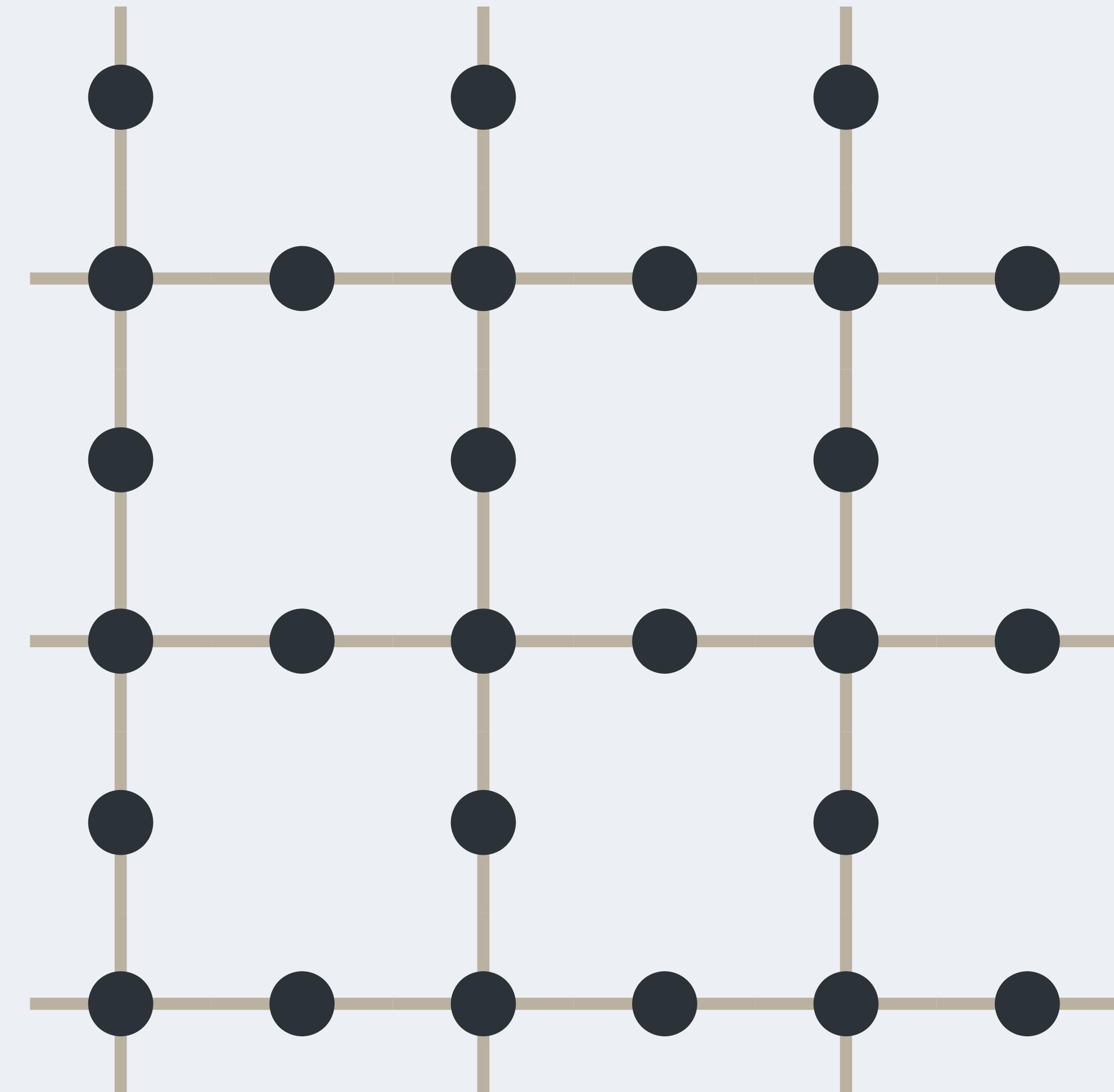
$$\begin{aligned} \beta H = & -K \sum_{i,j=0}^{N/2-1} \left[s_{2i+1,2j}(s_{2i,2j} + s_{2i+2,2j}) \right. \\ & \left. + s_{2i,2j+1}(s_{2i,2j} + s_{2i,2j+2}) \right] \\ & -K \sum_{i,j=0}^{N/2-1} \left[s_{2i+1,2j+1} \right. \\ & \left. (s_{2i,2j+1} + s_{2i+2,2j+1}) \right. \\ & \left. + s_{2i+1,2j} + s_{2i+1,2j+2} \right] \end{aligned}$$



