

Some fundamental properties of speckle*

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A speckle pattern formed in polarized monochromatic light may be regarded as resulting from a classical random walk in the complex plane. The resulting irradiance fluctuations obey negative exponential statistics, with ratio of standard deviation to mean (i.e., contrast) of unity. Reduction of this contrast, or smoothing of the speckle, requires diversity in polarization, space, frequency, or time. Addition of M uncorrelated speckle patterns on an intensity basis can reduce the contrast by $1/\sqrt{M}$. However, addition of speckle patterns on a complex amplitude basis provides no reduction of contrast. The distribution of scale sizes in a speckle pattern (i.e., the Wiener spectrum) is investigated from a physical point of view.

INTRODUCTION

Objects illuminated by light from a highly coherent cw laser are readily observed to acquire a peculiar granular appearance. Figure 1 shows a typical pattern observed in the image of a uniformly white reflecting object. This extremely complex pattern bears no obvious relationship to the macroscopic properties of the object illuminated. Rather it appears chaotic and unordered, and is best described quantitatively by the methods of probability and statistics.

The origin of this granularity was quickly recognized by early workers in the laser field.^{1,2} The vast majority of surfaces, synthetic or natural, are extremely rough on the scale of an optical wavelength. Under illumination by coherent light, the wave reflected from such a surface consists of contributions from many independent scattering areas. Propagation of this reflected light to a distant observation point results in the addition of these various scattered components with relative delays which may vary from several to many wavelengths, depending on the microscopic surface and the geometry (see Fig. 2). Interference of these dephased but coherent wavelets results in the granular pattern we know as speckle. Note that if the observation point is moved, the path lengths traveled by the scattered components change, and a new and independent value of intensity may result from the interference process. Thus the speckle pattern consists of a multitude of bright spots where the interference has been highly constructive, dark spots where the interference has been highly destructive, and irradiance levels in between these extremes. Accordingly, we observe a continuum of values of irradiance which has the appearance of a chaotic jumble of "speckles".

While the origin of speckle is perhaps easiest to discuss in the free-space reflection geometry of Fig. 2, with some additional work its appearance in the imaging geometry of Fig. 3 can also be explained. The image formed at a given point in the observation plane consists of a superposition of a multitude of complex amplitude spread functions, each arising from a different scattering point on the surface of the object. As a consequence of the roughness of this surface, the various amplitude spread functions add with different phases, resulting again in a complex pattern of interference, or a speckle pattern superimposed on the image of interest.

The appearance of speckle is not limited to imagery formed with reflected light. If a photographic trans-

parency is illuminated through a diffuser, the wave front passing through the transparency has a highly corrugated and extremely complex structure. In the image of such a transparency we again find large fluctuations of irradiance caused by the overlapping of a multitude of dephased amplitude spread functions. While most of the discussions in this paper are presented in terms of the reflecting geometries of Figs. 2 and 3, the conclusions apply equally well to the transmission geometry, provided the wave front transmitted by the transparency satisfies the same basic assumptions applied to the wave front reflected from a rough object.

While a detailed analysis of the properties of speckle patterns produced by laser light began in the early 1960's, nonetheless, far earlier studies of specklelike phenomena are found in the physics and engineering literature. Mention should be made of the studies of "coronas" or Fraunhofer rings by Verdet³ and Lord Rayleigh.⁴ Later, in a series of papers dealing with scattering of light from a large number of particles, von Laue⁵ derived many of the basic properties of specklelike phenomena.

In a more modern vein, direct analogs of laser speckle are found in all types of coherent imagery, including radar astronomy,⁶ synthetic-aperture radar,⁷ and acoustic imagery.⁸ In addition, statistical phenomena entirely analogous to speckle are found in radio-wave propagation,⁹ the temporal statistics of incoherent light,¹⁰ the theory of narrowband electrical noise,¹¹ and even in the general theory of spectral analysis of ran-

