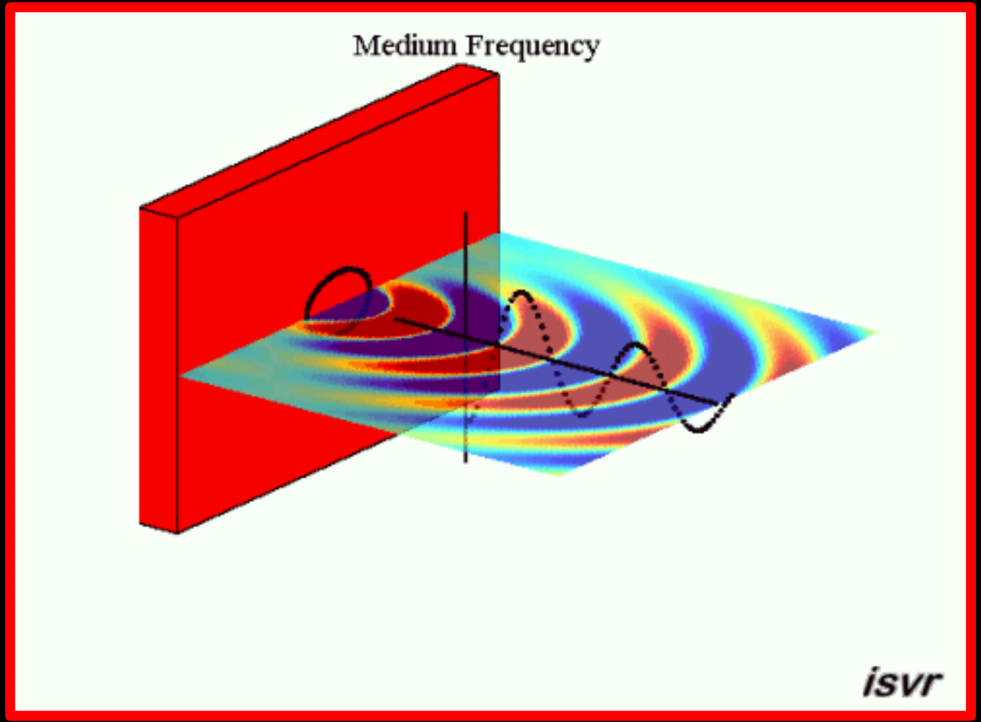
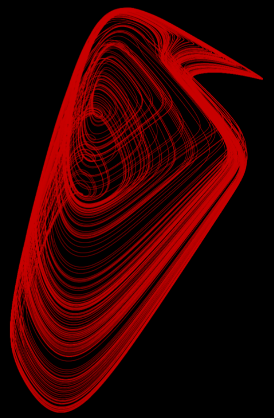
Radiacion y entorno



Conservacion de la masa

$$\frac{1}{c^2}\frac{\partial p}{\partial t} + \rho\, \nabla . \boldsymbol{v} = \rho q$$

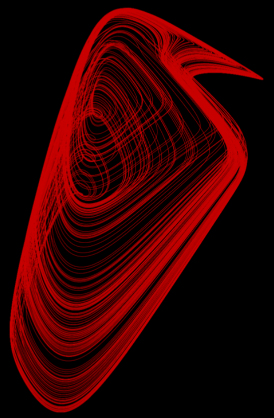
ρq flujo de masa entrante



Conservacion del momento

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} + \nabla\, p = \boldsymbol{f}$$

$\boldsymbol{f}$ densidad de fuerzas externas



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ρq flujo de masa

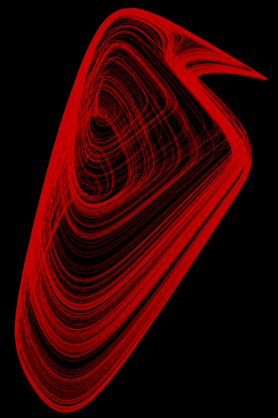
Conservacion del momento

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} + \nabla\, p = \boldsymbol{f}$$

$\boldsymbol{f}$ densidad de fuerza

Fuentes sonoras de tipo 1 — Hay una variacion temporal del flujo de materia

Fuentes sonoras de tipo 2 — Hay una variacion espacial de las fuerzas



Conservacion de la masa

$$\frac{1}{c^2}\frac{\partial p}{\partial t} + \rho\,\nabla.\boldsymbol{v} = \rho q$$