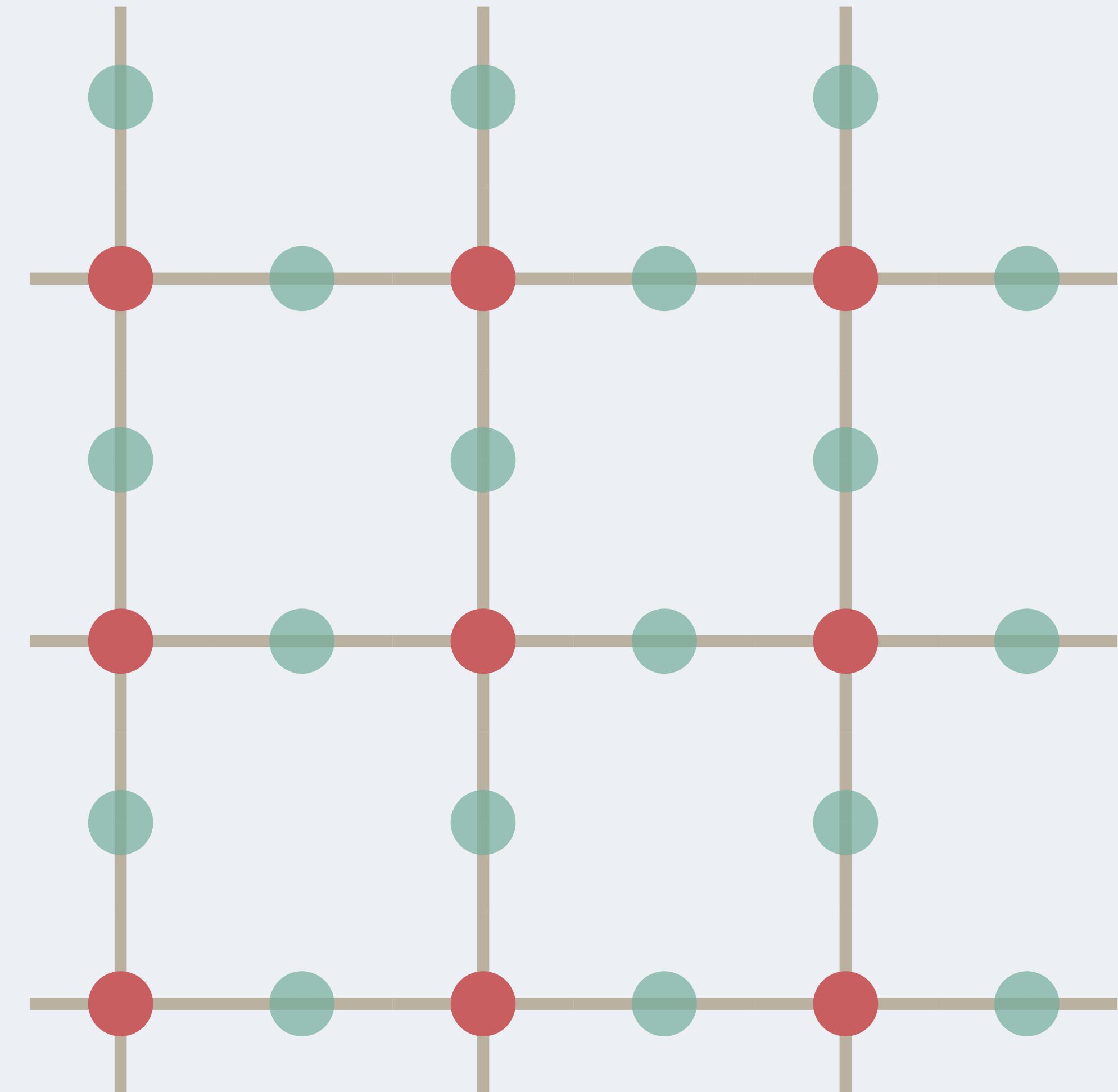
$$\beta H = -K \sum_{i,j=0}^{N/2-1} \left[s_{2i+1,2j}(s_{2i,2j} + s_{2i+2,2j}) + s_{2i,2j+1}(s_{2i,2j} + s_{2i,2j+2}) \right] - K \sum_{i,j=0}^{N/2-1} \left[s_{2i+1,2j+1}(s_{2i,2j+1} + s_{2i+2,2j+1}) + s_{2i+1,2j}(s_{2i+1,2j+1} + s_{2i+1,2j+2}) \right]$$



