REVIEW ARTICLE

Sonic crystals and sonic wave-guides

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Received 28 July 2004, in final form 17 November 2004 Published 11 April 2005 Online at stacks.iop.org/MST/16/R47

Abstract

A sonic crystal is a finite-size periodic array composed of scatterers embedded in a homogeneous material. It should have full band-gaps where any sound wave is not allowed to propagate but is reflected completely. It is actually a sonic version of a photonic crystal. Since similarities and differences between the photonic and electronic band structures were discussed and summarized in 1993 by Yablonovitch, photonic crystals have been intensively investigated from the physical and application-oriented points of view. In the same time frame, sonic and phononic crystals have been discussed to realize acoustic band-gaps, wave-guides and filters. Their first experimental realizations of full band-gaps were both reported in 1998. Two-dimensional sonic crystals of rigid cylinders in air have been recently revealed to be promising for acoustical coupled wave-guides constructed in a sonic-crystal slab. This review focuses on sonic crystals corresponding to photonic crystals, and reviews papers on the fundamental physical aspects, methods of theoretical analyses, experimental techniques to realize two-dimensional sonic crystals, wave-guides and coupled wave-guides, and finally ideas of sonic circuits built in sonic-crystal slabs.

Keywords: sonic crystal, sonic-crystal slab, periodic structure, full band-gap, evanescent field, wave-guide, coupled wave-guides, acoustic filters, sonic circuits

(Some figures in this article are in colour only in the electronic version)

1. Introduction

A 'sonic crystal' is an artificial crystal of a finite-size periodic array composed of sonic scatterers embedded in a homogeneous host material. It should have full band-gaps, or complete band-gaps, where any sound wave is not allowed to propagate into the crystal but is reflected completely by the crystal. The host material may be solid, then the term 'phononic crystal' is usually used [16-18, 29, 30, 38, 66, 69, 85, 86, 94] for the artificial crystals. In a solid host material, longitudinal waves and transversal shear waves may exist and may be coupled with one another. In contrast, sonic crystals are considered to be independent of the transversal

waves, although the scatterers are typically made of solid materials allocated in fluid [19, 21, 22, 67, 71, 68, 51-58]. This definition of 'sonic crystal' and 'phononic crystal' is adequate enough here.

A sonic crystal is actually a sonic version of a photonic crystal. Thus, developments of photonic crystals stimulated those of sonic crystals and vice versa. Since the similarities and the differences between photonic and electronic band structures were discussed and summarized by Yablonovitch [89] in 1993, photonic crystals have been intensively investigated with great physical interest as summarized by Joannopoulos et al [23] and by Johnson et al [24] as well as from the application-oriented points of view, as listed up in the bibliography [10]. In general, there are more than two directions of periodicity in an artificial crystal, and their periods are different. The most popular two-dimensional

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Figure 1. Two-dimensional periodic arrays of scatterers with square lattice. The parallel direction (*a*) corresponds to [100] or ΓX direction in the reciprocal space, and the diagonal direction (*b*) to [110] or ΓM direction [57].

square-lattice structure of sonic crystals is shown in figure 1. The period along the side of the lattice is equal to the lattice constant a, and the diagonal direction has a period of $\sqrt{2}a$. Consequently, the well-known Bragg reflections, namely band-gaps, occur at different frequencies proportional to 1/a and 0.707/a for the square lattice. If these two bandgaps are not wide enough, their frequency ranges do not overlap. However, it was found that these band-gaps can overlap due to reflections on the surface of thick scatterers as well as due to wave propagation in the scatterers. Then, any wave is reflected completely from this periodic structure in the frequency range where all the band-gaps for the different periodical directions overlap. This is the fundamental mechanism for the formation of a full band-gap which is required for artificial crystals, such as photonic crystals, sonic crystals and phononic crystals.

One of the major expectations of photonic crystals is their ability to suppress spontaneous emission of light [89]. The density of the final states is the density of optical modes available to the emission. If there is no optical mode available, there will be no spontaneous emission. For example, spontaneous emission determines fundamentally the maximum available output voltage in solar cells. In a full band-gap of a photonic crystal, optical modes, spontaneous emission and zero-point fluctuations are all absent.

On the other hand, the major expectations of sonic crystals, or of artificial crystals in general, are their ability to guide acoustic waves highly efficiently even without loss along the wave-guide constructed in the crystal by removing the scatterers along the pathway of the waves. One of the most exciting phenomena, which was noted by Mekis et al [48] first in photonic crystal, is a high transmission through sharp bends in photonic-crystal wave-guides. The well-known optical wave-guide is made of a layered structure of dielectric materials in which the refractive index of the core region is slightly higher than that of the cladding material. The optical waves are completely confined in the core region through the total reflection on the boundary, which is a smooth plane, between the core and the cladding. At any bend where the incident angle of an optical wave to the bending surface exceeds the critical angle for the total reflection, the optical wave is no longer confined in the core. Therefore, sharp bends are not allowed in conventional dielectric wave-guides. The complete reflection on the boundary of artificial crystals,

however, is due to the full band-gap property itself, and it occurs independent of the incident angle. This makes sharp bends of wave-guide in the artificial crystal possible. The evanescent field distributes across the boundary of the waveguide into the surrounding artificial crystal by several times the lattice constant, just like the evanescent field in the case of total reflection on the dielectric boundary. Since acoustic waves have no phenomena of total reflection, although a practical 100% reflection occurs on the flat boundary of a great difference of the acoustic characteristic impedances such as on a metal surface in air, the acoustic version of photonic-crystal wave-guides is a new structure for acoustic waves. Especially, a wave-guide with sharp bends is a completely novel acoustic device. From this point of view, the acoustic wave-guides constructed in artificial crystals are under investigation not only theoretically [26, 29, 30, 53, 55, 56, 58] but also experimentally [51, 59, 60].

A simple and very important property of artificial crystals in general is the scalability of the frequency characteristics from the low audible frequency to the terahertz range, i.e. from seismic waves to phonon waves. Thus, almost all investigations performed so far in the audible or ultrasonic frequency regions are fundamentally applicable to higher frequencies.

Mainly two-dimensional structures for sonic crystals are investigated, not only because they are constructed more simply and easily than the three-dimensional ones, but also because they have practically useful properties. Therefore this review paper only deals with two-dimensional sonic crystals.

Following this introductory section, the present status of sonic crystals and sonic-crystal wave-guides are reviewed in the following structure of sections. First, in section 2, physical fundamentals for the artificial crystals are summarized, and correspondences between sonic crystals and photonic crystals are discussed based on the published literatures. In section 3, theoretical methods are discussed that have been presented so far to calculate the band-gap structures of sonic crystals and the properties of the sonic-crystal wave-guides. Two representative and useful methods are reviewed. One is the classical method of plane-wave expansion (PWE), which is applicable to an infinite repetition of a fundamental inhomogeneous structure. The other is the finite difference time domain (FDTD) method, which is used as a powerful simulation method for the propagation of waves in arbitrary, inhomogeneous structures. The FDTD method seems to be indispensable in the theoretical investigations of waveguides, and the method has features which should be carefully noted in the applications. Therefore, the following points are discussed, namely the numerical dispersions, the appropriate application of absorbing boundary conditions, and the Courant convergence condition, which demands timeconsuming calculations in the case of high parameter contrasts; especially in the case of sonic crystals composed of rigid cylinders in air. In section 4, the investigations on band-gap structures of the two-dimensional sonic crystals are reviewed. The experimental realizations of the two-dimensional wave propagations are classified into two structures. One is a structure called 'bulk' (section 4.1) which is inhomogeneous cross-sectionally but homogeneous along the perpendicular direction. The other is a periodic structure built in a 'slab' wave-guide (section 4.2) which is by nature three-dimensional. In section 5, wave-guides constructed in the sonic crystals are discussed. Fundamental theoretical investigations for soniccrystal wave-guide are reviewed (section 5.1). Transient behaviour of the guided waves in the sonic-crystal waveguides, which will be an important aspect of the guided modes when applied in the high-speed signal processing devices, is reviewed in section 5.2. In section 6, one of the most important application-oriented devices made of sonic crystals, namely the sonic coupled wave-guide, is discussed.