ρq flujo de masa

Conservacion del momento

$$\rho\frac{\partial \boldsymbol{v}}{\partial t} + \nabla\,p = \boldsymbol{f}$$

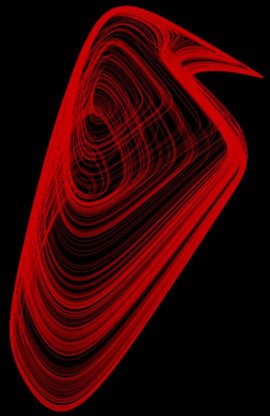
$\boldsymbol{f}$ densidad de fuerza

Eliminando la velocidad de las anteriores (derivando respecto del tiempo la primera, tomando divergencia en la segunda)

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2\,p = \frac{\partial(\rho q)}{\partial t} - \nabla.f$$

Fuentes sonoras de tipo 1

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\partial(\rho q)}{\partial t} - \nabla . f$$

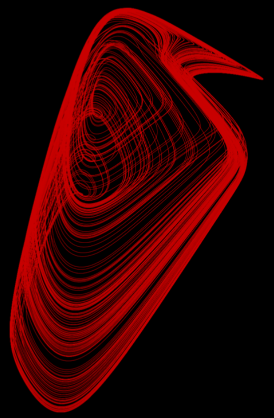


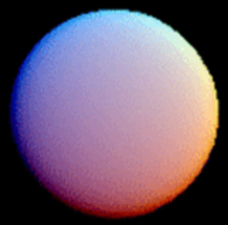
Fuentes sonoras de tipo 1

Fuentes sonoras de tipo 2

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\partial(\rho q)}{\partial t} - \nabla . f$$

Vamos a analizar algunas fuentes sencillas

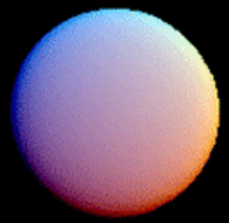


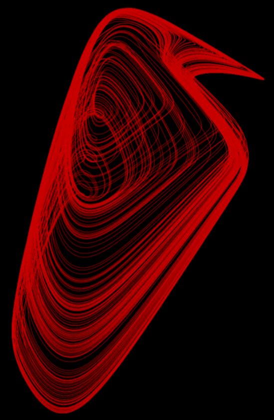


Esfera pulsante

La variacion temporal del flujo
se genera "a los empujones"

A veces es mas natural plantear las ecuaciones en la zona homogenea, y plantear en lugar de fuentes, condiciones de contorno



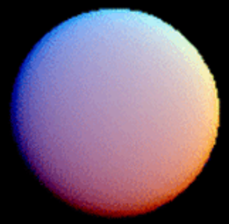


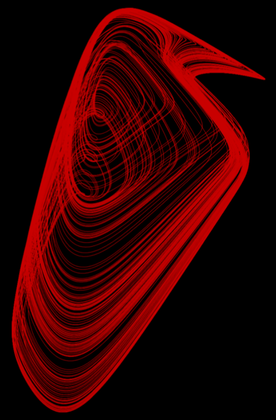
Afuera de la esfera, vale la ecuacion de ondas homogenea:

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = 0$$

Y en efericas,

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \left(\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r}\right)$$





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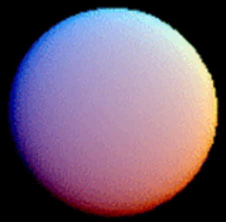
$$\frac{1}{c^2}\frac{\partial^2 (rp)}{\partial t^2} - \frac{\partial^2 (rp)}{\partial r^2} = 0$$



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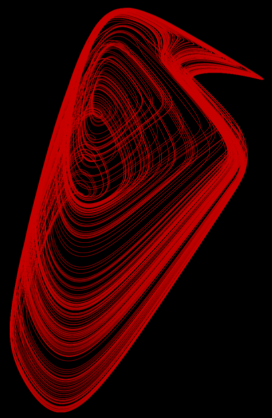
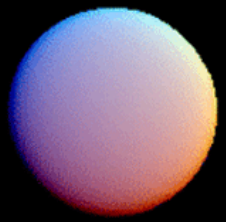
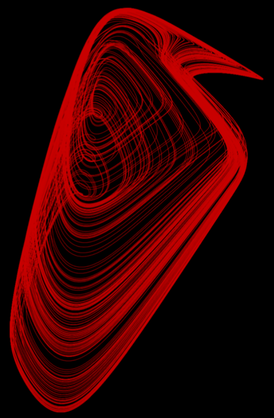


$$rp = f_1(ct - r) + f_2(ct + r)$$



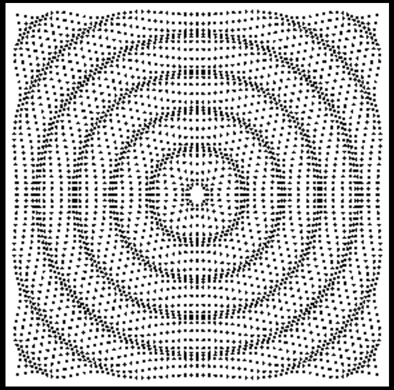
$$\frac{1}{c^2}\frac{\partial^2(rp)}{\partial t^2} - \frac{\partial^2(rp)}{\partial r^2} = 0$$