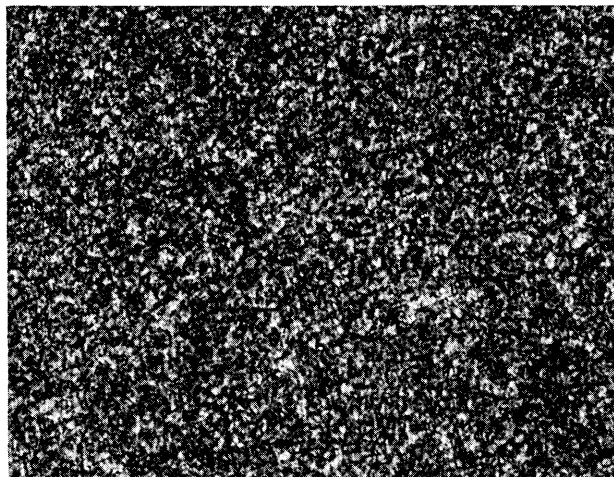


FIG. 1. Typical speckle pattern.

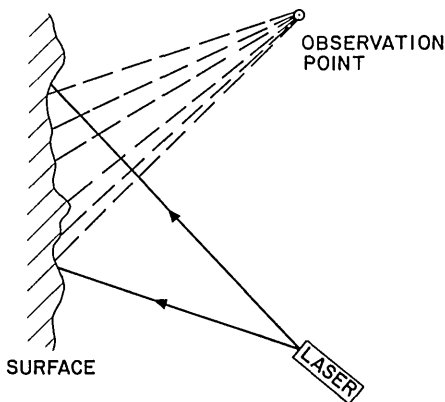


FIG. 2. Speckle formation in the free-space geometry.

dom processes.¹² As a consequence of the ubiquitous nature of the random interference phenomenon, the term "speckle" has taken on a far broader meaning than might have been envisioned when it was introduced in the early 1960's.

The purpose of this paper is to provide an introduction to the more important properties of speckle patterns, in hopes that this background will enable the reader to more fully enjoy and appreciate the many more detailed papers in this special issue on speckle. The reader wishing to acquire a more detailed understanding of the statistical properties of speckle patterns may consult Ref. 13. Ultimately, a completely rigorous understanding of laser speckle requires a detailed discussion of the properties of electromagnetic waves that have been reflected from rough surfaces.¹⁴ However, a good intuitive feeling for the properties of speckle can be obtained without an extremely detailed examination of the physics of the scattering process.

SPECKLE AS A RANDOM-WALK PHENOMENON

Due to our lack of knowledge of the detailed microscopic structure of the surface from which the light is reflected, it is necessary to discuss the properties of speckle patterns in statistical terms. The statistics of concern are defined over an ensemble of objects, all with the same macroscopic properties, but differing in microscopic detail. Thus if we place a detector at position (x, y, z) in the observation plane of Fig. 2 or 3, the measured irradiance is not exactly predictable, but we can describe its statistical properties over an ensemble of rough surfaces.

To aid in the development of a statistical model for speckle, we initially make some simplifying assumptions. Namely, we suppose that the field incident at (x, y, z) is perfectly polarized and perfectly monochromatic. Under such conditions we can represent this field by a complex-valued analytic signal of the form

$$u(x, y, z; t) = A(x, y, z) \exp(i2\pi\nu t), \quad (1)$$

where ν is the optical frequency and $A(x, y, z)$ is a complex phasor amplitude

$$A(x, y, z) = |A(x, y, z)| \exp[i\theta(x, y, z)]. \quad (2)$$

The directly observable quantity is the irradiance at

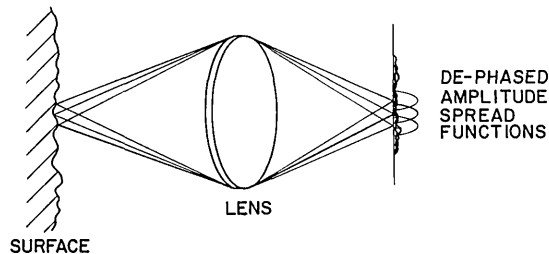


FIG. 3. Speckle formation in the imaging geometry.