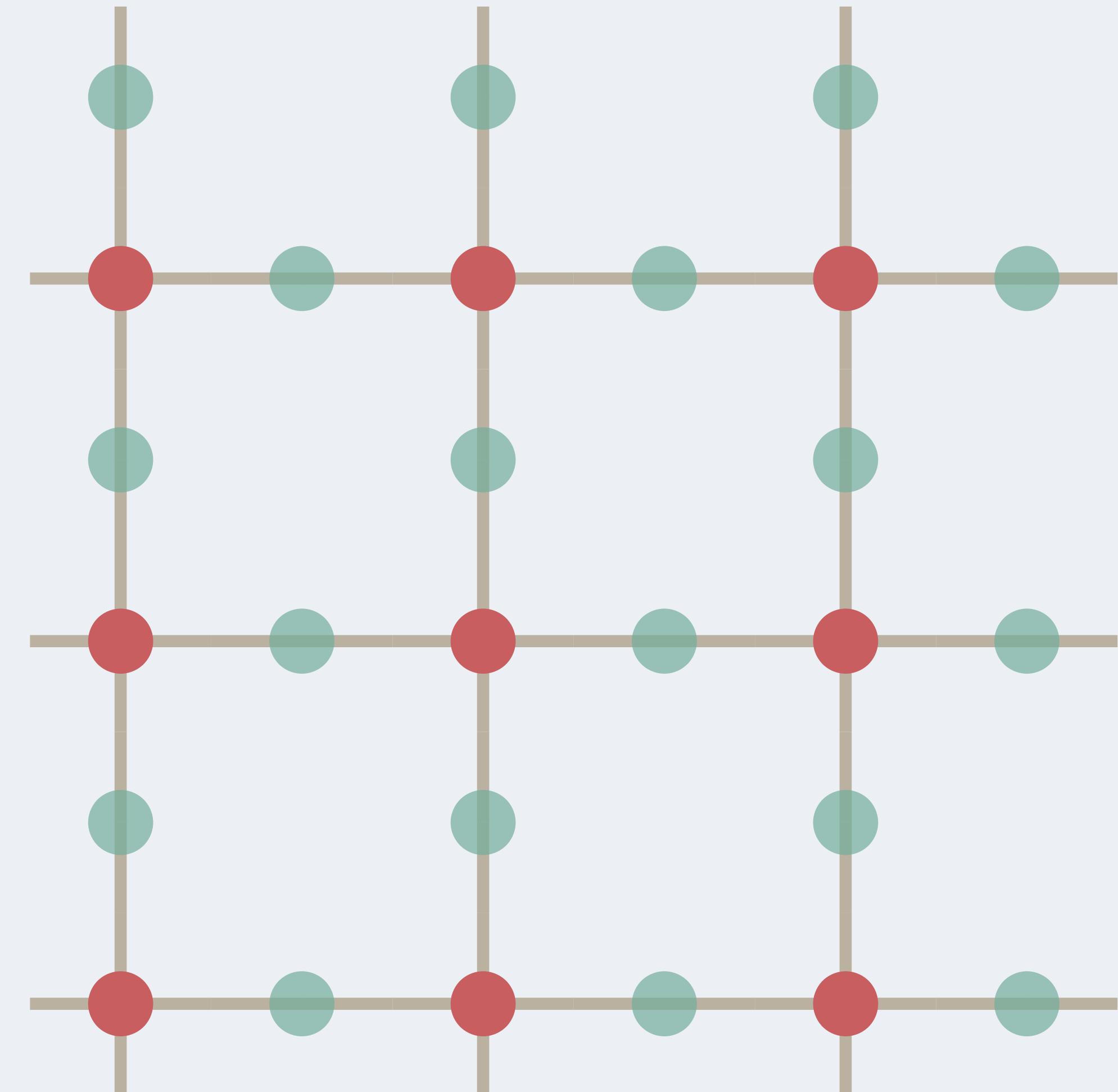
$$\beta H = -2K \sum_{i,j=0}^{N/2-1} \left[s_{2i+1,2j}(s_{2i,2j} + s_{2i+2,2j}) \right. \\ \left. + s_{2i,2j+1}(s_{2i,2j} + s_{2i,2j+2}) \right]$$
$$-K \sum_{i,j=0}^{N/2-1} \left[s_{2i+1,2j+1}(s_{2i,2j+1} + s_{2i+2,2j+1}) \right. \\ \left. + s_{2i+1,2j}(s_{2i+1,2j+1} + s_{2i+1,2j+2}) \right]$$