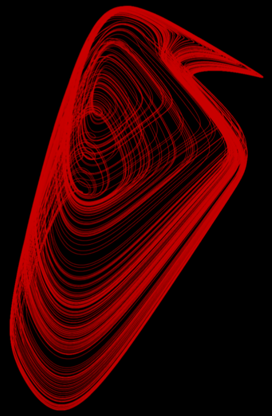
$$rp = f_1(ct - r) + f_2(ct + r)$$

$$p = \frac{1}{r} f_1(ct - r) + \frac{1}{r} f_2(ct + r)$$

La condicion de contorno en el infinito es que no hay reflexion, por lo tanto…



$$p = \frac{1}{r} f_1(ct - r)$$



$$p = \frac{1}{r} A e^{i(\omega t - kr)}$$

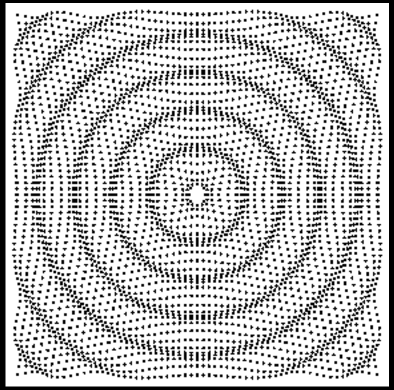
De la ecuacion de la conservacion del momento, sale que la velocidad puede derivarse de un gradiente

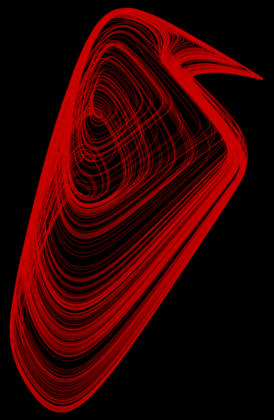
$$\boldsymbol{v} = \nabla \phi$$

Y reemplazando en la ecuacion de conservacion del momento,

$$p = -\rho_0 \frac{\partial \phi}{\partial t}$$

Donde el potencial
Va a satisfacer una ecuacion homogenea





$$\phi = -\frac{1}{i\omega\rho_0} p$$

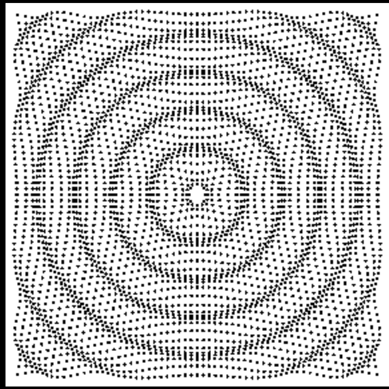
$$\boldsymbol{v} = \nabla\phi = (1 - \frac{i}{kr})\frac{p}{\rho_0 c}\hat{\boldsymbol{r}} \equiv \boldsymbol{v}\hat{\boldsymbol{r}}$$

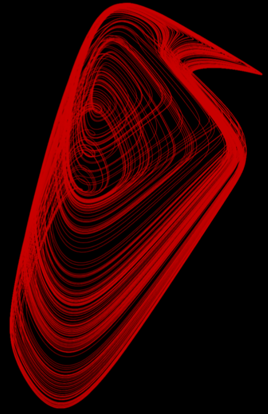
$$z \equiv \frac{p}{v}$$

Entonces, ponemos la condicion de contorno de que la velocidad de las particulas en la frontera de la esfera tienen la velocidad de la esfera, y usamos la impedancia para conectar la velocidad en la superficie con la presion alli. Luego, con esa condicion de contorno, recuperamos la presion en todos lados.