(x, y, z) , which is given by

$$I(x, y, z) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |u(x, y, z; t)|^2 dt = |A(x, y, z)|^2. \quad (3)$$

Perhaps the most important statistical property of a speckle pattern is the probability distribution of the irradiance I . How likely are we to observe a bright peak or a dark null in the irradiance at a given point? This question can be answered by noting the similarity of the problem at hand to the classical problem of the random walk, which has been studied for nearly 100 years.^{4, 15, 16}

The complex amplitude of the field at (x, y, z) may be regarded as resulting from the sum of contributions from many elementary scattering areas on the rough surface. Thus the phasor amplitude of the field can be represented by

$$A(x, y, z) = \sum_{k=1}^N |a_k| \exp(i\phi_k), \quad (4)$$

where $|a_k|$ and ϕ_k represent the amplitude and phase of the contribution from the k th scattering area and N is the total number of such contributions. Figure 4 illustrates this phasor addition.

Now we make two important assumptions about the contributions from the elementary scatterers.

(i) The amplitude a_k and the phase ϕ_k of the k th elementary phasor are statistically independent of each other and of the amplitudes and phases of all other elementary phasors (i.e., the elementary scattering areas are unrelated and the strength of a given scattered component bears no relation to its phase).

(ii) The phases ϕ_k of the elementary contributions are equally likely to lie anywhere in the primary interval $(-\pi, \pi)$ (i.e., the surface is rough compared with a wavelength, with the result that phase excursions of

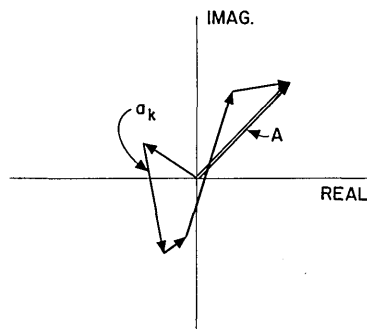


FIG. 4. Random walk in the complex plane.

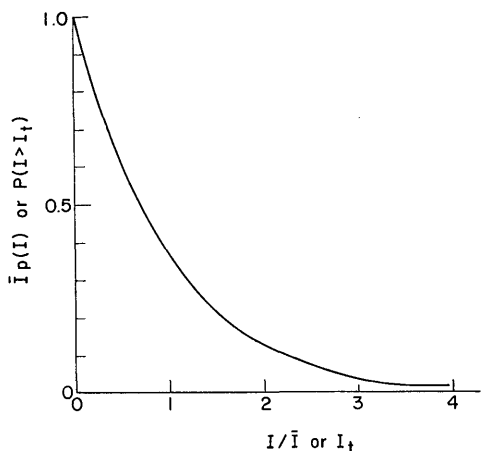


FIG. 5. Probability density function of a polarized speckle pattern.

many times 2π rad produce a uniform probability distribution on the primary interval).

With these two assumptions, the similarity of our problem to the classical random walk in a plane becomes complete. Without presenting the detailed mathematical proofs, which can be found elsewhere (e. g., Ref. 13), we simply state the conclusions of the random-walk analysis. Provided the number N of elementary contributions is large, we find (a) the real and imaginary parts of the complex field at (x, y, z) are independent, zero mean, identically distributed Gaussian random variables, and (b) the irradiance I obeys negative exponential statistics, i. e., its probability density function is of the form

$$p(I) = \begin{cases} (1/\bar{I}) \exp(-I/\bar{I}), & I \geq 0, \\ 0, & \text{otherwise,} \end{cases} \quad (5)$$

where \bar{I} is the mean or expected irradiance.

The probability density function of Eq. (5) is illustrated in Fig. 5. The probability that the irradiance exceeds a given threshold I_t is a similar function

$$P(I > I_t) = \exp(-I_t/\bar{I}), \quad I_t \geq 0. \quad (6)$$

A fundamentally important characteristic of the negative exponential distribution is that its standard deviation precisely equals its mean. Thus, the contrast of a polarized speckle pattern, as defined by

$$C = \sigma_I/\bar{I}, \quad (7)$$

is always unity. Herein lies the reason for the subjective impression that the variations of irradiance in a typical speckle pattern are indeed a significant fraction of the mean.

