The photoelectric determination of *h/e*: A new approach to an old problem

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A method is described for the photoelectric determination of h/e, based on Fowler's classic theory of the frequency-dependent photoelectric yield. In this method, the geometrical arrangement of the electrodes in the h/e phototube is found to be of fundamental importance, and great care was taken to ensure that plane-parallel geometry was maintained. This method, which provides a value of h/e to within $\frac{2}{3}$% of the accepted value, is objective and accurate and is considered to be a very suitable experiment for an advanced undergraduate physics laboratory.

I. INTRODUCTION

Perhaps more so than in most experiments offered in undergraduate teaching laboratories, accurate results in the photoelectric "h/e experiment" [1] require both good experimental data and a rational means of interpreting those data. By "data" here is meant the curves of photoelectric current versus retarding voltage—curves which are familiar to all who have carried out this classic laboratory exercise.[2] The data themselves will likely be good if the following experimental points have received due attention. (1) Absence of excessive leakage currents between the two electrode terminals on the phototube. (2) Purity of the photoemissive surface and its surrounding vacuum environment. (3) Absence of excessive dark currents, observable when the phototube is not illuminated. (4) Absence of excessive reverse currents due to photoelectric emission from the collecting electrode of the phototube. (5) Absence of deleterious space-charge effects,[3] due to the presence of electrons in the space between the two electrodes of the phototube. (6) Spectral purity of the light used to illuminate the phototube. (7) Constancy of light intensity during any given current–voltage run. (8) Accuracy and sensitivity of the equipment used for measuring the photocurrent.

Even when all the foregoing requirements are met, the data obtained might still not be easily interpretable for the purpose of determining h/e. This will indeed be the case if one is looking for a sharp and definitely located point on the voltage axis of the current–voltage plot where the photocurrent goes to zero. Such a condition can only be realized if the photoemitter is at the absolute zero of temperature.

Rational interpretation of the photocurrent data of a phototube is possible only if due attention is paid to the question of the *geometry* of the electrode structure in the phototube. This is equivalent to recognizing the crucial role which the form and arrangement of electrodes in a phototube plays in determining the shape of any current–voltage curve obtained with that tube.[4] For obtaining rationally interpretable current–voltage curves even when the emitter temperature is above absolute zero there exist three choices of electrode geometry in phototubes. These are the geometries of (1) concentric spheres, (2) coaxial cylinders, (3) parallel planes. The remainder of this discussion will be limited to the case of parallel planes.

Since the theory of the current versus voltage curve for the parallel-plane type of phototube has been given,[5] it will not be repeated here. Experimental results for parallel-planes phototubes have been reported by Du Bridge and Hergenrother,[6] however, the objective of their work was not the determination of h/e. An important feature of current–voltage curves obtained with parallel-planes phototubes is the fact that such curves, when the plot is of the form log (current) versus linear retarding voltage, have a uniform and essentially fixed shape which is the same for any and all irradiation frequencies which are energetically capable of causing electron emission by the surface photoelectric effect. Moreover, this shape coincides with Fowler's well-known theoretical "universal curve," [7] whose original purpose was to describe the frequency-dependent photoelectric yield of a photoemissive substance.