$$U_{0/\omega} \ll a$$

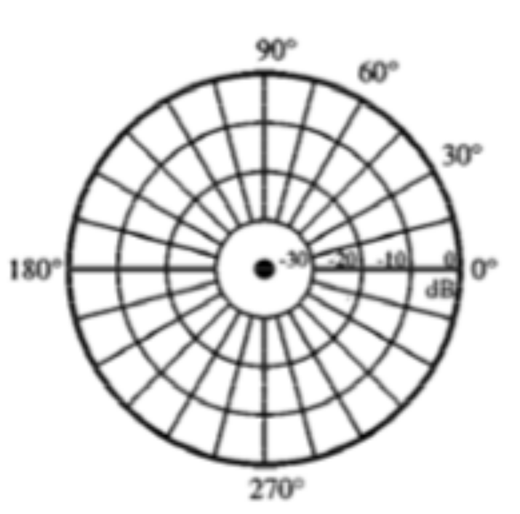
$$v(a,t) = U_0 e^{j\omega t}$$





$$P(r) = \frac{\rho_0 c a}{r} V_a \frac{ika}{1 + ika} e^{ik(r-a)}$$

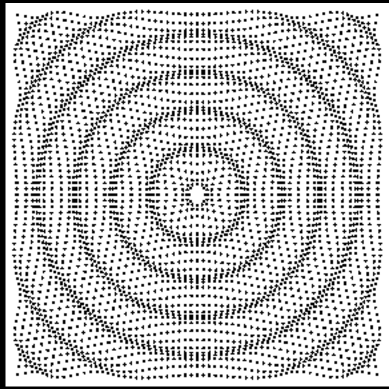
$$I(r) = \frac{1}{2} Re\big(P(r)V^*(r)\big) = (4\pi V_a)^2 c\rho_0 \frac{k^2 a^2}{4\pi r^2 2S(1 + k^2 a^2)}$$



$$\mathfrak{P} = (4\pi V_a)^2 c\rho_0 \frac{k^2 a^2}{2S(1 + k^2 a^2)}$$

Potencia

Que va como el cuadrado de la frecuencia si $ka \ll 1$, independiente de la misma si $ka \gg 1$



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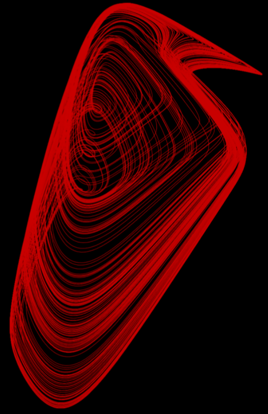

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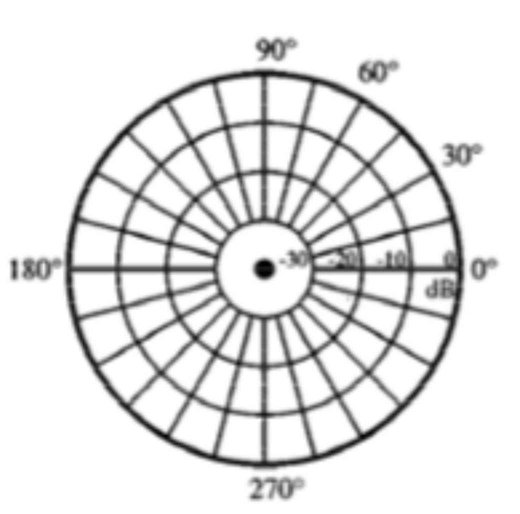
Pertinente para la conversacion sobre proyeccion



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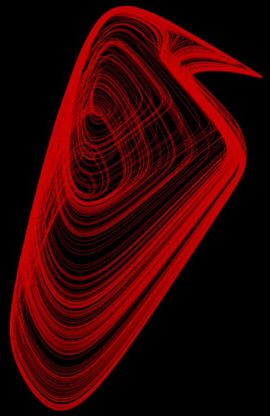
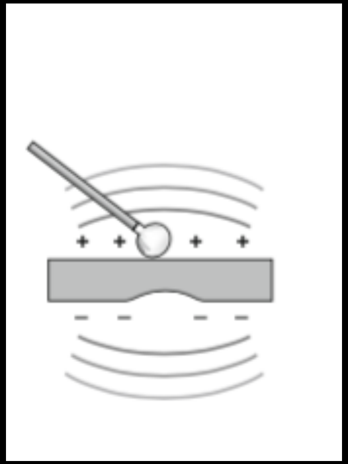


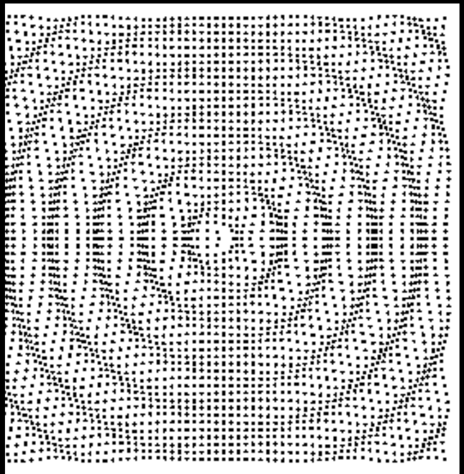


Un cuerpo que empuja entre
$r_1 \;\; y \;\; r_2$

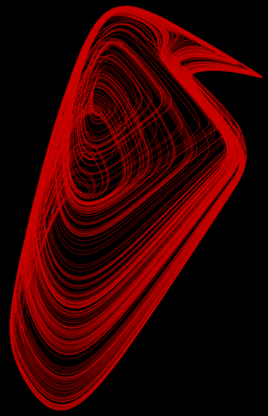
$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = -\nabla . f = f_0 \delta(r - r_1) - f_0 \delta(r - r_2)$$