$$\beta H \simeq -2K \sum_{i,j=0}^{N/2-1} \left[s_{2i+1,2j}(s_{2i,2j} + s_{2i+2,2j}) + s_{2i,2j+1}(s_{2i,2j} + s_{2i,2j+2}) \right]$$



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$$\left(\frac{d}{dx}\right)^2\phi(x)=\frac{1}{2}\left(\frac{d^2}{dx^2}\right)\left(\frac{\partial \phi}{\partial x}\right)$$

$$= \sum_{k=1}^{N-1} \sum_{j=1}^N \sum_{i=1}^N \frac{1}{2} \delta_{i,j} \delta_{i+1,j} \delta_{i+2,j} \delta_{i+3,j} \delta_{i+4,j} \delta_{i+5,j}$$

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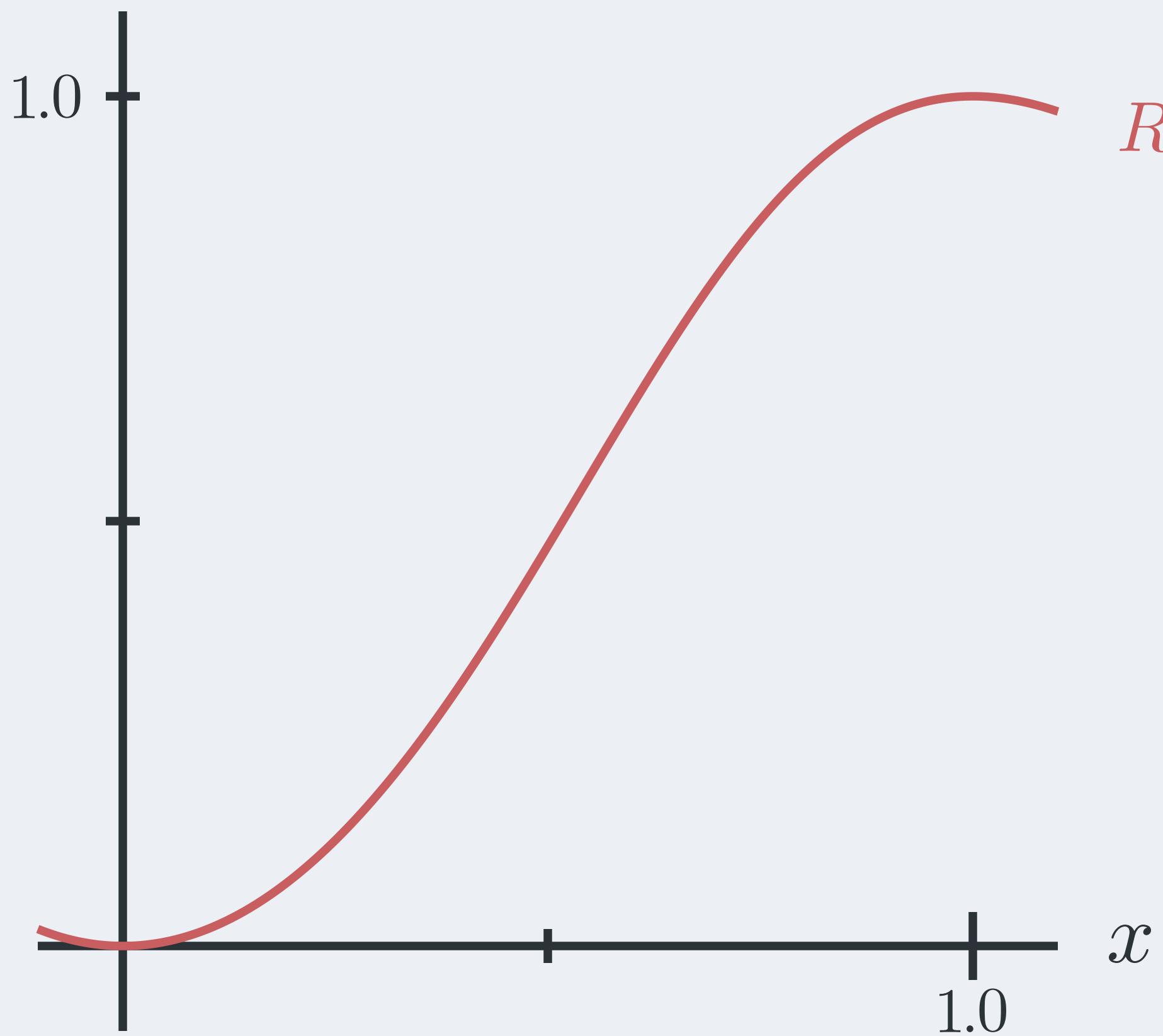
$$\left(\beta\,H\right)' = -K'\,\sum_{i,j=0}^{N/2-1} \left[s'_{i,j}\left(s'_{i+1,j} + s'_{i,j+1}\right)\right]$$