2. Two-dimensional artificial crystals: sonic and photonic crystals

The refractive index constant for x-rays is tiny, generally 10^{-4} . The forbidden x-ray stop bands are extremely narrow in frequency. As the refractive index constant is increased, the narrow forbidden gaps open up, eventually overlapping in all directions in the reciprocal space. Following Yablonovitch [89] we shall see that this requires an index contrast larger than or equal to 2. The refractive index depends on the dielectric constant as well as on the magnetic permeability. Due to their lossive properties, however, magnetic materials are usually not considered for photonic crystals. Therefore, the dielectric constant is the only material parameter relevant to photonic crystals. When we develop sonic crystals, the contrast of the bulk moduli of the scatterers and the host material and the contrast of the densities of the scatterers and the host material are the relevant material parameters. Characteristic impedance and refractive index will play important roles in sonic crystals, both of them being derived from bulk modulus and density. Although the contrast of refractive index is noted for photonic crystals from the above-mentioned situations, the contrast of characteristic impedance may be more important in the formation of full band-gaps for sonic crystals, as pointed out by Kee [28].

We summarize and compare the fundamental properties of the two-dimensional sound waves and electromagnetic waves [53] in this section. First, we review the normalized version of the equations of motion which describe the behaviour of sound waves and electromagnetic waves. The normalized physical quantities and material parameters introduced in this section are used in the following sections. Second, sonic crystals and photonic crystals are compared in a common parameter space.

2.1. Wave equations for sound waves and electromagnetic waves

Sound waves are described by the following simple equations:

$$\frac{\partial \dot{\boldsymbol{\xi}}}{\partial t} = -\boldsymbol{\nabla}p, \qquad \frac{\partial p}{\partial t} = -K\boldsymbol{\nabla}\cdot\dot{\boldsymbol{\xi}}.$$
 (1)

The physical quantities are particle velocity $\dot{\xi}$ and sound pressure p, which are the pressure deviation from the static pressure due to sound waves. The material parameters of the medium are bulk modulus K and density ρ . Since ξ denotes conventionally the position of the particle, the particle velocity is given by its time derivative $\dot{\xi}$. This notation is simplified by the following normalization process. Dissipations are not included here, because the dissipation term plays no fundamental role for the artificial crystals.

In order to compare these equations with the electromagnetic equations, we normalize them by the material parameters of a simple uniform medium which will be the host material of the sonic crystal. Let them be denoted with an index 0, namely density by ρ_0 , bulk modulus by K_0 , the speed of sound by $c_0 = \sqrt{K_0/\rho_0}$ and the acoustic characteristic impedance by $Z_0 = \rho_0 c_0 = \sqrt{\rho_0 K_0}$. The normalized physical quantities are $v = Z_0 \dot{\xi}$ for the normalized particle velocity which has the same dimension as the sound pressure, and $u = c_0 t$ for the normalized time which has the dimension of a distance and is equal to the travelling distance of a plane wave in the uniform medium in time t. The material parameters of the inhomogeneous structures are described by the normalized density, or the contrast of density $\rho_0/\rho = \rho$ and the normalized bulk modulus, or the contrast of bulk modulus $K/K_0 = \overline{K}$, respectively. Then the normalized equations of motion are given as [51]

$$\frac{\partial \boldsymbol{v}}{\partial \boldsymbol{u}} = -\underline{\rho} \boldsymbol{\nabla} \boldsymbol{p}, \qquad \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{u}} = -\overline{K} \boldsymbol{\nabla} \cdot \boldsymbol{v}.$$
 (2)

The FDTD method solves the inhomogeneous wave equations (2) for the theoretical investigation of acoustic wave propagation in a sonic crystal of an arbitrary finite shape [51]. A single second-order inhomogeneous differential equation for the sound pressure p is derived from equations (2) in the normalized form:

$$\nabla \cdot (\underline{\rho} \nabla p) - \frac{1}{\overline{K}} \frac{\partial^2 p}{\partial u^2} = 0.$$
(3)

This equation was solved to obtain the band-gap structure of sonic crystals by the method of Green's function [22], by the plane-wave expansion (PWE) method [38, 76], or by a variational method [68, 71] for an infinite array of scatterers.

The electromagnetic equations of motion in lossless media are normalized also in the same way as the equations of motion for sound waves. We denote the material parameters of a simple uniform medium, which will be the host material of the photonic crystal, with an index 0, namely the dielectric constant by ε_0 , magnetic permeability by μ_0 , the velocity of light by $c_0 = 1/\sqrt{\varepsilon_0\mu_0}$ and the characteristic impedance by $Z_0 = \mu_0 c_0 = \sqrt{\mu_0/\varepsilon_0}$. We use the same notation c_0 and Z_0 both for sound waves and for electromagnetic waves. The normalized physical quantities are: $G = Z_0 H$ for the normalized magnetic field, which has the same dimension as the electric field; $u = c_0 t$ for the normalized time,



Figure 2. Correspondence between field components of two-dimensional sound waves and electromagnetic waves in a vector diagram [54]. ©2002 The Japan Society of Applied Physics

 Table 1. A complete correspondence between two-dimensional sound waves and electromagnetic waves [53].

Sound waves	TE waves	TM waves	
v_x	E_{y}	$-G_{y}$	
v_y	$-E_x$	G_x	
р	G_z	E_z	
K	μ	<u></u>	
<u>ρ</u>	ε	$\underline{\mu}$	



Figure 3. Two-dimensional sonic crystals and photonic crystals in a common parameter space [55]. Phononic crystals are also included for comparison. ©2002 IEICE

which has the dimension of a distance; $\varepsilon_0/\varepsilon = \underline{\varepsilon}$ and $\mu_0/\mu = \underline{\mu}$ for the normalized dielectric constant and magnetic permeability, respectively. Then the normalized equations of electromagnetic waves are given as follows [51]:

$$\frac{\partial \boldsymbol{E}}{\partial u} = \underline{\boldsymbol{\varepsilon}} \boldsymbol{\nabla} \times \boldsymbol{G}, \qquad \frac{\partial \boldsymbol{G}}{\partial u} = -\underline{\boldsymbol{\mu}} \boldsymbol{\nabla} \times \boldsymbol{E}. \tag{4}$$

2.2. Correspondence of physical quantities and material parameters between two-dimensional sound waves and electromagnetic waves

Equations (2) for sound waves include only operations of gradient or divergence, whereas those for electromagnetic waves (4) are described by vector operation of rotation. These equations seem to be completely different in nature and have no correspondences. In the two-dimensional case, however, a complete correspondence was found between them in Cartesian coordinates [53], as shown in table 1. From this correspondence, it is very simple and also helpful to give an overview of sonic crystals and photonic crystals, by drawing a vector diagram as shown in figure 2. The magnetic field Hof the TE wave as well as the electric field E of the TM wave behave like the sound pressure p of scalar quantity. The transversal components E of the TE wave as well as H of the TM wave are perpendicular to the direction of propagation, while the particle velocity of the sound waves is parallel with the propagation. This simple consideration makes clear the physical situations of sonic crystals. In the typical two-dimensional sonic crystals, sound waves in the host fluid material are almost completely reflected on the surfaces of the scatterers, which look like pillars illustrated in figure 2, due to the large impedance mismatch, or contrast. No transversal elastic waves are excited in the scatterers [55], and the scenario of two-dimensional sound waves is valid for sonic crystals and corresponds completely to the scenario of either TE waves or TM waves [53].