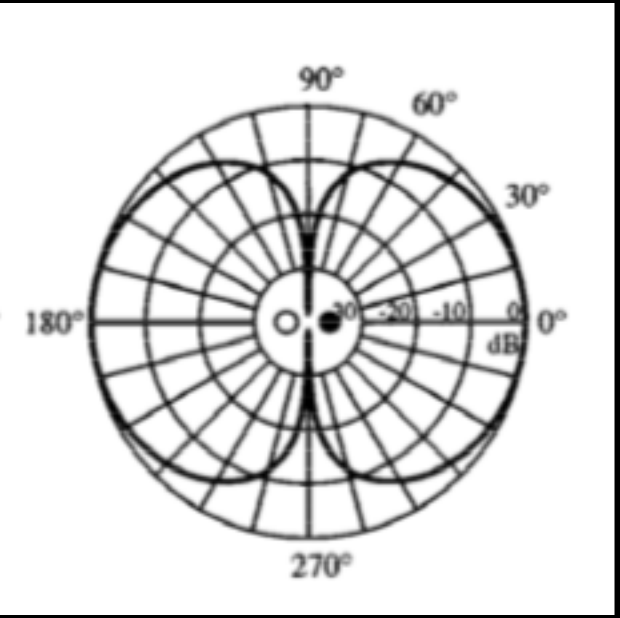
$$f_0 = i\omega\rho_0 U$$

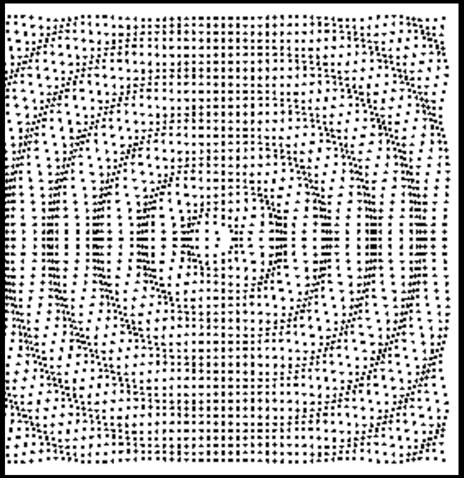
(En la zona en que se empuja, usamos Newton)



$$\mathfrak{P} = (4\pi V_a)^2 c\rho_0 \frac{k^4 d^2}{24\pi c^3}$$







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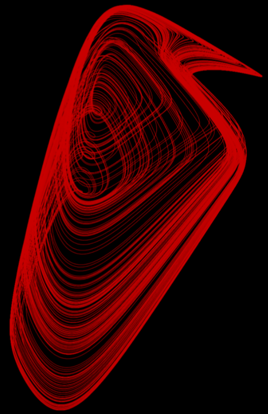
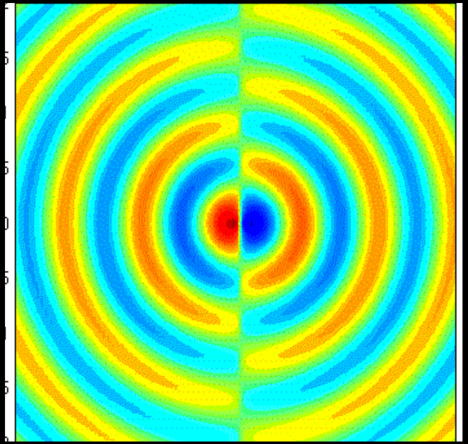
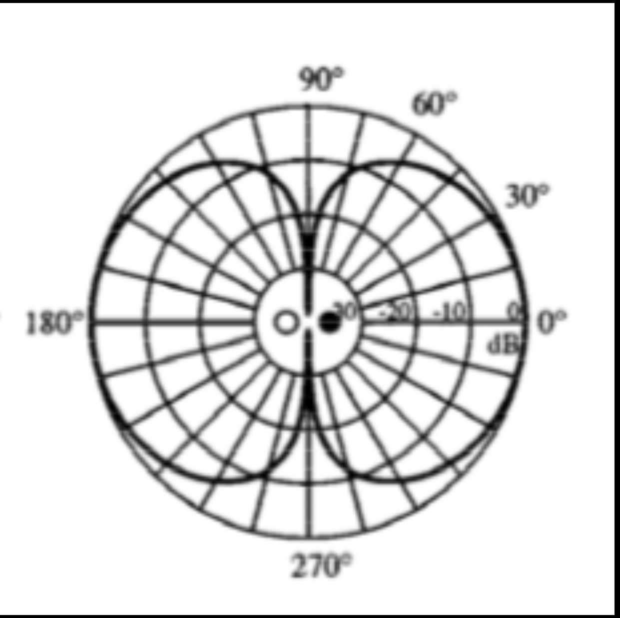
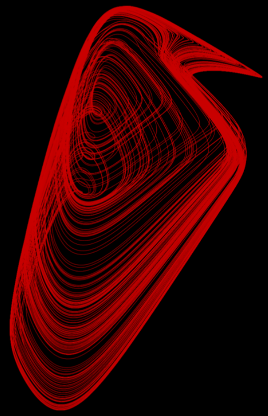