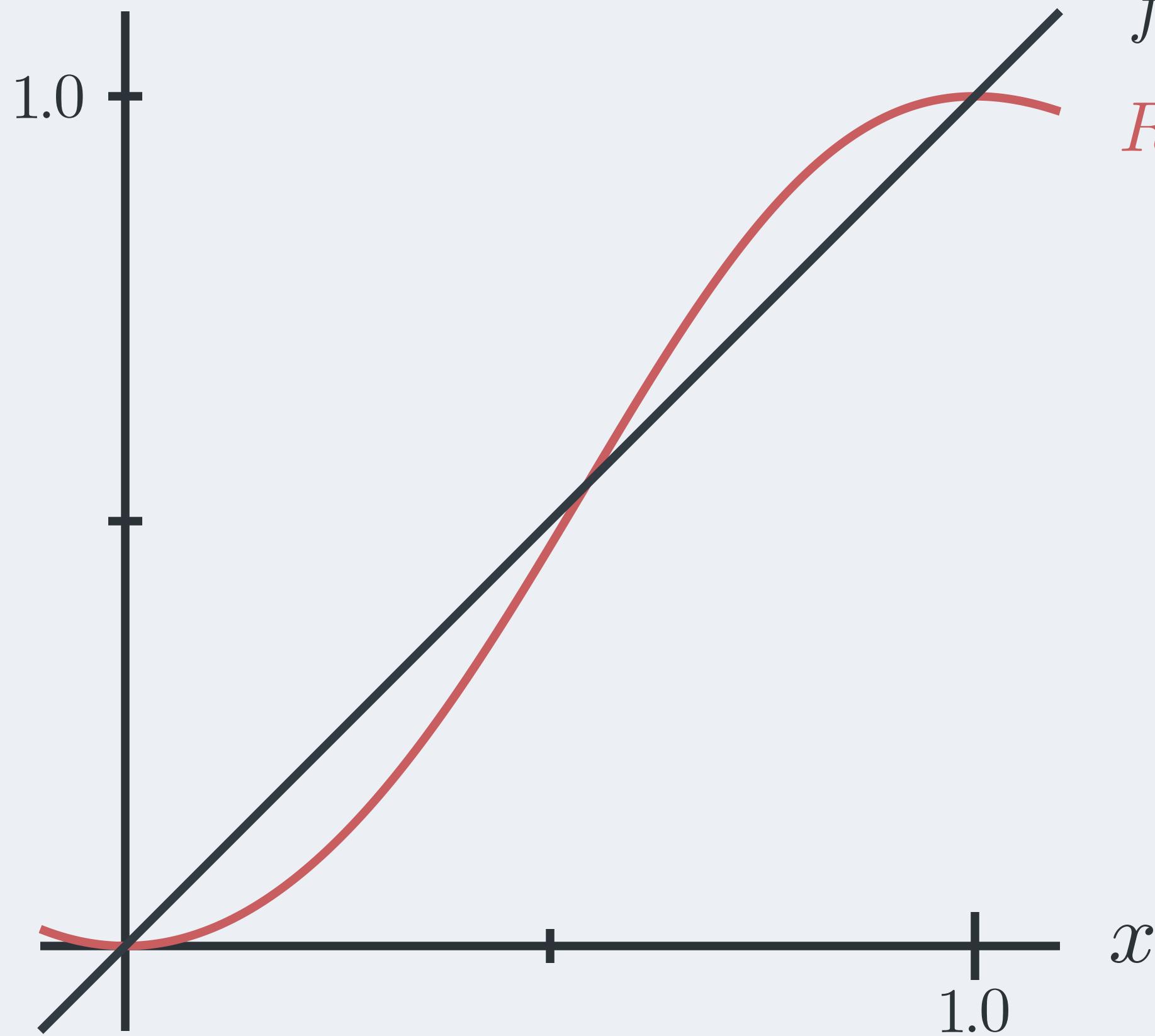
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$$R(x) = 2x^2/(1 + x^4)$$