2.3. Sonic crystals and photonic crystals in a common parameter space

The complete correspondence shown in table 1 represents a common material-parameter space for the two-dimensional sonic crystals and photonic crystals [55], as shown in figure 3. The ordinate describes the normalized bulk modulus \overline{K} or dielectric constant $\underline{\varepsilon}$, and the abscissa describes the normalized density $\underline{\rho}$ or magnetic permeability of the scatterers for sonic crystals and photonic crystals, respectively. The dynamic range of the normalized parameters extends over 4 to 5 orders in magnitude so that both axes are drawn on a logarithmic scale. The centre of the space (1, 1) corresponds to a uniform space made of the host material without scatterers.

The typical and representative photonic crystals, made of dielectric cylinders of GaAs in air and of air cylinders in GaAs, are plotted at (1.0, 0.0865) and (1.0, 11.56) in the figure, respectively [23]. These are both about one order apart from the uniform point (1, 1). It should be noted that all promising photonic crystals investigated so far are dielectric and nonmagnetic, lying on a line of $\mu = 1.0$. On the other hand, sonic crystals distribute diversely in the parameter space. For example, the typical sonic crystal of acrylic resin cylinders in air [54] is at (1.1 \times 10⁻³, 3.8 \times 10⁴) and, thus, far from the typical photonic crystals. Although phononic crystals should be plotted on a three-dimensional parameter space of \overline{K} , ρ and shear modulus G, some of them are projected on figure 3 for comparison. The first experimental observation of an ultrasonic full band-gap in periodic two-dimensional composites for the longitudinal wave mode was made by Montero de Espinosa et al [61] in a unique structure of an aluminium alloy plate with a square periodic arrangement of cylindrical holes filled with mercury. This crystal is positioned at (0.198, 0.368) in the close neighbourhood of the typical photonic crystals. A phononic crystal of steel cylinders in water reported by Khelif *et al* [31] lies between the typical sonic and photonic crystals. No full band-gap has been observed, yet, for periodic structures of acrylic resin cylinders in water because of the too small parameter contrasts. A quasi-sonic crystal of air cylinders in agar host [56] is at (790, 6.5×10^{-5}) far from the photonic crystals.

All promising sonic crystals and photonic crystals are noted to be located around $\overline{c} = \sqrt{\overline{K}\rho} = 10 \text{ or } 10^{-1} \text{ in figure 3}$. Yablonovitch has estimated this index contrast \overline{c} or $\underline{c} = 1/\overline{c}$ to be larger than 2 to realize overlapping band-gaps, namely to realize full band-gaps [89]. As is evident already from the above discussions, typical sonic crystals have a peculiar feature of high characteristic impedance contrast $\overline{Z} = \sqrt{\overline{K}/\rho}$, ranging between 10³ and 10⁴. From this feature, differences between sonic and photonic crystals arise, especially with respect to field distribution and wave propagation in the wave-guides, as discussed in later sections.

3. Theoretical methods of sonic crystals and sonic-crystal wave-guides

Theoretical methods that may be used to analyse the propagation of sound waves in the sonic crystal are classified into four groups. The first is (1) the plane-wave expansion (PWE) method which is valid for periodic structures of an infinite size [11, 71, 68], (2) the FDTD method for arbitrary structures of a finite size [53, 79], (3) the wave propagation method which solves slowly varying envelope functions of waves in an arbitrary structure [33], and (4) a method which gives analytical solutions for some specific structures [27]. All these methods are used in the analysis of photonic crystals, whereas methods (3) and (4) have never been applied to sonic crystals. Especially, the FDTD method is powerful and inevitable for the theoretical and numerical analysis of sonic-crystal wave-guides.

3.1. Plane-wave expansion method with Floquet–Bloch theorem

A harmonic form for the sound pressure $p(\mathbf{r}, u) = e^{jk_0 u} p(\mathbf{r})$ is assumed. Here, the wave number in the homogeneous space is denoted by $k_0 = \omega/c_0$. Then equation (3) is transformed to the following equation²:

$$\nabla \cdot (\underline{\rho}(\boldsymbol{r})\nabla p(\boldsymbol{r})) + \frac{k_0^2}{\overline{K}(\boldsymbol{r})}p(\boldsymbol{r}) = 0.$$
 (5)

A typical model of a sonic crystal with an infinite periodic array of scatterers is adopted. Applying the Floquet-Bloch theorem for the periodic system, the spatial function of the pressure is described by $p(r) = u_k(r) e^{ik \cdot r}$, where $u_k(r)$ is a periodic function according to the periodicity of the sonic crystal. Equation (5) is to be solved for the periodic function p(r).

² The electromagnetic equations for the analysis of photonic crystals are described in terms of the magnetic field to construct a Hermitian operator as discussed by Joannopoulos *et al* [23], and derived from equations (4) into the following normalized form. Usually $\mu(r) = 1$ for photonic crystals.

$$\nabla \times \left(\underline{\varepsilon}(\mathbf{r})\nabla \times \mathbf{G}(\mathbf{r})\right) + \frac{k_0^2}{\mu(\mathbf{r})}\mathbf{G}(\mathbf{r}) = 0.$$

According to the PWE method, Economou *et al* [11] and Sigalas [78] expanded $u_k(r)$ as well as the material parameters $\underline{\rho}(r)$ and $\underline{K}(r) = 1/\overline{K}(r)$ in Fourier series with the corresponding coefficients u_{k+G} , \underline{K}_G and $\underline{\rho}_G$, respectively, where *G* are the vectors of the reciprocal lattice. In terms of these coefficients, equation (5) is transformed into an infinite matrix equation for u_G .

$$\sum_{G'} [\omega^2 \underline{K}_{G-G'} - (k+G)(k+G')\underline{\rho}_{G-G'}]u_{G'} = 0.$$
(6)

Keeping a finite number M of Fourier components in the expansion, an appropriate $M \times M$ matrix equation was solved. Economou *et al* [11] reported that usually a value of M around 350 was adequate to achieve a 1% accuracy. Sigalas [78] mentioned that the eigenvalues do not change by more than 5% when using more reciprocal vectors than M = 100 per unit cell, and he noted further an interesting fact that much better convergence can be achieved with equation (5) rather than with the equivalent wave equation, where $\overline{K}(r)$ appears as a multiplicative factor in the first term of the equation.