In the free space geometry of Fig. 2, the assumption that the number N of elementary contributions is large is generally well satisfied. However, in the imaging geometry of Fig. 3, this assumption can be violated if the imaging system is nearly capable of resolving the elementary scattering areas. Nonetheless, in the vast majority of practical applications the number N is indeed very large. The predicted negative exponential statistics of irradiance have been well verified experimentally.¹⁷⁻¹⁹

It is perhaps also worth emphasizing that, once the

surface has a roughness which is sufficient to produce phase excursions comparable with 2π rad, negative exponential statistics result, and further increases of roughness produce no perceptible changes of irradiance statistics. Thus to a good approximation, all surfaces that are rough on the scale of a wavelength produce the same form of irradiance statistics, regardless of just how much rougher than this limit they may be.

SUPPRESSION OF SPECKLE

Although a number of beneficial uses can be made of speckle, it usually is more of a hindrance than a help. The presence of speckle in an image reduces the ability of a human observer to resolve fine detail. The presence of speckle in the signal detected by an optical radar system can reduce the probability of target detection and/or cause the radar system to lose track. Thus in most cases of practical interest, suppression of speckle is a goal towards which we aspire. How is it possible to reduce the fluctuations present in a detected speckle pattern?

The answer to this question follows from the fundamental result of probability theory that the sum of M identically distributed, real-valued, uncorrelated random variables has a mean value which is M times the mean of any one component, and a standard deviation which is \sqrt{M} times the standard deviation of one component. Thus, if we add M uncorrelated speckle patterns on an irradiance basis, the contrast of the resultant speckle pattern is reduced in accord with the law

$$C = \sigma_I/\bar{I} = 1/\sqrt{M}. \quad (8)$$

Uncorrelated speckle patterns can be obtained from a given object by means of time, space, frequency, or polarization diversity. For example, reflection from a surface such as nonglossy paper generally involves multiple scattering, with a considerable amount of depolarization resulting. For such a surface, the speckle patterns observed through a polarization analyzer will change detailed structure dramatically as the analyzer is rotated through 90° . The irradiance in a speckle pattern is, of course, the sum of the irradiances contributed by two orthogonal linear polarization components. Hence when complete depolarization of the reflected wave occurs, the contrast of the total speckle pattern is $1/\sqrt{2}$, rather than unity.

Pure spatial diversity occurs, for example, when a reflecting surface is illuminated by several different lasers from different angles. If the angles of illumination are sufficiently separated, the path length delays experienced by each of the reflected beams will be different enough to generate uncorrelated speckle patterns.²⁰ Since the lasers are mutually incoherent on the time scale of this experiment, the M speckle patterns add on an irradiance basis, with a consequent reduction of contrast by $1/\sqrt{M}$. This particular example is suggestive of the more general fact that illumination of an object by a sufficiently extended incoherent source suppresses speckle.

A second way of changing optical paths (in wavelengths) traveled by a reflected wave is to change the

optical frequency of the illumination.²¹ Thus, if the surface is illuminated by light with M separate frequency components of equal strength, and if the separation of these frequency components is sufficiently great, M uncorrelated speckle patterns will result, with addition on an irradiance basis. In a reflection geometry, with angles of incidence and reflection near normal to the surface, the separation $\Delta\nu$ required to produce uncorrelated speckle is approximately²²

$$\Delta\nu \cong c/2\sigma_z, \quad (9)$$

where c is the velocity of light and σ_z is the standard deviation of the surface height fluctuations. This particular example is suggestive of the more general fact that illumination of an object with sufficiently extended spectral bandwidth suppresses speckle.

Time diversity is most readily applied in the transmission geometry. If a transparency object is illuminated through a diffuser, then motion of that diffuser results in a continuous changing of the speckle pattern in the image. A time exposure in the image plane then results in the addition, on an intensity basis, of a number of uncorrelated speckle patterns, thus suppressing the contrast of the detected speckle pattern.

We close this discussion of speckle suppression by describing an important negative result. Suppose that, with a single ideally monochromatic laser, we simultaneously illuminate two separated regions on a rough reflecting object in the free-space geometry of Fig. 2. These regions are physically separated by a distance large compared to the correlation distance of the surface itself, so the microscopic surface structures in the two areas can be considered to be independent. Either of these two areas illuminated individually will produce a speckle pattern with unity contrast. If the two areas are illuminated simultaneously by the same monochromatic source, will the superposition of the two independent speckle patterns reduce the contrast of the resultant pattern?

