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The foci can be interchanged, and the above construction is still valid.

APPENDIX II. GEOMETRIC RELATIONSHIPS

According to Kepler's second law, the planet speed at a point P is in inverse ratio to the perpendicular from the center of force to the tangent to that point. For an ellipse (Fig. 6),

$$(FN)(F'N')=b^2,$$

where (FN) and $(F'N')$ are the perpendiculars from the foci on the tangent to P. It then follows that the magnitude of the velocity of the planet at P is proportional to $(F'N')$.

Hence, the velocity V, proportional to $(F'N')$ in magnitude and normal to $(F'N')$ in direction, can be resolved into two components: V_h, proportional to (CA) in magnitude and normal to (FP), and V_p, proportional to $(F'C)$ in magnitude and normal to $(F'C)$.

The magnitude of the velocity at P is

$$V=(2K/b^2)(F'N'),$$

and it follows directly that the velocity component normal to FP is

$$V_h=(2k/b^2)(CA)=2ka/b^2,$$

and that the component normal to the major axis is

$$V_p=(2k/b^2)(CF')=(2ka/b^2)e,$$

for all positions in the elliptic orbit.

Interference-Fringe Counting as an Aspect of the Doppler Shift

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Experiments suggest that the effect observed with Manchester's microwave Doppler-shift apparatus may be interpreted as a counting of interference fringes. This fringe-counting frequency is the same as that obtained from the Doppler-shift lower beat frequency, and does not invalidate the use of the apparatus for velocity measurements. In reflection from a moving mirror, and other Doppler-effect experiments where interference is possible, fringe counting may be regarded as an aspect of the Doppler effect; the connection between the fringe-counting frequency and the Doppler-frequency shift is provided by the fact that a linearly time-varying phase is equivalent to a constant-frequency shift.

I. INTRODUCTION

A SIMPLE apparatus using microwaves for demonstrating the Doppler-frequency shift due to reflection from a moving mirror has been described by Manchester.[1] It was suggested that this apparatus, shown schematically in Fig. 1(a) with some additional instrumentation used in the present work, might be used to measure velocities in other experiments, such as the terminal velocity of a Fletcher's trolley. In Manchester's experiment a fixed-frequency microwave is reflected from a metal mirror moving relative to the source. The reflected signal, Doppler-shifted in frequency, is mixed with the fixed frequency of the source, yielding a difference frequency in the audio or the subaudio region.

In our experiments with a similar apparatus, we have obtained results which suggest that, at least for short ranges (inside a laboratory), the effect observed with this apparatus may be interpreted as interference-fringe counting. Figure 1 shows that this apparatus may be considered as a form of an amplitude-division interferometer, and the fringes should be "visible" provided the

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[1] F. D. Manchester, Am. J. Phys. 33, 499 (1965).

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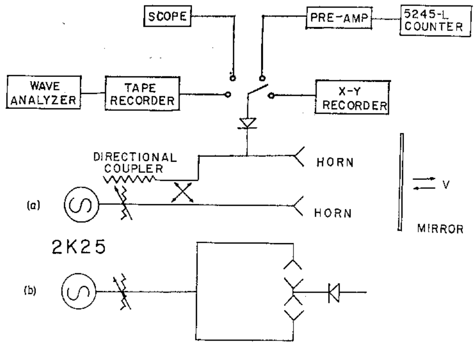

FIG. 1. (a) Doppler/interference shift apparatus with auxilliary equipment used in this experiment. For routine work, only oscilloscope or vacuum-tube voltmeter is required for detector readout. (b) Froome-type of interferometer.

round trip to the mirror is less than the coherence length of the wave packets involved. Since the klystron oscillator used as the source is highly coherent and monochromatic, one might expect to find interference effects. One can compare this interferometer with the one used by Froome[2] to measure the velocity to light [shown schematically in Fig. 1(b)]. In Fig. 1(a), the detector sees a fixed phase and amplitude from one source and a variable phase (including a constant phase shift due to reflection) and amplitude from another source; while in Fig. 1(b), the detector moves, seeing variable phases from both sources (and variable amplitudes if the wavefronts are not perfectly plane). Thus, a demonstration using this Doppler-shift apparatus must join the list[3] of those Doppler-effect demonstrations which are complicated by interference effects. In the present case the complication is particularly simple, and it clearly brings out another aspect of the Doppler effect which may be present when interference is possible.