Sánchez-Pérez *et al* [71] and also Rubio *et al* [68] employed a variational method in which the pressure function $u_k(r)$ is expanded as a superposition of a finite number N of localized functions, which account for the periodicity of the Bravais lattice of the system, and each localized function is defined by a product of one-dimensional cubic B-splines. Using the standard procedure, the differential equation (5) is transformed to a finite and sparse matrix equation for the coefficients of the expansions, namely the variational parameters. They have shown that N = 100 functions are enough to get results similar to those obtained by the PWE method. The band structures of a sonic crystal made of aluminium cylinders in air obtained by this variational method were used in figure 5.

Interpreting equation (5) as an operator equation

$$L(\mathbf{r})p(\mathbf{r}) = \omega^2 p(\mathbf{r}), \tag{7}$$

James *et al* solved the transmission spectrum of a onedimensional periodic structure of finite size using a Green's function method [22]. The results agreed well with the experimentally measured transmission spectra of the layered structures.

3.2. Two-dimensional FDTD method for sonic crystals

Since analytical methods cannot practically solve the propagation of waves in a finite size periodic structure, the FDTD method is an important numerical tool, especially for the analysis of wave-guides with bends, and for simulations of the propagation of waves in more complicated acoustic circuits. The FDTD method has, however, a few crucial drawbacks. So we should briefly overview the essence of the method especially for sound waves, and point out the applicabilities here [53].

3.2.1. Finite difference time domain equations. The equations to be solved in a two-dimensional region where sonic crystals are included are given by equations (2) in the Cartesian coordinate components (v_x, v_y) of v and the scalar pressure p. Let Δu denote the finite difference of the normalized time u, and Δx , Δy those of spatial coordinates x, y. Taking the



Figure 4. Spatial sampling points for two-dimensional FDTD method applied to sound waves.

temporal sampling points of the sound pressure *p* at $u = n\Delta u$, and those of the particle velocities v_x , v_y at $u = (n + \frac{1}{2})\Delta u$, the spatial sampling points of those quantities are illustrated in figure 4.

As in the Yee algorithm, which was developed to solve electromagnetic equations, both temporal and spatial derivatives in the acoustic equations (2) are approximated by the central symmetric finite differences. Then finite difference equations are obtained, which solve $v^{n+\frac{1}{2}}$ and p^{n+1} mutually progressively, increasing *n* [53].

$$v_x^{n+\frac{1}{2}}\left(i+\frac{1}{2},j\right) = v_x^{n-\frac{1}{2}}\left(i+\frac{1}{2},j\right) - \underline{\rho}\left(i+\frac{1}{2},j\right) R_x\{p^n(i+1,j) - p^n(i,j)\},$$
(8)
$$v_y^{n+\frac{1}{2}}\left(i,j+\frac{1}{2}\right) = v_y^{n-\frac{1}{2}}\left(i,j+\frac{1}{2}\right)$$

$$-\underline{\rho}\left(i, j + \frac{1}{2}\right) R_{y}\{p^{n}(i, j + 1) - p^{n}(i, j)\},$$
(9)
$$p^{n+1}(i, j) = p^{n}(i, j)$$
$$-\overline{K}(i, j) R_{y} \left\{y^{n+\frac{1}{2}}(i + \frac{1}{2}, i) - y^{n+\frac{1}{2}}(i - \frac{1}{2}, i)\right\}$$

$$-\overline{K}(i,j)R_{y}\left\{v_{y}^{n+\frac{1}{2}}\left(i,j+\frac{1}{2}\right)-v_{y}^{n+\frac{1}{2}}\left(i,j-\frac{1}{2}\right)\right\}.$$
(10)

Here the following normalized parameters,

$$R_x = \frac{\Delta u}{\Delta x}, \qquad R_y = \frac{\Delta u}{\Delta y}$$
 (11)

are introduced. The parameters R_x and R_y denote the ratios of the distances that an ideal plane wave propagates in the host medium in a time difference Δt , namely a normalized time difference $\Delta u = c_0 \Delta t$, to the spatial differences Δx and Δy , respectively.

Sound sources are given by either the sound pressure $p^n(i, j)$ or the particle velocity $(v_x^{n-\frac{1}{2}}(i+\frac{1}{2}, j), v_y^{n-\frac{1}{2}}(i, j+\frac{1}{2}))$. The normalized coefficients $\underline{\rho}(i+\frac{1}{2}, j), \underline{\rho}(i, j+\frac{1}{2})$ and $\overline{K}(i, j)$ are equal to unity in the host material by definition. For the scatterers, the coefficients represent the contrasts of densities and bulk moduli between the scatterers and the host material. Equations (8)–(10) simulate fairly exactly the temporal evolution of sound waves in a two-dimensional space, if a few important conditions described in the next section are fulfilled.

3.2.2. Important conditions of the FDTD method. There are a few important conditions to which the actual applications of the FDTD method are subjected inevitably. Although these have already been discussed in the field of electromagnetic waves, the actual situations are very different for sound waves in sonic crystals.

(1) Numerical stability (Courant condition). Finitedifference wave equations have a well-known numerical stability condition called the 'Courant condition'. In the case of sonic crystals, normalized material parameters ρ and \overline{K} , which appear in equations (8)–(10), multiplied with R_x and R_y are much larger than one, and the Courant condition should be fulfilled including these parameters. Taking $R_x = R_y$ and $\mathcal{M}_{\rho K}$ as the maximum value of ρ and \overline{K} in the region of analysis, the Courant condition is given as follows [53]:

$$R_x = R_y < 1/(\sqrt{2}\mathcal{M}_{\rho K}).$$
 (12)

This condition imposes very difficult restrictions on the numerical analysis of wave propagation in the sonic crystals. In the case of the typical photonic crystal of GaAs dielectric cylinders in air, the parameter contrasts between the host and the scatterer's $\underline{\varepsilon}$ and μ , which correspond to \overline{K} and ρ , respectively, are less than or equal to unity, and the Courant condition is easily fulfilled. In contrast, a typical sonic crystal of acrylic cylinders in air [54] or aluminium cylinders in air [68, 57] has a parameter contrast in the order of 10^4 – 10^5 as shown in figure 3. Even if a sub-grid method is applied to the scatterers of high contrast parameter value, any direct fulfilment of the Courant condition requires 10⁴ times longer calculation time than the wave propagation in free space. Such a long calculation time is only acceptable for supercomputers. This restriction has been escaped skillfully. Kee [28] and Miyashita [54] noted that the characteristic impedance contrast \overline{Z} practically determines the wave propagations in sonic crystals made of rigid cylinders in air, and Miyashita has reduced the bulk modulus contrast to $\overline{K} = 8.0$ keeping the value of \overline{Z} exact.

(2) Numerical dispersion. The next important property is called numerical dispersion, or grid dispersion, which makes the velocity of waves in the FDTD calculation dispersive between [100] and [110] directions and also dependent on frequency. It has been reported that the minimum dispersion is obtained at the equal limit of the Courant condition, namely at $R_x = R_y = 1/\sqrt{2}$, and a measure of the dispersion is given as [54, 55]

$$k \cong k_0 \left\{ 1 + \frac{(k_0 \Delta u)^2 (1 - \sin^2 2\theta)}{24 - 4(k_0 \Delta u)^2 (2 - \sin^2 2\theta)} \right\},$$
 (13)

where k is the wave number, k_0 the wave number of the homogeneous space filled with the host material and θ the propagation angle of the sinusoidal plane wave.