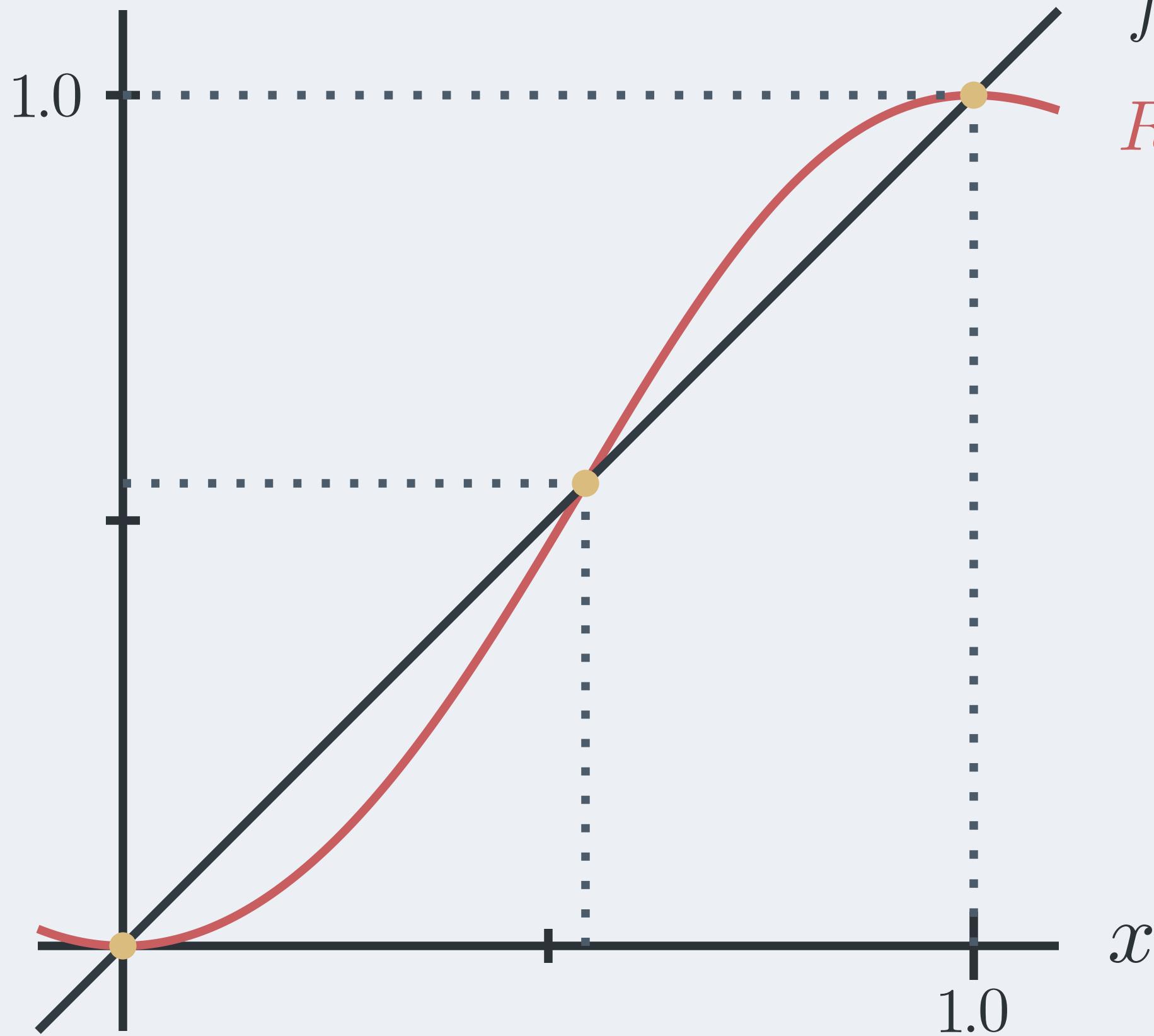




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Un poco de SymPy

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2 from IPython.display import display
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8 display(sp.factor(eq))
```

$$\Rightarrow x(x - 1)(x^3 + x^2 + x - 1)$$

```
1 # ...
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3 sols = sp.solve(x**3 + x**2 + x - 1, x)
4 for s in sols:
5     display(s)
```

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$\Rightarrow 0 : \dots, 1 : \dots$

$$\Rightarrow 2 : -\frac{1}{3} - \frac{2}{9 \sqrt[3]{\frac{17}{27} + \frac{\sqrt{33}}{9}}} + \sqrt[3]{\frac{17}{27} + \frac{\sqrt{33}}{9}}$$

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```
1 xs_val = sols[2].evalf()
2 display(xs_val)
```

$\Rightarrow 0.543689012692076$

Algunas cosas de RG

$$x' = R(x), \quad x^* = R(x^*)$$

$$R(x^* + \delta x) = x^* + \frac{dR}{dx} \Bigg|_{x^*} \delta x = x^* + \lambda \delta x$$

$$\delta x' = \lambda \delta x, \quad \delta T' = \lambda \delta T$$

$$\nu = \frac{\ln l}{\ln \lambda}$$

```
1 R = 2*x/(1+x**4)
2 display(R.diff(x))
```

$$\Rightarrow -\frac{8x^4}{(x^4+1)^2} + \frac{2}{x^4+1}$$

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```
1 xs = symbols('x^*')
2 lam = R.diff(x).subs(R, xs).subs(x, xs))
3 display(lam)
```

$$\Rightarrow -2(x^*)^3 + \frac{4x^*}{(x^*)^4+1}$$

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$$\Rightarrow -\frac{8x^4}{(x^4+1)^2} + \frac{2}{x^4+1}$$

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3 display(lam)
```

$$\Rightarrow -2(x^*)^3 + \frac{4(x^*)^4}{(x^*)^4+1}$$

```
1 display(lam.subs(xs, xs_val)) # xs_val = 0.5437...
2
```

Eso es todo