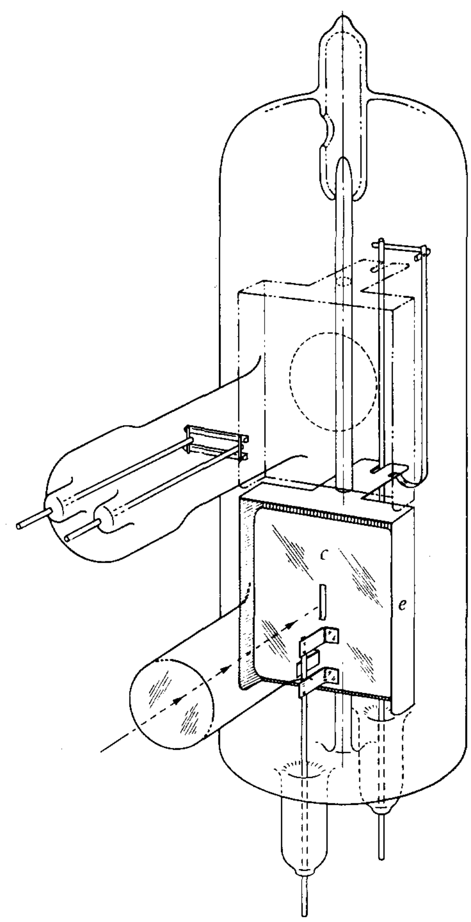

Fig. 1. The h/e phototube.

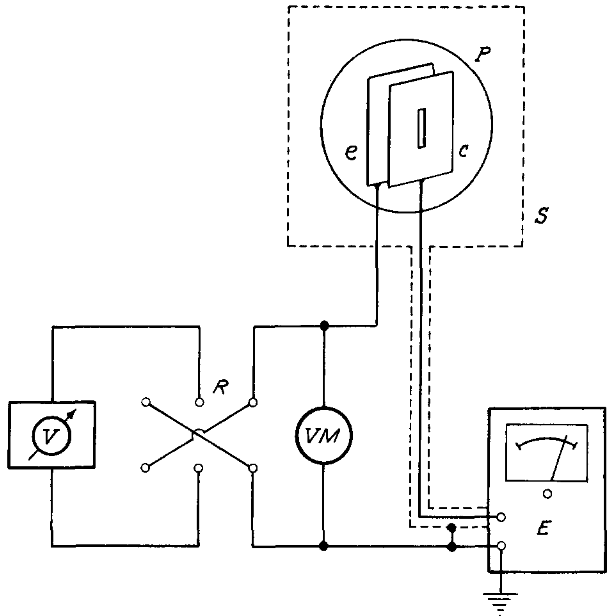

Fig. 2. Schematic diagram of the electric circuit. V is a variable source of dc retarding voltage; R is a reversing switch. VM is an autoranging digital voltmeter (Fluke Model 8800). The phototube is at P, with e the emitting electrode and c the collector. S is an electrostatic shield, and E is a Keithley Model 610C electronic electrometer for measuring the phototube collector current.

We have found that when Fowler's curve is matched to a set of experimental data points obtained with a parallel-planes phototube, the position of the curve along the axis of retardation voltages varies linearly with the frequency of the light irradiating the phototube. This linearity is a key factor in our method of photoelectrically determining h/e.

II. APPARATUS

Our parallel-planes type phototube, made specially for advanced-lab type h/e determinations, is shown in Fig. 1. The electrode material is sheet nickel, but the emitting electrode has a thin overlay of vacuum-evaporated barium (work function ~ 2.2 eV), which serves as the photoemitter. The collecting electrode has a small rectangular opening for admitting monochromatic irradiating light to the emitter. All is enclosed in a highly evacuated and hermetically sealed glass envelope.

Figure 2 shows the electric circuit, which is standard. Photocurrents were measured with a Keithley Model 610C electronic electrometer. Photocurrents ranged from a few femtoamperes to about 100 pA.

The light source was a GE H100 A4/T mercury-vapor lamp, operated on direct current at 0.5 A. Monochromatic light was obtained with a grating-type monochromator.

III. METHOD

The experimental procedure followed for the collection of current versus voltage data is straightforward and essentially standard. However, the procedure for arriving at a value of critical retardation voltage for each irradiation frequency is novel and will be described. Using published data[8] and convenient scale units, one makes a careful plot on a sheet of graph paper of Fowler's $\Phi(\delta)$ against δ,[9] as shown in Fig. 3. On a second sheet of graph paper one plots values of the logarithm of photocurrent (experimentally measured) against retardation voltmeter reading. The latter are interpreted as electron-volt energy values, and may