It is clear that plane waves propagating in the direction of $\theta = \pi/4$ do not experience numerical dispersion with a free space dispersion relation $k = k_0$. On the other hand, plane

Table 2. Phase errors induced in the propagation of ten times wavelength.

$\lambda/\Delta x$	10	20	30	40
Phase error $[2\pi]$	0.088	0.021	0.0092	0.0052

waves propagating along the *x* or *y* axis suffer from numerical dispersion. Phase differences induced between these two directions of propagation are calculated as shown in table 2. In the typical cases of sonic-crystal analyses, the spatial region of analysis is about 10 wavelengths square, so a 1% phase error will be obtained with a grid-spacing finer than 30 grids in the wavelength. Sigalas and Garcia used about 60 grids for the band-gap calculation [79]. This numerical dispersion reduced with a new FDTD algorithm called 'non-standard FDTD method' developed by Cole [9] with a sacrifice of programming complexity. No application of the nonstandard FDTD method to the numerical investigation of sonic crystals is known, yet.

(3) Absorbing boundary condition (ABC). In the analysis of wave propagation by the finite element method (FEM), open-space conditions are normally encountered. One way to deal with this problem is the so-called combination method, where the whole unlimited space is divided into two regions [50]. One is a bounded space which includes all the non-uniform objects, where the wave propagation is described by the FEM. The other is the complementary space of the former one, and it is described by the boundary element method (BEM). Combining the formulations for these two regions on the boundary, the open-space wave propagations are solved.

In the past ten years, an efficient method for open-space wave propagation has been developed introducing an artificial absorbing boundary condition (ABC) to the FDTD method [62]. Especially the perfectly matched layer (PML) developed by Berenger [3] is most effective and most frequently used in the analysis of electromagnetic and sound wave propagations. Plane waves which propagate normally to the boundary of PML are not reflected on the boundary as if the impedances were perfectly matched. Actually they are absorbed to the desired degree of typically about 100 dB during the propagation in the PML. On the other hand, the plane waves propagating along the boundary experience no attenuation, and they are absorbed by the other perpendicular parts of the PML boundary. This PML scheme has been applied also to the numerical analysis of acoustic wave propagation in non-elastic media [54, 93].

4. Two-dimensional sonic crystals: full band-gaps

Photonic crystals for microwaves were first experimentally discussed in 1993 pursuing the analogy between the behaviour of electromagnetic waves in three-dimensional periodic structures of dielectric materials and the familiar behaviour of electron waves in natural crystals [89]. Almost at the same time, the band-gap structure of acoustic waves propagating in periodic composite media was discussed, and the existence of full band-gaps was predicted theoretically. Kushwaha *et al* [36] reported in 1993 the first full band-gap calculations for periodic elastic composites composed of an array of straight infinite cylinders embedded in an elastic background.

An experimental report appeared in 1995 about bandgap structures of a one-dimensional periodic array of two to ten perspex plates in water by James *et al* [22]. They compared the experimental results with the theoretical bandgap structures of a finite number (between 2 and 10) of unit cells. Including the defect states³, good agreements were obtained in the cut-off frequencies of the passband. The first experimental observation of an ultrasonic full band-gap in a two-dimensional phononic crystal for the longitudinal wave mode was reported in 1998 by Montero de Espinosa *et al* [61]. Their novel structure consists of an aluminium alloy plate with a periodic square arrangement of cylindrical holes filled with mercury. A full band-gap was observed between 1.00 MHz and 1.12 MHz.

Experimental band-gap structures of two-dimensional sonic crystals were first reported in 1998 by Sánchez-Pérez et al [71], and also by Robertson and Rudy [67]. They obtained an effect, in their words, considered as a fingerprint of the existence of a full band-gap. The former experiments were refined in 1999 by Rubio et al [68], and full band-gap spectra of the two-dimensional sonic crystals of rigid cylinders in air were depicted after the critical comparison between the theoretical predictions and the experimental results. A full band-gap transmission spectrum was experimentally obtained by Miyashita and Inoue [52] for a two-dimensional sonic crystal of acrylic resin cylinders in air. Let us call this type of sonic crystal composed of a two-dimensional periodic array of long scatterers for a two-dimensional plane-wave propagation as 'sonic-crystal bulk' in this review. Recently, a novel periodic array of sonic scatterers was constructed by Miyashita [57], built in a thin air layer between two parallel metal plates as an alternative to the two-dimensional soniccrystal bulks, which will be referred to as a 'sonic-crystal slab'. This slab structure is expected to be preferable and promising for the sonic crystals with respect to the realization of integrated acoustic circuits. These experimental reports are reviewed together with the methods used for the theoretical considerations in the following sections.

4.1. Experimental sonic crystal, sonic-crystal bulks

Experimental observations of the band-gap structures of sonic crystals have been reported since 1998. Robertson and Ruddy presented the experimental observation of acoustic stopbands in a two-dimensional periodic array of rigid cylindrical scatterers in air [67]. The scatterer is made of an electrical conduit of 2.34 cm in diameter and 1 m in length. A square array was 6 rows deep \times 10 rows wide with a lattice constant of 3.7 cm and a filling factor of 0.31. Also a triangular lattice was built with a filling factor of 0.366. They measured the transmission spectrum of the crystals, applying an impulse signal through a speaker, where the most precise measurement was achieved using a sound waveform generated by taking the second derivative of a Gaussian. The Fourier transforms of the acoustic signals arriving at the microphone with and without the scatterers were compared. A temporal windowing processing was used to reduce the effect of the reverberant The measured and calculated Fourier amplitude fields.

³ A pass-band which appears in the band-gap of the original crystal due to a lack (defect) of scatterers.

transmitted along the ΓX direction⁴ showed clearly a stop band both for the square lattice and for the triangular lattice. It was difficult, however, to identify a clear band-gap from the amplitude of the waves transmitted either along the ΓM direction of the square lattice or along the ΓJ direction of the triangular lattice. They tried to inspect the phase information of the Fourier transforms, namely the phase dependence on the frequency, and found a frequency range where the phase delay exhibits anomalous dispersion with positive slope, and they interpreted it as a clear indication of the presence of a bandgap. They concluded together with the phase information that their experimental results showed a full band-gap for sound waves propagating in the triangular lattice. They noted also that a simple two-dimensional array will be of limited value for most practical applications, because the stop-band exists only for propagation in a plane, which is perpendicular to the cylinders and contains the central axes of the speaker and the microphone. This was overcome later by the introduction of a sonic-crystal slab [57].

Sánchez-Pérez *et al* have investigated experimentally the band-gap structure of a two-dimensional periodic array of rigid, stainless steel and wood cylinders in air with two different geometrical configurations, square and triangular [71]. Their experiments were performed in an anechoic chamber. They concluded that there was an overlap between the attenuation peaks measured along the two high-symmetry directions of the Brillouin zone (BZ) in the range of audible frequencies in both configurations above a certain filling ratio, and that this is considered as a fingerprint of the existence of a full acoustic band-gap.