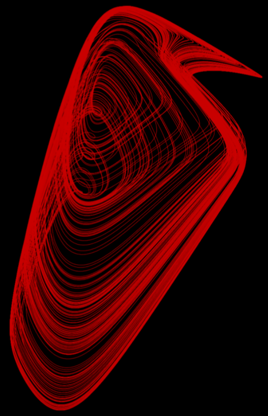




Nuestro dipolo mas cercano



Notar que ocurre si aislo el lado de adelante del de atras

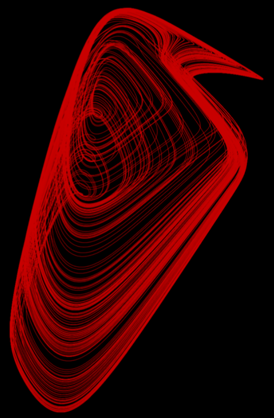


Notar que ocurre si aislo el lado de adelante del de atras

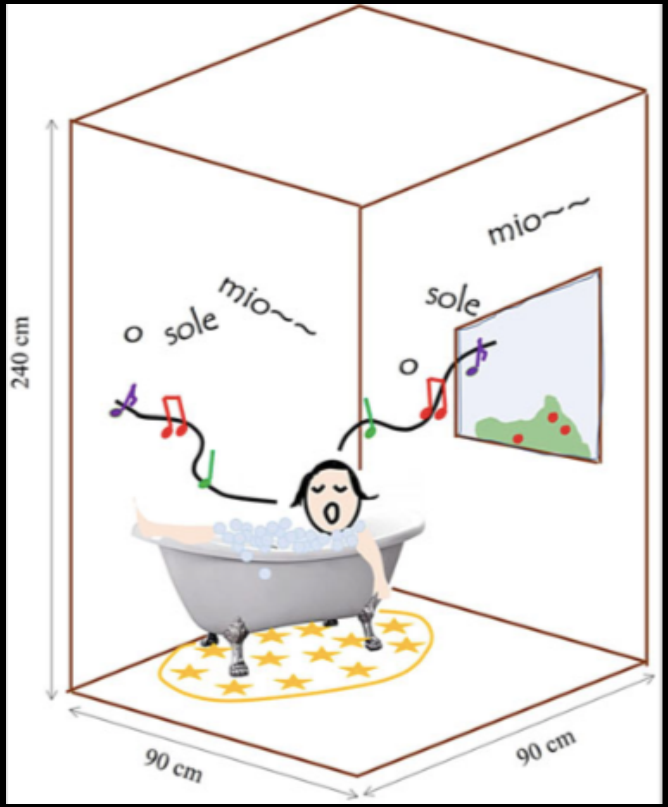
k^2 vrs k^4

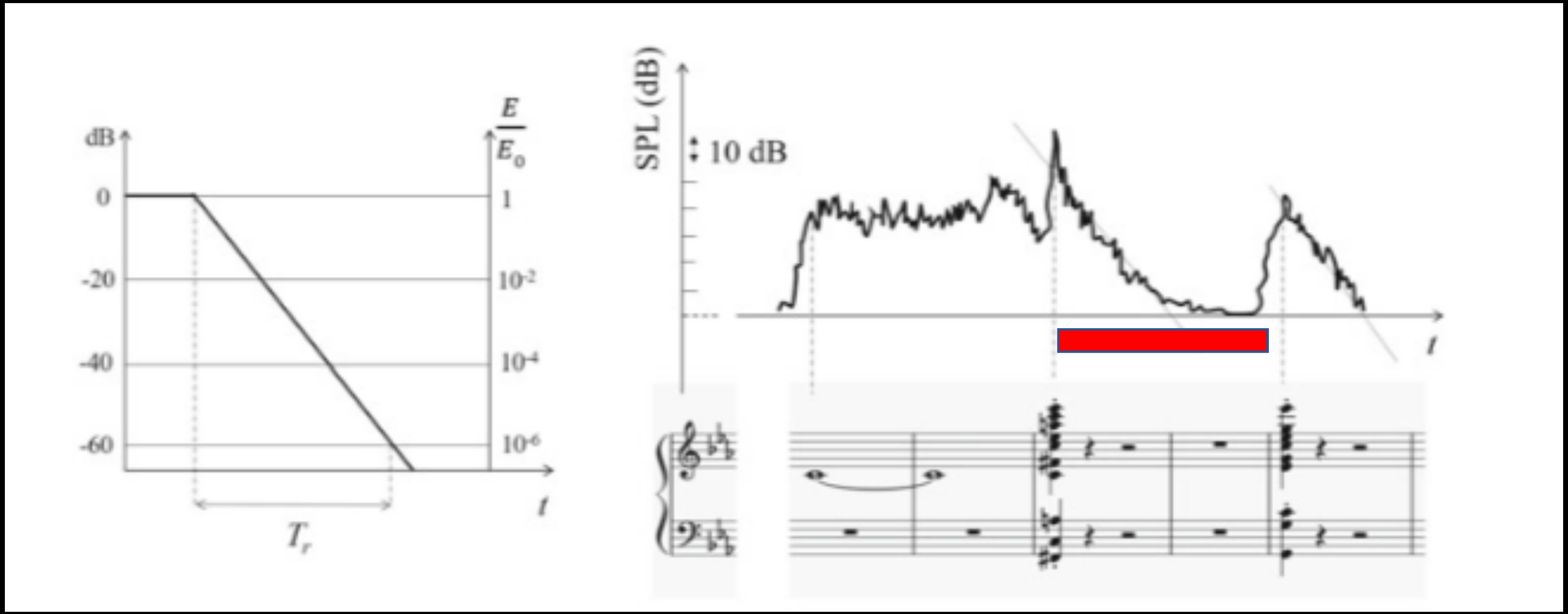
Dos dipolos… el cuadrupolo

Ademas de la radiacion… a donde darian

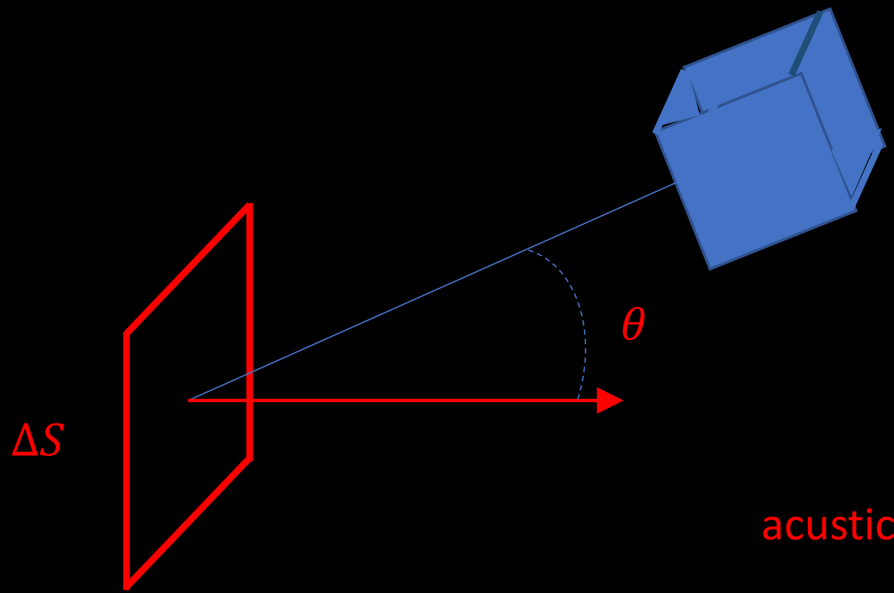


Ademas de la radiacion… a donde darian





Reverberation time

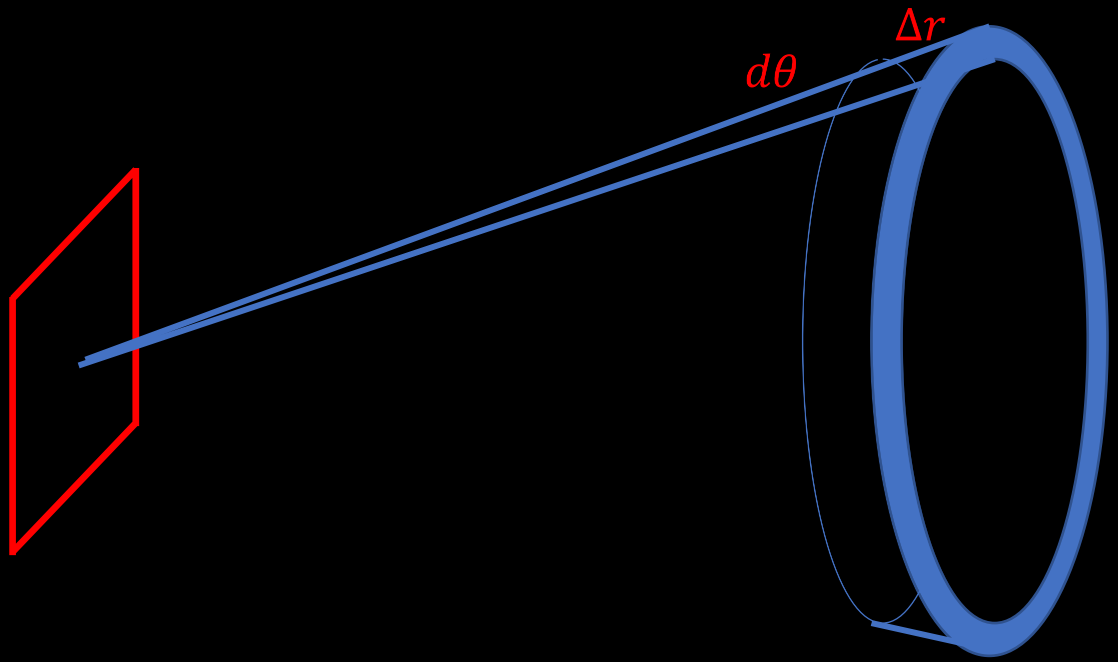


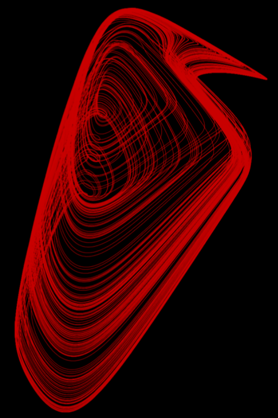
Vamos a establecer una relacion entre la energia acustica en el cuarto, y el flujo de energia a traves de as paredes de la habitacion (ΔS).