When the mirror moves radially relative to the source, one would expect the detector output to vary as the maxima and minima pass through the crystal. The frequency of the detector output can be very simply calculated. (It is clear that, for velocities of interest here, relativistic concepts need not be employed.) It is assumed that, in Fig. 2, the velocity v of the mirror is constant and either toward or away from the detector, and we denote the free-space wavelength, frequency, and velocity of the wave launched from the oscillator by λ_0, f_0, and c. Then, in time $t=\lambda_0/v$, the mirror moves one free-space wavelength. The detector output yields a low frequency, two periods of which correspond to the time necessary for the mirror to move one free-space wavelength. Therefore, one may write

$$t=\lambda_0/v=2\tau=2/f, \tag{1}$$

where τ and f are, respectively, the period and the frequency of the detector output. From Eq. (1) it follows immediately that

$$f=2v/\lambda_0=2\,vf_0/c. \tag{2}$$

The fact that the detector is actually in waveguide does not invalidate this result since the maxima and minima move past the detector at virtually the same rate as they move outside the waveguide, as in the Froome type of interferometer. Equation (2) is, of course, just the frequency *shift* to be expected from the Doppler effect—and, therefore, the *difference* frequency to be observed if the Doppler-shifted reflected wave were mixed with the direct wave from the source. Complete nulls are not obtained because at only one position is it possible to obtain equal amplitudes for both waves (Fig. 2 is an idealization). We note that the maxima and minima are correlated with positions of the mirror and the detector output is, in general, not zero when the mirror is stationary. Such phenomena are not usually associated with the concept of the Doppler shift, which is zero when there is no relative motion and whose maxima and minima are not ordinarily correlated with particular positions of the source and observer.

Since the moving mirror produces a changing phase at the detector, the connection with the usual treatment of the Doppler effect is obvious. If the mirror moves with constant velocity the changing phase (increasing or decreasing) is a linear function of time and, therefore, equivalent to a constant frequency (above or below the original frequency). Hence, the Doppler shift can

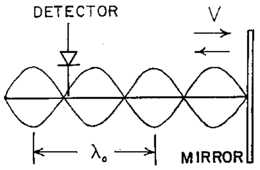

FIG. 2. Standing wave moves past detector with velocity of moving mirror.

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[2] K. D. Froome, Proc. Roy. Soc. (London) **A213**, 123 (1952); **A223**, 195 (1954).

[3] W. C. Michels, Am. J. Phys. **24**, 51 (1956).

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be regarded as interference-fringe counting in this experiment. An alternative viewpoint is to regard fringe counting as an aspect of the Doppler effect, applicable to similar cases where interference effects are possible.

A simple but important technical point should be noted regarding this experiment. If the output of the detector is observed by an indicator that is ac coupled, the observer will not be aware that the maxima and minima are present when the mirror is stationary; the ac component will appear only when the mirror moves, as one would expect from the Doppler shift. However, when the indicator, such as an oscilloscope, is dc coupled, the observer can determine that the maxima and minima are present even when the mirror is stationary; and it is easy to see that the maxima occur at regular, repeatable intervals corresponding to definite mirror positions.

II. EXPERIMENTAL RESULTS

In order to study the reflection from a moving mirror with some degree of precision, the mirror was mounted securely on the carriage of a large lathe. The microwave equipment was essentially that described by Manchester. The output of the crystal could be fed directly to a Data Equipment model 600 x–y recorder (with time on the x axis) or the output could be measured on a Hewlett–Packard 5245L digital counter. Alternatively, the crystal output could be recorded, after preamplification, on a Wollensak tape recorder for spectrum analysis at a later time. For tape recording it was necessary to provide a 1-kc/sec subcarrier to be amplitude-modulated by the low-frequency Doppler-interference shift, since the tape recorder could not record this low frequency directly. The 1-kc/sec carrier could be demodulated during playback to recover the crystal output when desired. The lathe-carriage velocity and the x–y recorder were calibrated with the same time standard, and the klystron frequency was measured with a cavity wave meter.

The data concerning fringe counting (or Doppler shift) obtained from this experiment consistently lay within 0.5% of the theoretical values given by Eq. (2) and are within the expected experimental error.

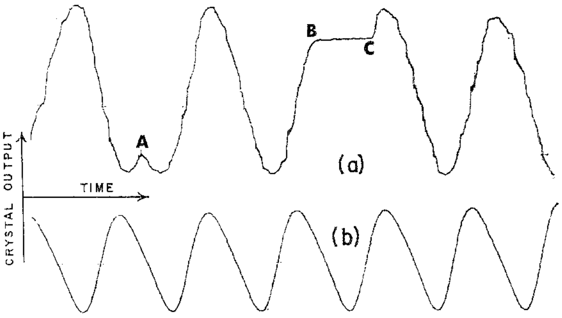

FIG. 3. (a) Amplitude vs time record taken directly from crystal output. At point A the mirror velocity was reversed. Mirror was stopped at B; started again in same direction at C. (b) Typical crystal output at higher mirror velocity. Time scales in (a) and (b) are different.