Rubio et al have tried to interpret the experimental existence of full band-gaps in two-dimensional sonic crystals through the critical comparison between theory and experiment [68]. They have solved equation (5) applying Bloch's theorem and introducing a novel variational method for the transmission of sound waves through infinite size sonic crystals of square and triangular lattices in air. A velocity ratio of 17.2 was used in the theoretical calculations corresponding to the case of aluminium rods in air, and a density ratio of 200 was considered, although the actual ratio is of the order of 10⁵. They pointed out that no valuable differences exist in the calculated values even with higher density ratios. The transmission measurements of the sound waves through the sonic crystals made of aluminium or wood rods in air were combined with the phase dispersion measurement of the sound waves. Following Robertson and Ruddy [67], they supposed that the anomalous phase dispersions coincide with the borders of the band-gaps. Discussing the critical comparison between the theoretical dispersion relations obtained by the above-mentioned variational method and the experimental attenuation bands, they characterized the band-gap structures. Their summary is shown in figure 5. Along the ΓX directions in both lattices, a good agreement between theoretical and experimental values of the band-gap borders can be noted. Nevertheless, along ΓM (square) and ΓJ (triangular), the experimentally obtained attenuation bands appear in frequency regions where no band-gaps exist in the theoretically obtained dispersion relation. They believed this effect to be an artefact of the experimental technique. With a



Figure 5. Theoretical and experimental band structures by Rubio *et al* [68]. (*a*) Gap map calculated for square lattices of rigid cylinders in air. The dotted (full) lines define the limits of the first pseudo-gap along the ΓX (ΓM) direction in BZ. A full sonic band-gap is the overlap of both pseudo-gaps (shaded zone). The black dots (diamonds) are the limits of the attenuation peaks measured along the ΓX (ΓM) directions. (*b*) The same for a triangular lattice, where the diamonds refer to the ΓJ direction. The reduced units are $\omega a/2\pi c_0$, where c_0 is the speed of sound in air and *a* the lattice constant. ©1999 IEEE.

critical comparison between the theory and the experiments, they concluded that the square lattice with a filling fraction of 0.41 has a full band-gap in the region [3099–4093] Hz, and that the result agrees well with the theoretical value [3331–3899] Hz.

Miyashita and Inoue performed a comprehensive theoretical and experimental study on a sonic crystal made of acrylic cylinders in air, and presented evidence for the existence of a full band-gap in the transmission spectra both theoretically and experimentally [54]. They derived FDTD equations for acoustic longitudinal wave propagations especially in the two-dimensional space [53], applying the method of FDTD analysis for electromagnetic waves [92] Temporal solutions of the FDTD to the acoustic case. equations were used directly to simulate the waveform of sound in sonic crystals and also sonic-crystal wave-guides, including the transient phenomena. In contrast to the other theoretical methods of PWE or the variational one, the FDTD method can analyse arbitrary inhomogeneous structures of finite size corresponding to the experimental situations. To precisely calculate the wave propagation in the limited region of analysis, they used the PML, and achieved a residual reflection coefficient of -105 dB on the boundary of analysis [53].

Bulk modulus contrast \overline{K} , or ratio of scatterers (solid) to host material (air), is much larger than unity for sonic crystals,

 $^{^4~}$ The reader may refer to the notation $\Gamma X,$ ΓM and ΓJ in figure 5.



Figure 6. Sonic-crystal bulk of acrylic resin rods in air by Miyashita *et al* [57]. Materials are all acrylic resin. The lattice constant is 24.0 mm, the radius of the scatterers 10.2 mm and their length about 30 cm. The top and bottom sides both have sufficient amounts of glass wool to effectively absorb the sound waves of oblique wavefronts. A straight wave-guide as well as a bending wave-guide was constructed in this sonic-crystal bulk, as reviewed in section 5.3.1.

as shown in figure 3. The FDTD calculation needs much computation time, for it is proportional to the large parameter ratio \overline{K} to fulfil the Courant condition as stated in section 3.2.2. Miyashita and Inoue considered that the high characteristic impedance ratio \overline{Z} of the sonic crystal mainly determines the sonic-wave distribution in the sonic crystals as discussed in section 2. They calculated the band-gap structures with several computation-tolerant parameter values of \overline{K} and ρ , keeping the characteristic impedance ratio $\overline{Z} = \sqrt{\overline{K}/\rho}$ to the actual value of 5.9 × 10³. They found no more practical variations in the region of the band-gap spectra calculated with $\overline{K} = 8.0$ are shown in figure 7 together with the measured transmission spectra.

Based on the numerical results, Miyashita and Inoue constructed a sonic crystal of a two-dimensional array of 10×11 acrylic resin cylinders with a radius of 10.2 mm and a lattice constant of 24.0 mm in air, resulting in a filling ratio of 0.567 [51, 54]. To realize a two-dimensional sonic crystal, and to measure the transmission spectra of the two-dimensionally propagating plane waves, the following two points were carefully considered as shown in figure 6.

First, the length of the acrylic resin cylinders was designed to be long enough to assure uniformity of the sonic crystal in



Figure 7. Experimental and theoretical band-gap structures of a sonic crystal of acrylic resin rods in air by Miyashita [54]. ©2002 The Japan Society of Applied Physics

the third dimension. Second, a sufficient amount of glass wool was placed so that the oblique plane waves may not be reflected at the top and bottom of the scatterers. They used a short tone-burst with a time duration of 8 ms, in order to measure the transmitted sound waves in the normal laboratory circumstances independent of the undesired echos in the time domain. The steady-state parts of the transmitted waveform were extracted, and their amplitude and phase were determined relative to those of the source signal by means of digital or numerical homodyne detection. The measurements were finally normalized by the same experimental measurements without the sonic crystal. The experimental results show a clear and deep full band-gap in the transmission spectra, as shown in figure 7. The measured full band-gap was 6.8-9.5 kHz, or 0.475-0.67 in normalized frequencies with a transmission ratio smaller than -25 dB. The experimental band-gap was 20% wider than the theoretical one.

It should be noted that the existence of the full bandgaps of two-dimensional sonic crystals was first observed by carefully realizing two-dimensional plane-wave propagation in the crystals. Alternatively, to assure two-dimensional wave propagation in the sonic crystal, Miyashita *et al* have invented a periodic structure within a thin acoustic wave-guide slab [57]. The idea leads to a sort of integrated acoustic circuit made of wave-guides in a thin sonic-crystal slab, as reviewed in the following sections.

4.2. Experimental sonic-crystal slab

A periodic array of acoustic scatterers constructed in an acoustic wave-guide slab, i.e. in a thin space between two parallel rigid plates, is expected to be a good candidate for a practical sonic crystal for two-dimensional sound waves unlike the sonic-crystal bulk. This type of sonic crystal was investigated recently by Miyashita *et al* [57, 59, 60]. A sound wave propagating in the sonic-crystal slab is not a two-dimensional wave, whose wavefront should be uniform and perpendicular to the slab structure. However, it is expected to behave approximately like a two-dimensional wave due to the mode selection properties of a thin wave-guide slab.



Figure 8. Sonic-crystal slabs [57]. (a) For direction [100], (b) for direction [110].



Figure 9. Measured band-gap structures of a sonic-crystal slab [57].

The sonic-crystal slab is constructed, as shown in figure 8, by Miyashita et al [57] between a pair of aluminium metal plates with 15 mm spacing, which is smaller than the wavelength of the sound waves in the full band-gap. The lattice constant of the sonic-crystal slab is 12.0 mm, the radius of the scatterers is 5.0 mm and the number of scatterers is 16×12 . The measured transmission spectra are shown in figure 9 [57]. The band-gap structure is similar to those of sonic-crystal bulk shown in figure 7 as expected, but not as sharp as expected. The transmission in the lower frequency range is too small in the [100] direction, whereas in the higher frequency range it is too small in the [110] direction. The full band-gap determined from figure 9 is between 15.1 kHz and 18.8 kHz, or between 0.52 and 0.65 in normalized frequencies, with a transmission ratio smaller than -20 dB. The scaling property of the artificial crystal [23] is naturally fulfilled between the full band-gap of this sonic-crystal slab and that of the sonic-crystal bulk shown in figure 7, i.e. their ranges are almost the same in the normalized frequencies.