$\mathcal{E}$ densidad de energia acustica

$$dV\mathcal{E}\;\frac{\Delta S}{4\pi r^2}\cos(\theta)$$

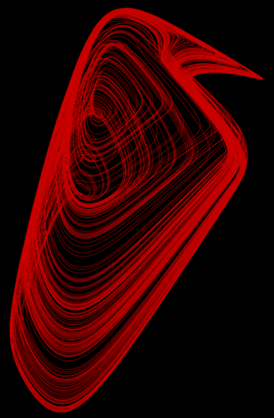
De la energia en el volumen,
La fraccion que pasara por ΔS





La energia que pasa por el area de toda la cascara ΔE

viniendo del volumen $dV = 2\pi \ sin\theta \ r \ \Delta r \ d\theta$



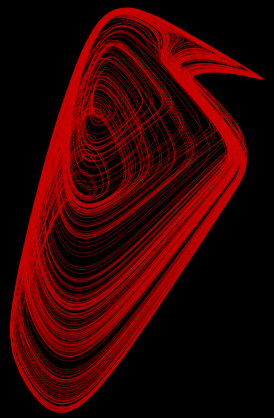
$$\Delta E = \frac{\mathcal{E}\,\Delta S\,\Delta r}{2}\int_0^{\pi/2} \sin\theta\,\cos\theta\,d\theta = \frac{\mathcal{E}\,\Delta S\,\Delta r}{4}$$

Como la energia ΔE llega en un tiempo $\Delta t = \Delta r/c$

$$\boxed{\frac{dE}{dt} = \frac{\mathcal{E}c}{4}}$$