The answer is an emphatic *no*, for the two speckle patterns add on a complex amplitude basis, rather than the intensity basis described earlier. Each of the complex fields contributed by one of the surface areas is a random walk in the complex plane. The addition of two random walks simply results in a third random walk with more steps. As long as the number of steps in each of the component random walks is large, the first-order statistics of the combined random walk are indistinguishable in form from those of the two components. Thus the statistics of the irradiance fluctuations remain negative exponential, and we conclude that addition of speckle patterns on a complex amplitude basis does not reduce the speckle contrast.

DISTRIBUTION OF SCALE SIZES IN A SPECKLE PATTERN

A second property of a speckle pattern which greatly influences its effects on optical system performance is the coarseness of the granularity in the pattern. In actuality, a speckle pattern consists of peaks and nulls of many different scale sizes, so any measure of

coarseness must of necessity indicate an average distribution. A very suitable measure is the so-called Wiener spectrum of a speckle pattern, which is a measure of the average strengths of all possible spatial frequency components of the pattern. The usual way to find the Wiener spectrum is to first calculate the autocorrelation function of the pattern, and then to Fourier transform that pattern in accord with the Wiener-Kinchine theorem.²³ Here we prefer to take a more physical approach to the problem.

Considering first the free-space propagation geometry of Fig. 2, we suppose that the speckle pattern of concern is detected in a plane with coordinates (x, y) . Any specific detected speckle pattern consists of a continuum of Fourier components. Focusing attention on a specific vector spatial frequency, $\nu = (\nu_x, \nu_y)$, if we square the amplitude of this Fourier component and average over an ensemble of microscopically different objects, the resulting number is the value of the Wiener spectrum at frequency ν .

An ideal Fourier component of the intensity may be thought of as a sinusoidal fringe component of the speckle pattern. Now we ask how a sinusoidal fringe of vector frequency $\nu = (\nu_x, \nu_y)$ can be generated in the speckle pattern. The answer is that such a fringe must of necessity arise by simple interference of light reflected from two points on the object separated by a vector spacing

$$\mathbf{s} = (s_x, s_y) = (\lambda z \nu_x, \lambda z \nu_y), \quad (10)$$

where z is the distance from the reflecting surface (assumed approximately planar) to the observation plane.

As illustrated in Fig. 6, there are many pairs of points on the object separated by a given vector spacing \mathbf{s} ; all such pairs contribute a fringe component at the same frequency ν . Assuming that the correlation area of the surface itself is small compared with the size of the illuminated spot on the object, to an excellent approximation these many fringes at frequency ν add with random spatial phase. Over the ensemble of reflecting objects, the mean-square strength of the fringe at ν is simply the sum of the mean-square strengths of the component fringes. Thus, for a uniformly bright object on a dark background, the value of the Wiener spectrum at ν is proportional to the number of ways the vector spacing \mathbf{s} is embraced by the object. An equivalent statement, which also holds for objects of nonuniform brightness, is that the shape of the Wiener spectrum is identically the same as the shape of the autocorrelation function of the object radiance distribution. A slight qualification of this statement must be made for the zero-spatial-frequency component of spectrum. Since irradiance is a non-negative quantity, the speckle pattern always rides on constant mean irradiance level, with the result that the Wiener spectrum contains a δ -function component at the origin. A more-detailed analysis shows that the Wiener spectrum $\mathcal{W}(\nu)$ is given mathematically by

$$\mathcal{W}(\nu) = (\bar{I})^2 \left[\delta(\nu) + \iint_{-\infty}^{\infty} R(\xi)R(\xi - \lambda z \nu) d\xi / \left(\iint_{-\infty}^{\infty} R(\xi) d\xi \right)^2 \right], \quad (11)$$

where \bar{I} is the mean irradiance in the speckle pattern,

$R(\xi)$ represents the radiance of the object at coordinate ξ , and $\delta(\nu)$ is a two-dimensional Dirac δ function at the origin of the ν plane.