Figure 3(a) shows an x–y record made directly from the crystal output. The important points are:

(1) At point A the velocity of the mirror was reversed. The switching transient is visible. Note that the "mirror image" of the curve is produced.

(2) At point B the mirror was stopped, and at point C the mirror resumed travel in the same direction. Note that the output does not fall to zero, and the curve at C joins smoothly with the curve at B.

Figure 3(a) was made at a very slow speed; some of the noise appearing on the record was due to persons moving in the room; some other was due to intentional vibration at several points. This noise was not ordinarily present. There is some additional nonremovable noise due to voltage and temperature changes at the klystron. As has been pointed out by Michels,[2] changes in the path of propagation will also appear as a Doppler shift and, therefore, random changes in the index of refraction will appear as noise. A typical record at higher speed is shown in Fig. 3(b).

The record in Fig. 3(a) is characteristic of an interference phenomenon but not of the usual concept of a Doppler shift. The maxima and minima are correlated with positions of the mirror when the mirror moves, and they are stationary in space and time when the mirror is stationary. This observation may also be performed with a dc voltmeter or oscilloscope instead of an x–y recorder.

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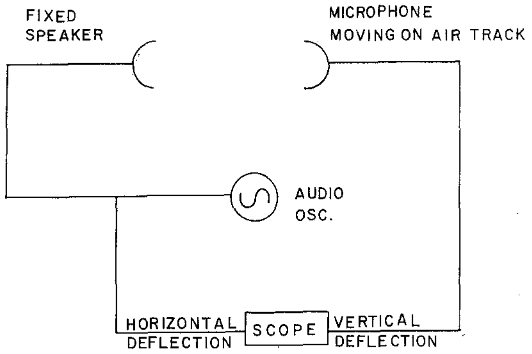

FIG. 4. Typical air-track arrangement used to demonstrate Doppler shift with sound. Alternative arrangement uses fixed receiver and moving transmitter.

The output of the crystal showed considerable harmonic content (nonsinusoidal content) when the mirror was close to the microwave apparatus. Most of this was traced to multiple-path interference, particularly from the lathe bed. This problem was negligible when the mirror was more than 2 m from the microwave horns.

III. NONINTERFERENCE DEMONSTRATIONS OF THE DOPPLER SHIFT

The arrangement shown schematically in Fig. 4 is sometimes used to demonstrate an acoustic Doppler shift.[3] It differs in principle from our previous discussion in that no beat frequency can be obtained by mixing with the source. There is phase and amplitude comparison with the source by means of the Lissajous figure on the oscilloscope, but only by stretching one's imagination can one think of the system as an interferometer. One would expect to find varying phase and amplitude due to the path difference. A particular amplitude and phase difference are associated with each distance separating the loudspeaker and the microphone, so one is confronted with a variant of the problem discussed earlier.

Typical results for this experiment are stationary ellipses representing fixed phases and amplitudes corresponding to different stationary microphone positions. When the microphone is in motion the resulting nonstationary Lissajous figure may be interpreted as a time-varying phase difference or an equivalent frequency shift.

There are other simpler demonstrations of the acoustic Doppler shift, such as a rotating whistle, which do not rely on phase comparison with the source.

IV. CONCLUSIONS

Since the effect observed with the microwave apparatus may be alternatively described as interference-fringe counting or Doppler shift, this experiment has considerable student value beyond the mere demonstration of the Doppler effect.

For example, this experiment may serve as a point of departure for bringing forth the concepts of coherence length and time, Fourier analysis, fringe visiblitiy, and their interrelations.

One way of observing the microwave Doppler shift with more complex apparatus would be to spectrum-analyze the reflected signal *without* mixing with the source as local oscillator. In principle it should be possible to observe the upward and downward shifts in frequency, as with sonar. Unfortunately, owing to technical difficulties, the dispersion of contemporary microwave spectrum analyzers is insufficient to resolve the Doppler shift produced by slowly moving objects if the propagation is in air or vacuum.

The problem of Doppler shift and interference-fringe counting is also present in the usual Doppler-shift demonstrations with a laser,[4] wherein one mirror of a Michelson interferometer is moved and the difference frequency observed after mixing in a photomultiplier. The same frequency can be observed with an incoherent source as long as the fringes are visible. Since the laser coherence length is so great, one is practically always confronted with interference-fringe counting in this sort of table-top demonstration.

ACKNOWLEDGMENT

We would like to express our appreciation to J. A. Fleming for his aid in the shop.

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[4] T. J. Perkins, *Experiments in Physical Optics Using Continuous Laser Light* (Optics Technology, Inc., Belmont, Cal., 1964), p. 37.

06 November 2023 20:19:27