Si la absorcion es una fraccion de esa energia que llega,

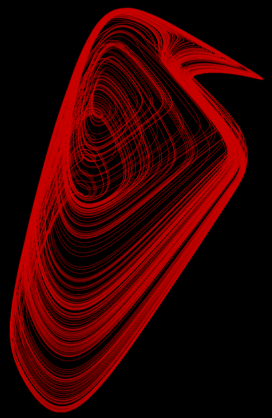
Absorcion= A $\frac{\mathcal{E}c}{4}$



Si Π es la tasa de produccion de energia acustica,

$$V\frac{d\mathcal{E}}{dt}+\frac{Ac}{4}\mathcal{E}=\Pi$$

$$\tau_{\mathcal{E}}=\frac{4V}{Ac}$$



Si Π es la tasa de produccion de energia acustica,

$$V\frac{d\mathcal{E}}{dt}+\frac{Ac}{4}\mathcal{E}=\Pi$$

$$\tau_{\mathcal{E}}=\frac{4V}{Ac}$$

Poca absorcion, grandes volumenes, grandes $\tau_{\mathcal{E}}$
(cantos gregorianos con notas largas)

$$\tau_{\varepsilon} = \frac{4V}{Ac}$$

	Optimum T_r (s)
Cabaret	0.8
Lecture, play	1.0
Chamber music	1.4
Opera	1.3–1.6
Concert	1.7–2.0
Organ music	2.5