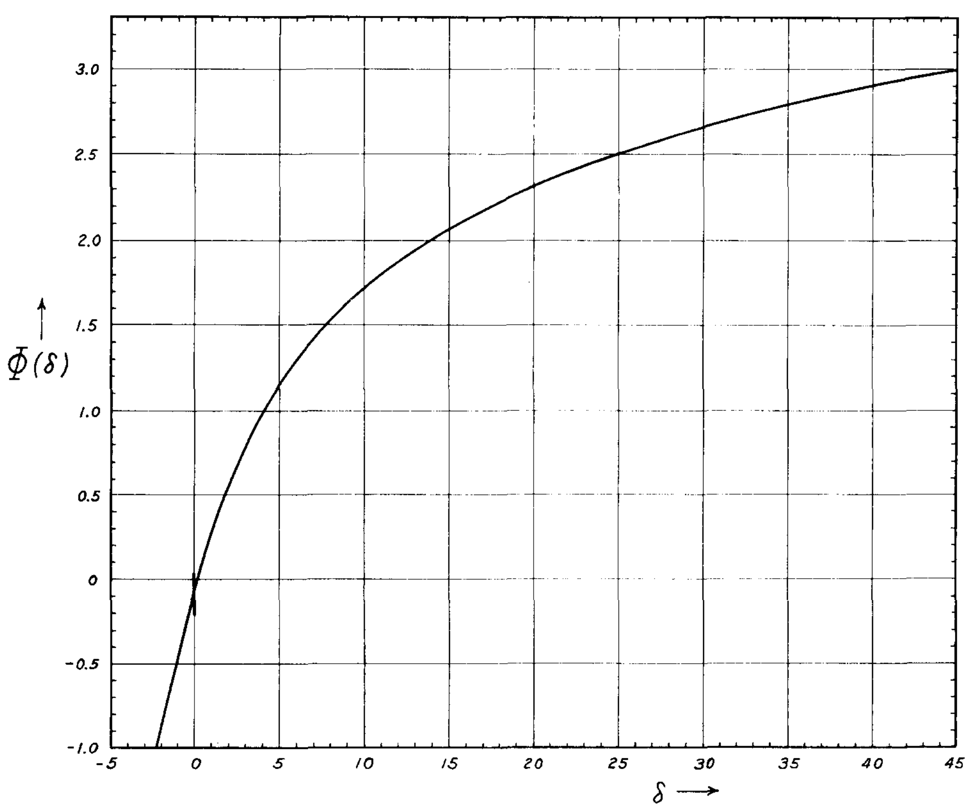

Fig. 3. Graph of Fowler's theoretical curve.