5. Sonic-crystal wave-guide

Once a full band-gap has been realized for a sonic crystal, a good confinement of the sound waves is expected in a waveguide which is constructed by removing a line of scatterers in the sonic crystal. This expectation comes naturally also from the successful development of the sharp bending photonic wave-guides by Mekis *et al* [48].

5.1. Theoretical investigations of acoustic waves in sonic-crystal wave-guides

The propagation of acoustic waves through two-dimensional periodic composites consisting of solid cylinders in air was investigated by Sigalas [78] using the PWE method for two cases of single and line defects of the periodic alignment. Line defects can act as a wave-guide for acoustic waves, while single defects can be used as acoustical filters. Similar cases of defects have been studied for electromagnetic waves in photonic crystals [47, 64, 88]. Two methods are used for the theoretical analysis of the periodic structure with defects. One is direct numerical simulation of the wave propagations with the FDTD method reviewed in section 3.2. The other is solving acoustic wave-equations with the PWE method reviewed in section 3.1. The latter method requires an infinite periodic structure of the object. In order to fulfil the requirement, the idea of super-cells has been introduced. In the supercell calculations, the object is assumed to be composed of an infinite periodic repetition of a super-cell which is itself a minimum periodic structure with a single defect or a line of defects. It is desirable that the wave propagations in the spatially repeated super-cells are practically independent of each other.

Sigalas studied the defects mode in the band structure of acoustic waves propagating in a typical sonic crystal, namely a two-dimensional square lattice of solid cylinders periodically placed in air. The filling ratio of the cylinders in a unit cell is 0.548 and the full band-gap extends from 0.46 to 0.65 in the



Figure 10. Initial buildup of the waves in the sonic and photonic 'straight wave-guides' [56].

reduced frequency a/λ , which corresponds closely to the case of the sonic-crystal bulk studied by Miyashita et al [57] and reviewed in section 4.1. First, two different super-cells were studied consisting of 3×3 and 5×5 cylinders, respectively. A defect is introduced by decreasing the radius $r_{\rm d}$ of one cylinder at the centre of the super-cell. A defect band, a passband which appears in the band-gap of the original crystal without defects, emerged from the upper edge of the band-gap and moved towards the centre of the band-gap. For $r_{\rm d} = 0$, the centre of the defect band was at $a/\lambda = 0.584$. It was also found that, although the centre of the defect band is almost the same, the width of the defect band is about five times smaller in the case of super-cells with 5×5 cylinders, due to the smaller overlap of the wave-fields located around neighbouring defects. Second, line defects were introduced using a super-cell of 5×5 cylinders. These defects can act as a straight acoustic wave-guide. Three defect modes were found in the projected band structure along the direction of the line defect for $r_{\rm d}/r = 0.8$. The mode which emerges from the lower edge of the band-gap with increasing wave vector k in the $\omega - k$ diagram, had a poor concentration of the acoustic field along the line defect at k = 0.5. In contrast, two modes which come from the upper edge of the band-gap with almost the same frequency for every k are more localized along the line defect. Reducing the radius of the cylinders in the line defect to $r_{\rm d}/r = 0.6$ and to $r_{\rm d}/r = 0.4$, the three modes are separated and changed their frequencies within the band-gap.

These theoretical results indicate various possibilities of sonic-crystal wave-guides that confine and guide acoustic waves well along the line defects, and also of sonic crystals that act as narrow tunable filters for acoustic waves in air, by changing the radius of the cylinder of the single defect.

5.2. Transient behaviour of sound waves in the wave-guides

Numerical investigations of sound waves in sonic crystals and especially in sonic-crystal wave-guides have been elaborately performed by using the FDTD method. The most preferable feature of the FDTD methods is that we can simulate exactly the temporal evolution of the wave propagation in an arbitrary inhomogeneous medium. So this section is devoted to reviewing the investigations of the transient behaviour of the wave propagation in sonic wave-guides.

In the application of sonic-crystal wave-guides to functional sonic circuits, the transient behaviour of the guided waves is to be investigated. Numerical inspections were reported by Miyashita [55, 56]⁵, being compared with a typical photonic-crystal wave-guide. Two sonic crystals and a photonic crystal were investigated. (1) 'Sonic crystal A'--acrylic resin cylinders in air, $r/a = 8/20, \overline{K} = 8.0, \rho =$ 2.3×10^{-7} , full band-gap: 0.47 < a/λ < 0.62. Here r denotes the radius of the scatterers, a the lattice constant, λ the wavelength in a uniform space of the host material and a/λ the normalized frequency. (2) 'Sonic crystal B'—air cylinders in agar gel, r/a = 6/20, $\overline{K} = 6.6 \times 10^{-7}$, $\rho = 8.0$. (3) 'Photonic crystal C'-GaAs dielectric cylinders in air, $r/a~=~6/20, \underline{\varepsilon}~=~0.865, \underline{\mu}~=~1.0,$ full band-gap: 0.38 ~< $a/\lambda < 0.475$. The initial buildup of the waves injected from the left inlet of the straight wave-guide was presented, as shown in figure 10, and studied comparing the three types of crystals. Conclusions were as follows: (1) the straight waveguide of sonic crystal A needs about five periods before the establishment of the steady state, (2) sonic crystal B has a very short rise time which is shorter than a period, (3) photonic

⁵ The CD attached to [55] includes movie files of the temporal evolution of the waves in the wave-guides.



Figure 11. Transient behaviour of the waves in the sonic and photonic 'bending wave-guides' [56].

crystal C needs about three periods. Next, the initial buildup of the waves injected from the left inlet into the two-times bending wave-guide was also investigated for the three types of crystals, as shown in figure 11, and concluded as follows: (1) In the bending wave-guide of sonic crystal A, the guided mode is observed to be reflected from the first bend, which occurred independent of the establishment of the steady state of the guided waves interfering with the waves reflected from the surrounding scatterers. Although the distribution of the guided waves is wide and complicated being accompanied by the evanescent fields, the waves are well guided along the bending wave-guides. (2) Sonic crystal B has a very short transient phenomenon in spite of relatively strong reflections from the first bend. A mode conversion is observed clearly at the first bend and also at the second. (3) Photonic crystal C shows also the establishment of the steady state of the guided waves independent of the reflections at the bends which excite standing waves along the straight parts of the wave-guide.

5.3. Experimental realization of 2D sonic-crystal wave-guides

It will not be as easy as in the photonic crystals for microwaves or light-waves to realize high-precision experiments on propagation of sound waves in sonic crystals, especially in the wave-guides. Only a few experimental investigations have been reported for sonic crystals so far, although many numerical investigations are reported.

