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Doppler beats or interference fringes?

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Doppler beats or interference fringes?

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The article by Remy on the optical Doppler shift[1] makes no mention of a simple and potentially very precise version of the same experiment which can be carried out using a Michelson interferometer with a moveable mirror. To obtain audible beat frequencies, the mirror can be cranked at speeds much less than 1 mm/sec by using a variable speed motor drive. Mirror speed is determined to three or four figures by using a stopwatch to time motion over distances measured by the interferometer scale. The recombined beams from the interferometer arms falling on a photocell detector can easily be made to generate beat frequencies covering the entire range of audible sound. Students can hear the beats while seeing them on the oscilloscope. Entertaining effects are obtained by cranking the mirror manually.

There is a different point of view to take about these Doppler beats, namely that the variations of light intensity arriving at the detector are caused by the ordinary interference fringes of the interferometer as they move by. Interference fringes are of course present whether the interferometer mirror is moving or not, and are not in most discussions associated with Doppler shift or beat frequency. However, when one mirror is moved new fringes must appear (or disappear) at the rate of one for each half wavelength of mirror displacement. A simple calculation shows that the number of fringes per second expected to be observed by moving the mirror at speed v is given by exactly the same formula $f = 2Fv/c$ that results from the Doppler shift theory. (F is the light frequency.)

By placing the photocell detector behind a hole in a screen the moving fringes can be observed while the beat frequency is being heard. The demonstration of two superficially different explanations converging in one experiment has great pedagogical value and has been observed to awaken interest not only in students but in well motivated faculty members.

[1]F. Remy, Am J. Phys. **46**, 763 (1978)

Note on properties of spin-1 matrices

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In a recent paper in this journal,[1] Weaver has demonstrated strong similarities between certain spin-1 and spin-½ problems. Much of his development depends on his Eq. (16),

$$S_iS_jS_k + S_kS_jS_i = \delta_{ij}S_k + \delta_{jk}S_i. \quad (1)$$

The steps which lead to the above result depend on a familiarity with the properties of the spin tensor under Lorentz transformations and the relation between the spin tensor and the usual vector spin matrices. Although this powerful approach may be used for dealing with any spin value, many readers, particularly students who would benefit most from Weaver's later discussion, do not have the background to follow these arguments. In the following paragraphs, we provide an alternate proof of Eq. (1).

We will deal with a 3 × 3 matrix representation of the spin-1 operators. Since a 3 × 3 matrix has nine components, the most general such matrix can be expanded in terms of nine linearly independent basis matrices. The nine possible quadratic combinations of our spin-1 matrices form such a basis, so that we can write for a general matrix M,

$$M = C_{ij}S_iS_j. \quad (2)$$

(A repeated index indicates summation over that index.) The spin matrices themselves can be expanded simply in such a series because of their commutation relations,

$$S_iS_j - S_jS_i = i\epsilon_{ijk}S_k. \quad (3)$$

A particular example would be

$$S_3 = i(S_2S_1 - S_1S_2). \quad (4)$$

Similarly, the unit matrix expansion follows from the result:

$$S_1^2 + S_2^2 + S_3^2 = 2. \quad (5)$$

A product of three or more spin-1 matrices may always be reduced to the quadratic form given in Eq. (2). In particular, we have

$$S_iS_jS_k = \mathcal{A}_{ijkmn}S_mS_n, \quad (6)$$

where the expansion coefficients must be imaginary constants. This latter property emerges if we observe that Eq. (6) must remain invariant under the time reversal operation which changes the sign of a spin operator, and replaces a c number by its complex conjugate. The adjoint of Eq. (6) is (since the S_i are Hermitian, $S_i^\dagger = S_i$)

$$(S_iS_jS_k)^\dagger = S_kS_jS_i = \mathcal{A}^*_{ijkmn}(S_mS_n)^\dagger$$
$$= -\mathcal{A}_{ijkmn}S_nS_m. \quad (7)$$

Adding Eqs. (6) and (7), we obtain

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