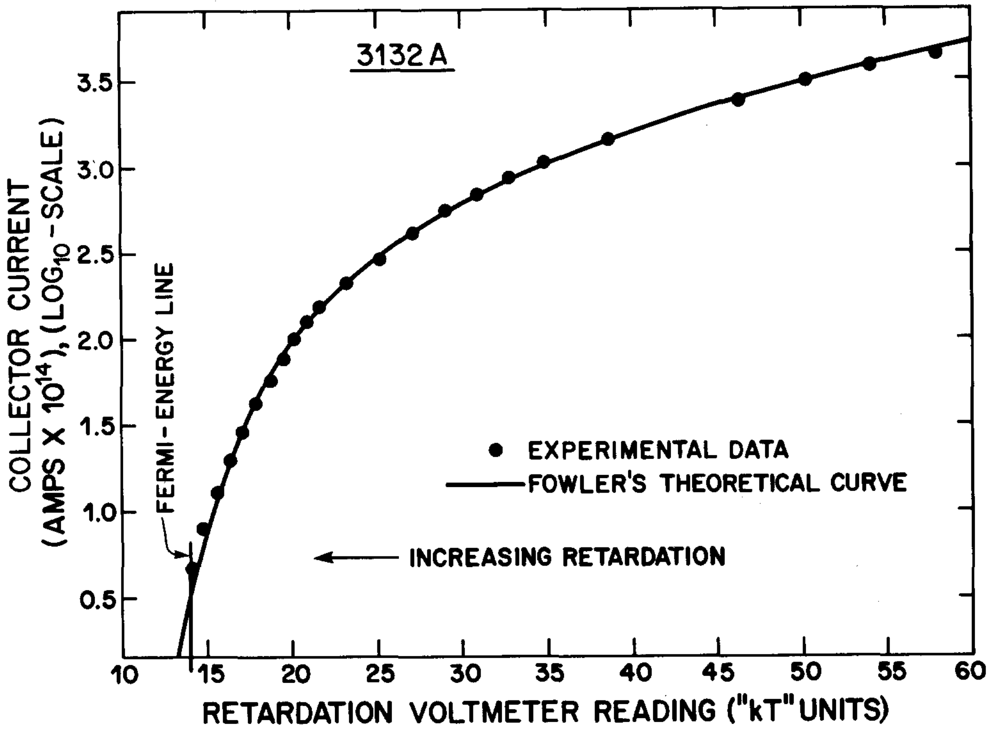

Fig. 4. Typical semilog plot of experimental collector-current data (points), to which Fowler's curve (solid line) has been fitted. Irradiation wavelength: 3132 Å.

be conveniently expressed in "kT" energy units. Here k is Boltzmann's constant and T is the absolute temperature of the photoemitter. The respective horizontal and vertical scales on the two graphs must be equivalent. Note that $\Phi(\delta)$ is usually $\log_{10}\varphi(\delta)$, so that in making the experimental plot, one should take $\log_{10}$ of the photocurrent, and not the natural logarithm.

The sheet containing the plotted experimental data points is now placed over the sheet having the Fowler curve. One then goes with the two sheets to a lighted window or light box and moves the upper sheet over the lower one in straight horizontal or vertical translations (no rotations!) until the best fit of the experimental points to the Fowler curve is obtained. (See, for example, Fig. 4, which represents results obtained at an irradiation wavelength of 3132 Å.) When the desired fit has been achieved, one marks carefully on the upper graph sheet the location of the $\delta = 0$ line (i.e., the "Fermi-energy" line) of the Fowler graph. The abscissa of the line marked on the upper graph sheet is the value of critical retardation energy which applies for the