5.3.1. Sonic-crystal wave-guide bulk. Miyashita and Inoue reported experiments on a straight sonic wave-guide as well as on a sonic wave-guide with a sharp bend [51] constructed in a sonic-crystal bulk, which is shown in figure 6, similar to photonic wave-guides with sharp bends reported by Mekis [48]. A quasi-plane wave was irradiated from a standard audio-speaker, and the sound waves guided along the sharp bend to the outlet were measured by a microphone and compared with the sound waves which propagated straight out of the crystal

as leakage sound-waves. The ratio of the guided waves to the leakage waves was about 30 dB in the lower half region of the full band-gap of the sonic-crystal bulk [51].

5.3.2. Sonic-crystal wave-guide slab. Mivashita et al introduced a sonic-crystal slab for a flexible platform for various shapes of acoustic wave-guides [57, 59, 60]. They have built a wave-guide with a sharp bend in the soniccrystal slab, whose inside view is shown in figure 12. For flexible constructions of various shapes and types of coupled wave-guides, there is always a small supporting rod of radius of 1.5 mm at each lattice point, and each scatterer may have a collar of outer radius of 3 mm to 6 mm around the supporting rod. In the figure, all collars have an outer radius of 5 mm. The inlet and outlet of the wave-guide are tapered in order to achieve better impedance matching. The sound waves were injected into the wave-guide by a audiotweeter (Fostex: FT7RP) having a rectangular aperture of 12 mm \times 80 mm and a frequency band of 3–45 kHz. The guided sound-waves were detected in front of the outlet by an omnidirectional microphone (Earthworks: QTC-1) having a small aperture of 10 mm diameter and a wide, flat frequency response from 10 Hz to 40 kHz. The leakage soundwaves, which propagated straight from the inlet without turning along the bend, were also detected outside the sonic crystal. The measured amplitudes of the guided and leakage waves as well as the guided-to-leakage ratio (GLR) are shown in figure 13 as a function of frequency. Please note the following points. Especially, (1) at the low end of the full acoustic band-gap (ABG) around 16.0 kHz, the leakage waves were weakest with a GLR of 30 dB over a relatively wide frequency range of 0.3 kHz. On the other hand, (2) at the centre of the ABG around 16.9 kHz, over a broad bandwidth of 0.5 kHz, the intensity of the guided wave was highest with a GLR of more than 20 dB. These bands will be useful for practical applications, although (3) at high end of the ABG around 17.9 kHz, a high GLR of 30 dB was also measured with a narrow bandwidth.



Figure 12. Inside view of an acoustic wave-guide with a sharp bend constructed in the sonic-crystal slab. The upper plate, which composes the slab structure together with the lower plate seen in the picture, is removed for the inside view [59, 60].



Figure 13. Measured guided waves and leakage waves for a wave-guide with a sharp bend built in a sonic-crystal slab [59, 60].

6. Sonic coupled wave-guides

The guided waves in the sonic-crystal wave-guides are accompanied by evanescent fields which extend to the periodic array of the scatterers surrounding the wave-guide, like electromagnetic waves in the dielectric wave-guides. These phenomena occur clearly in the case of the sonic crystal of acrylic resin cylinders in air, as shown in figures 10(a) and 11(a). Therefore, it is strongly expected that mode coupling arises easily between adjacent wave-guides in sonic crystals.

6.1. Experimental sonic coupled wave-guides

A preliminary successful experiment was recently reported by Miyashita *et al* [59, 60] on the mode coupling between two wave-guides constructed in a sonic-crystal slab of aluminium rods in air whose inner view is shown in figure 14. The acoustic wave input from the lower left inlet travels the lower wave-guide turning at the bend to the right outlet, generating coupled modes in the upper wave-guide which has no inlet but an outlet. The selected modes of the sound waves excited in the coupled wave-guide grow cumulatively travelling to the right outlet. The output of the coupled modes was measured and compared with the output from the lower wave-guide, as shown in figure 15. A remarkable result is that the ratio of the coupled wave to the input wave is -3 to -4 dB around the lower frequency part of 15.6 kHz to 16.8 kHz in the full band-gap of the sonic crystal.

The above experimental results on the mode coupling between the sonic-crystal wave-guides show that the soniccrystal slab is a promising platform for a new two-dimensional acoustic integrated circuit. The elements of acoustic circuits will be sonic-crystal slabs, straight and bending wave-guides, coupled wave-guides, ring resonators constructed by bending wave-guides, coupled filters formed by a combination of ring resonators and coupled wave-guides, wave-splitters, and so on [59, 60]. These sonic crystal devices are expected to be developed in the near future.

7. Summary

Following an introduction to the properties of artificial crystals in general, their physical backgrounds, theoretical methods for the analyses of sonic crystals and sonic wave-guides, the present status of theoretical and experimental sonic crystals, sonic-crystal wave-guides, and finally the most recent development of sonic crystals for coupled wave-guides were reviewed.

First, the fundamental equations used in the literature for the analyses of sonic crystals were reviewed, being compared with the electromagnetic equations for photonic crystals. Typical sonic crystals consisting of a periodic array of ridged cylinders in air were compared with typical photonic crystals on a common parameter space.

The theoretical methods presented so far to calculate the band-gap structures of sonic crystals were discussed comparatively. One is the classical and analytical method of plane-wave expansion (PWE), which is applicable to an infinite repetition of an inhomogeneous structure. The other is the finite difference time domain (FDTD) method, which



Figure 14. An inner view of a sonic coupled wave-guide constructed in the sonic-crystal slab [59, 60].



Figure 15. Measured input and coupled waves in the sonic coupled wave-guides constructed in a sonic-crystal slab [59, 60].

is also a powerful simulation method for wave propagation in an arbitrary structure of inhomogeneity. The FDTD method seems to be indispensable for the theoretical investigations of wave-guides. However, the method has some features which should be carefully noted in the applications as reviewed here.

Theoretical and numerical investigations of the band-gap structures of two-dimensional sonic crystals were reviewed. Although there are various reports which discuss sonic crystals theoretically, only two or three groups have presented experimental techniques and results. Two experimental techniques to determine the full band-gap were also reviewed. One combined the amplitude transmission spectrum and the phase dispersion. The other determined a full band-gap simply from the amplitude transmission spectra. The realizations of two-dimensional sonic crystals so far were classified into two types, i.e. bulk and slab structures. The latter was considered here to be promising for the future applicationoriented developments of sonic crystals.

Acoustic wave-guides have been investigated so far mainly theoretically for those constructed in elastic or

phononic crystals. A few but important reports of experiments on wave-guides in sonic crystals were reviewed. Transient behaviour of the guided waves in the sonic wave-guides was also reviewed, which will be an important nature of the guided modes when applied in high-speed signal processing devices.

One of the most important application-oriented devices made of sonic crystals will be sonic coupled wave-guides constructed in a sonic-crystal slab. With a promising experimental result on unidirectional mode coupling, this review paper was closed.

Acknowledgments

The author wishes to acknowledge many researchers in the world who are engaged in the related research fields and who contributed to fruitful discussions at international conferences. Especially he would like to acknowledge Professor Hauptmann at Otto-von-Guericke-University Magdeburg for his invitation to write a review paper about sonic crystals and sonic-crystal wave-guides. The author gratefully acknowledges Professor Saito at Ryukoku University for his introduction of this interesting phenomenon of sonic crystals, and Mr Inoue for his collaboration and his elaborate survey of the related papers in the fulfilment of his master course, and also all former students in the laboratory for their contributions. Finally, the author would like to thank Dr Wilkening at Ruhr-University Bochum for his help in correcting the English of the manuscript and for his many suggestions to refine this paper.

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