For the case of a uniformly bright square object of width L , the form of the Wiener spectrum is

$$w(\nu) = (\bar{I})^2 \left[\delta(\nu) + \left(\frac{\lambda z}{L}\right)^2 \Lambda\left(\frac{\lambda z}{L} \nu_x\right) \Lambda\left(\frac{\lambda z}{L} \nu_y\right) \right], \quad (12)$$

where $\Lambda(x) = 1 - |x|$ for $|x| \leq 1$, and zero otherwise. This spectrum is shown in Fig. 7. Note that the speckle pattern contains no frequency components higher than $L/\lambda z$ in the ν_x and ν_y directions, a consequence of the fact that no spacings larger than L can be embraced by the object in these two directions.

For the imaging geometry of Fig. 3, the argument leading to the Wiener spectrum must be changed slightly. In this case we note that a fringe of spatial frequency ν is generated in the image speckle pattern by interference of light from two points *on the lens aperture* separated by vector spacing $\mathbf{s} = (\lambda z \nu_x, \lambda z \nu_y)$, where z is now the distance from the lens to the image plane. Thus the relative mean-square strengths of fringe patterns of different spatial frequencies are found by determining the degree to which corresponding vector spacings are embraced by the lens aperture. The result is a continuous component of the Wiener spectrum which is proportional to the diffraction-limited optical transfer function of the imaging system. For a circular lens of diameter D , the Wiener spectrum of the image speckle pattern is¹³

$$w(\nu) = (\bar{I})^2 \left(\delta(\nu) + 2 \left(\frac{\lambda z}{D}\right)^2 \cdot \frac{2}{\pi} \left\{ \cos^{-1} \left(\frac{\lambda z}{D} \nu\right) - \left(\frac{\lambda z}{D} \nu\right) \left[1 - \left(\frac{\lambda z}{D} \nu\right)^2 \right]^{1/2} \right\} \right) \quad (13)$$

for $\nu \leq D/\lambda z$, and zero otherwise, where $\nu = |\nu|$.

The general conclusions to be drawn from these arguments are that, in any speckle pattern, large-scale fluctuations are the most populous, and no scale sizes are present beyond a certain small-size cutoff. The distribution of scale sizes in between these limits depends on the autocorrelation function of the object radiance distribution in the free-space geometry, or on the autocorrelation function of the pupil function of the imaging system in the imaging geometry.

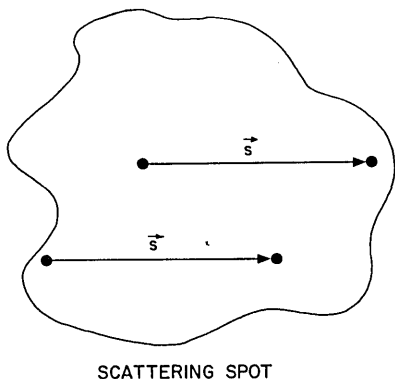


FIG. 6. Single vector spacing \mathbf{s} is embraced by the scattering spot in many different ways.

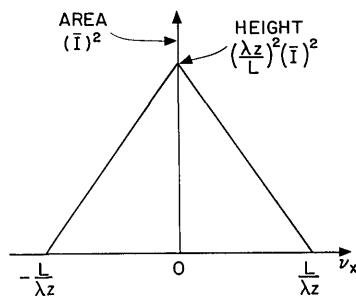


FIG. 7. Cross section of the Wiener spectrum of a speckle pattern arising from a rectangular scattering spot.

CONCLUDING REMARKS

This paper has touched upon only a few fundamental properties of speckle patterns. Many interesting areas have not been discussed, including: second- and higher-order statistics of speckle patterns; statistics of sums of speckle patterns, with and without a coherent background; properties of speckle patterns generated by relatively smooth surfaces; and applications of speckle patterns to information processing, nondestructive testing, and astronomy. Many of these subjects are covered in detail in a recent book on the subject of speckle.¹³ Many others are covered in this special issue. Hopefully, the background provided by this introductory paper will be helpful to the reader in his further investigation of this subject.

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