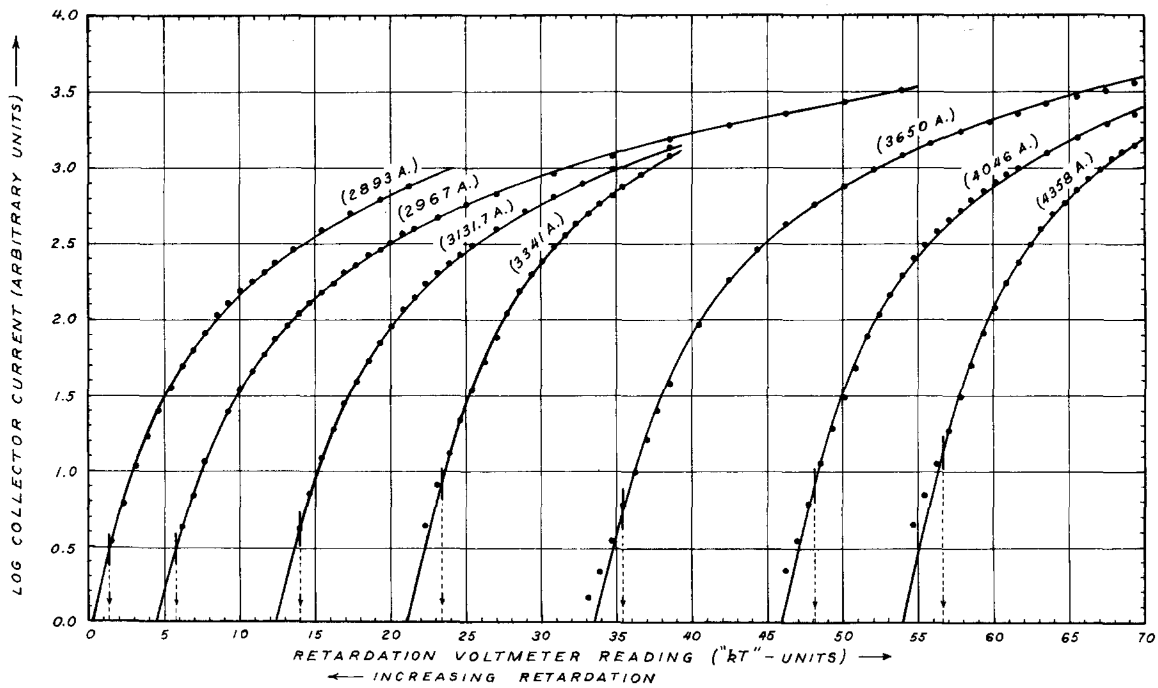

Fig. 5. Plots of log (collector current) versus retardation voltmeter reading (linear scale) for seven different irradiation wavelengths. Solid lines are Fowler's curve (fitted). Vertical arrows indicate critical retarding voltage for each irradiation wavelength.

irradiation frequency that was used in obtaining the plotted experimental data points.

The process of matching plots of experimental data to the Fowler curve is repeated for all other irradiation wavelengths used. Each such matching operation yields a pair of data values consisting of (a) the critical retardation voltmeter reading and (b) the irradiation frequency. These data are now plotted on an ordinary linear graph (see Fig. 6). The magnitude of the slope of the resulting straight line gives h/e.

IV. RESULTS

In principle, the method which has been described for determining h/e could be tested by employing only two different wavelengths of irradiating light. We have made a more thorough test by working with seven wavelengths, ranging from 2893 to 4358 Å. All are lines of the mercury spectrum. Plots of $\log_{10}$ (collector current) versus voltmeter reading of retardation voltage are given in Fig. 5. From one set of data to another the current units are variable, but within any given set the unit is fixed. The solid lines, which were drawn in after plotting the points, are Fowler's curve. Each has a short vertical line indicating $\delta = 0$. Critical retardation voltage is read off the scale of retardation voltmeter readings by extending each $\delta = 0$ line vertically downwards (dashed lines with arrows) to the horizontal scale.

Assembled in Table I are the data, obtained from Fig. 5, which are needed for making the straight-line plot of critical retardation voltage versus irradiation frequency that is shown in Fig. 6. The straight line shown in Fig. 6 could have been drawn by eye through the seven plotted points. It was felt however that a more objective approach would be to make a least-squares linear regression fit to the data. On the assumption that all the uncertainty of the measurements resides in the voltage values (4th column in Table I), one obtains by least-squares analysis the following expression for the best straight line:

$$V = -4.11 \times 10^{-15} f + 4.30 \text{ V},$$

where f is irradiation frequency in hertz. The quantity multiplying f is the slope of the line, and represents our value for h/e. To three significant figures the accepted value of h/e is 4.14×10^{-15} J s/C, so our value is seen to differ from the accepted value by about 3 parts in 414, or by about $\frac{2}{3}$ of 1%.

Table I. Data used for plotting the graph in Fig. 6. Second and fourth columns are data plotted.

Irradiation wavelength (Å)	Light frequency (Hz)	Scale location of Fermi energy (kT units)	(VM volts)
4358.34	6.879×10^{14}	56.7	1.473
4046.56	7.409×10^{14}	48.1	1.249
3650.15	8.213×10^{14}	35.4	0.919
3341.48	8.972×10^{14}	23.4	0.608
3131.70[a]	9.573×10^{14}	14.0	0.364
2967.28	10.103×10^{14}	5.8	0.151
2893.6	10.361×10^{14}	1.3	0.034

[a] The average of 3131.56 and 3131.84.

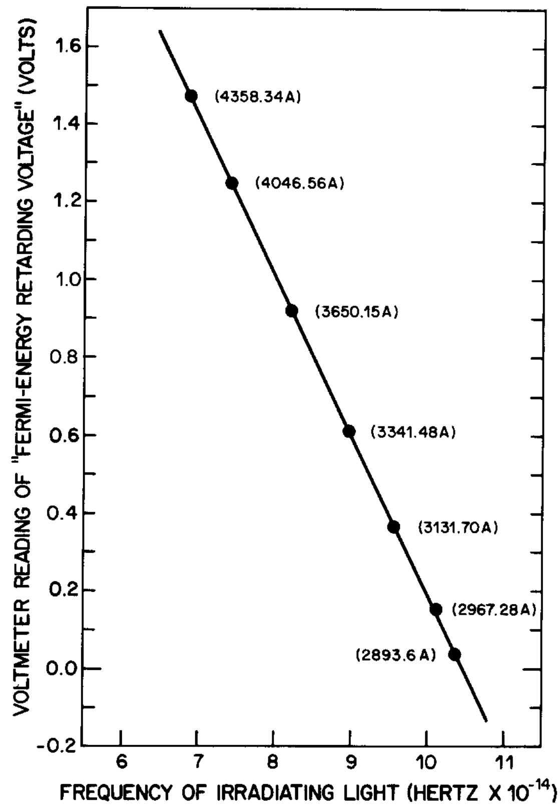

Fig. 6. Linear h/e plot. The location of the straight line was arrived at by least-squares calculation. Slope of line gives h/e.

The self-consistency of the data for determining h/e can be estimated by calculating the uncertainty in the slope.[10] This turns out to be 1.3×10^{-17} J s/C, which is 1.3 parts in 411, or about 1/3 of 1%. It seems reasonable to say that the precision of this method of determining h/e should be ample for the needs of today's advanced undergraduate teaching labs.

V. CONCLUSION

Although the determination of h/e by means of the photoelectric effect has for a long time ceased to occupy the interests of experimental specialists, teachers have steadfastly retained an interest in the h/e experiment, mainly because of its apparent simplicity and its fundamental position in the history and fact of quantum phenomena. That there have, however, been difficulties most instructors who have had direct contact with this experiment will attest. The present technique apparently removes most of the former difficulties, notably the one of accurately locating the elusive "zero-current" point on current-voltage curves. Lack of space in this presentation has resulted in "method" being emphasized over ""theory," but the pedagogic opportunities provided by the present technique should not be missed. The starting point remains the famous equation of Einstein, but this is in fact only a starting point as it does not provide a satisfactory, or at any rate complete, explanation of experimental observations. Historically, the full picture only began to emerge when the circumstances of the photoemissible electrons within a metal began to be better understood. Important papers in this development are those of Sommerfeld,[11] Nordheim,[12] and Fowler.[13] Fowler's paper marks the beginning of what might be called the modern era of photoelectricity. The publications which we have cited of L. A. Du Bridge are virtually indispensible to a well-rounded understanding, both theoretical and experimental, of this subject. Obtaining a plot like Fig. 4 is most gratifying, for the close agreement of experimental results with theory confirms the essential correctness of such ideas as the quantum-mechanical density of states, Fermi–Dirac statistics, the free-electron theory of metals, etc. All these ideas are accessible to upper-level undergraduates, while the experiment provides the confirmation.

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[1] K. T. Compton, Philos. Mag. **23**, 579 (1912); O. W. Richardson and K. T. Compton, Philos. Mag. **24**, 575 (1912); R. A. Millikan, Phys. Rev. **7**, 355 (1916); P. Lukirsky and S. Prilezaev, Z. Phys. **49**, 236 (1928).

[2] Reference 1 regards early specialist work in photoelectric h/e determinations. For discussions of h/e determinations in undergraduate labs, the interested reader is referred to Refs. 1–13 in the article of R. A. Powell, Am. J. Phys. **46**, 1046 (1978), and also to the article itself, which reflects a modern point of view of photoelectricity.

[3] Robt. N. Varney, Am. J. Phys. **49**, 425 (1981).

[4] In most phototubes designed for commercial applications, the concern is for efficient light collection rather than for the analysis of the kinetic energy of the photoelectrically emitted electrons. Unfortunately, no commercially available phototube seems to have an electrode structure which lends itself to the kind of straightforward analysis of electron kinetic energy that is needed for h/e determinations.

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[6] L. A. Du Bridge and R. C. Hergenrother, Phys. Rev. **44**, 861 (1933).

[7] L. A. Du Bridge, Phys. Rev. **39**, 108 (1932); L. A. Du Bridge, *New Theories of the Photoelectric Effect* (Hermann & Cie, Paris, 1933).

[8] L. A. Du Bridge, Phys. Rev. **39**, 108 (1932); A. L. Hughes and L. A. Du Bridge, *Photoelectric Phenomena* (McGraw-Hill, New York, 1932), p. 248; H. Simon and R. Suhrman, *Der Lichtelektrische Effekt* (Springer, Berlin, 1958), pp. 30–32. If desired, one can calculate one's own values of $\phi(\delta)$. One uses the following formulas (Ref. 9):

$$\phi(\delta) = \frac{4\pi m k^2 T^2}{h^2}\left(e^{\delta} - \frac{e^{2\delta}}{2^2} + \frac{e^{3\delta}}{3^2} - \cdots\right) \quad (\delta \leqslant 0),$$

$$\phi(\delta) = \frac{4\pi m k^2 T^2}{h^2}\left[\frac{\pi^2}{6} + \frac{1}{2}\delta^2 \times\left(e^{-\delta} - \frac{e^{-2\delta}}{2^2} + \frac{e^{-3\delta}}{3^2} - \cdots\right)\right] \quad (\delta \geqslant 0).$$

For most purposes δ values ranging from about -3 to $+45$ will be found to be adequate. Either of the series indicated above can easily be evaluated through the first four terms by means of a pocket calculator. In the expressions above, m is the electron mass, k is Boltzmann's constant, h is Planck's constant, and T is the absolute temperature of the photoemitter. In Fowler's theory, $\delta = (h\nu - \chi)/kT$, where ν is photon frequency and χ is the work function of the emitter. The curve shown in Fig. 3 is a plot of $\Phi(\delta)$ against δ, where $\Phi(\delta)$ is simply the logarithm of $\phi(\delta)$.

[9] R. H. Fowler, *Statistical Mechanics*, 2nd ed. (Cambridge University, Cambridge, 1936), p. 360, but see also pp. 341–342.

[10] J. Higbie, Am. J. Phys. **46**, 945 (1978); N. C. Barford, *Experimental Measurements: Precision, Error, and Truth* (Addison-Wesley, Reading, MA, 1967), p. 62.

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Quantum theory of an electron in external fields using unitary transformations

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The technique of using unitary transformations to solve quantum-mechanical problems involving nontrivial time-dependent Hamiltonians is discussed. After introducing the basic ideas in simple examples, the technique is used to solve the problem of an electron in crossed magnetic and electric static fields plus laser (plane wave) field.

I. INTRODUCTION

The theory of unitary transformations is explained in most of the textbooks in quantum mechanics.[1] In particular, the translation operators $\exp(i\mathbf{a}\cdot\hat{p}/\hbar)$ and $\exp(i\mathbf{b}\cdot\hat{x}/\hbar)$, where $\mathbf{a}(t)$ and $\mathbf{b}(t)$ are vectors, are discussed even in introductory books.[2] However, there are few examples where unitary transformations are actually exploited to solve relevant problems. In this paper we show that unitary transformations are quite useful to solve Schrödinger's equation with time-dependent Hamiltonians and nonuniform potentials. The technique is then applied to solve the quantum-mechanical problem of an electron in crossed electrostatic and magnetostatic fields under the influence of a laser (plane wave) field. This extends the previous work of Seely[3] and Miranda.[4]

We initially recall the translation of the position $\hat{x}$ and of momentum $\hat{p}$ operators under unitary transformations. Let

